Long-range correlations in Dirac liquids

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UNC-Chapel Hill

Trento, 13/04/10

Motivation

•What are the effects of long-range (unscreened) Coulomb interactions on Dirac (semi)metals?

•What are the effects of magnetic field in the presence of Coulomb interactions?

•What are the effects of long-range-correlated disorder?

Outline

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- Effects of Coulomb and e-ph interactions on Dirac fermions

PRL 87, 246802 (2001);
NPB 642, 515 (2004);
PRB 73, 115104 (2006) ;
PRB 74, 161402(R) (2006);
J. Phys.: Condens. Matter, 21, 075303 (2009).

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•Effects of magnetic fields on (interacting) Dirac fermions

PRL 87, 206401 (2001);
PRB 75, 153405 (2007)

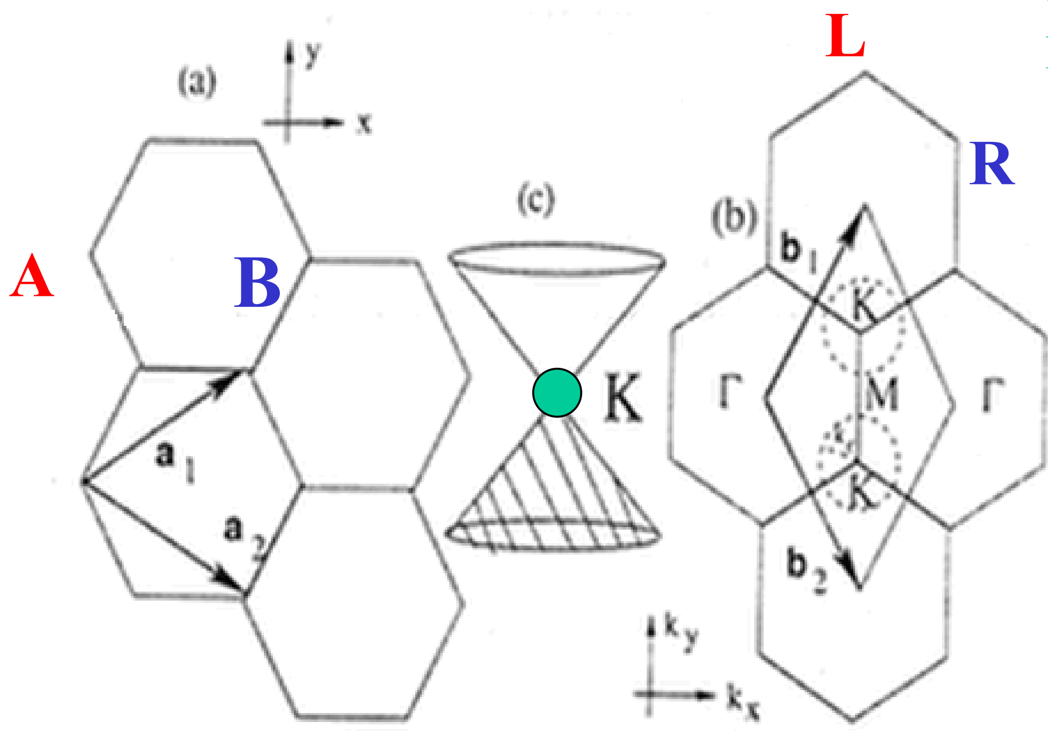
•Effects of (long-range-correlated) disorder

PRL 96, 027004 (2006);
PRB 75, 241406 (R) (2007);
EuroPhys.Lett. 82, 57008 (2008)

Massless Dirac fermions in graphene

- Nodal quasiparticle excitations

P.R. Wallace, '47
G. W. Semenoff, '84;
E. Fradkin, '86



- Dirac (bi-) spinors: $\Psi = (\psi^{\sigma}{}_{LA}, \psi^{\sigma}{}_{LB}, \psi^{\sigma}{}_{RA}, \psi^{\sigma}{}_{RB})$

- Massless Dirac Hamiltonian: $H = \psi^{+} i\, v_F \gamma \partial\, \psi$

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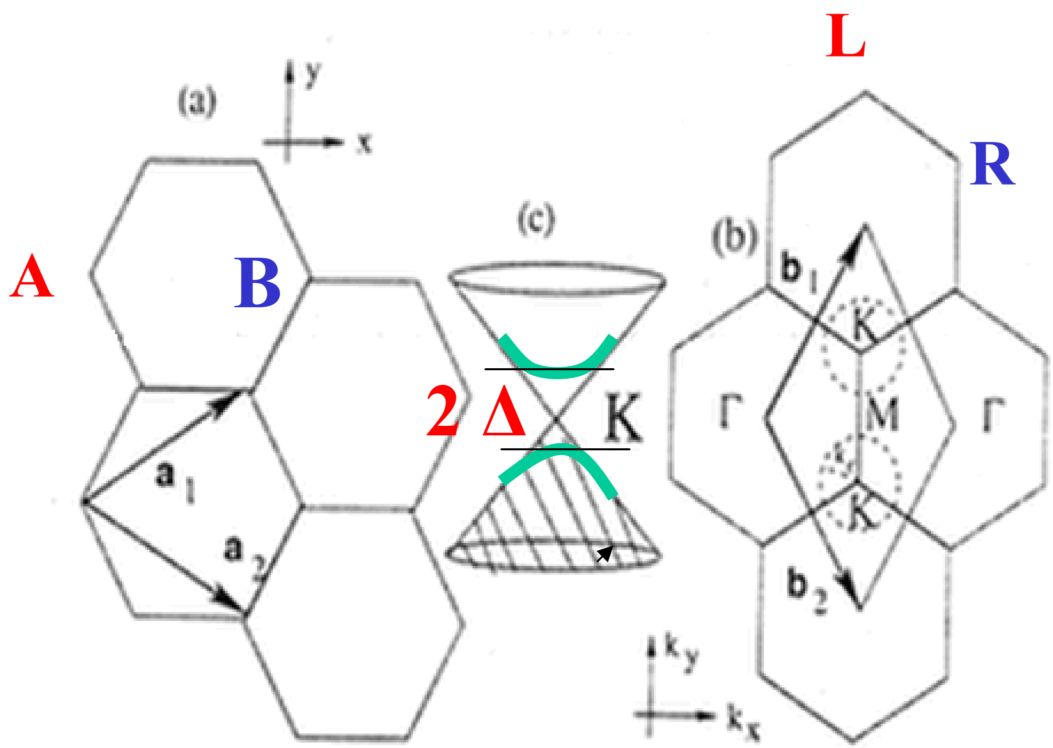

•Dirac (bi-) spinors: $\Psi = (\psi^{\sigma}_{LA}, \psi^{\sigma}_{LB}, \psi^{\sigma}_{RA}, \psi^{\sigma}_{RB})$

• Massive Dirac Hamiltonian: $H = \psi^{+}(i\, v_F \gamma \partial + \Delta)\, \psi$

Different Dirac fermion masses

Different Dirac fermion masses

- 4-spinor wave functions:

$$\Psi(p) = (\psi_{C,n,\alpha}(p), \tau_2^{nm} s_2^{\alpha\beta} \psi^\dagger_{C,m,\beta}(-p))$$

$$\psi = (\frac{1+\tau_3}{2} + i\frac{1-\tau_3}{2} \otimes \sigma_2)(A_1, B_1, A_2, B_2)^T$$

$$\rho_n \otimes \sigma_a \otimes \tau_i \otimes s_\alpha$$

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- Dirac mass terms (p-h):

$$\psi^\dagger \sigma_3 \otimes \tau_1 \otimes s_0 \psi = A^\dagger_{L\alpha} B_{R\alpha} + B^\dagger_{L\alpha} A_{R\alpha} + h.c.$$

$$\psi^\dagger \sigma_3 \otimes \tau_3 \otimes s_0 \psi = \sum_{i=L,R} (A^\dagger_{i\alpha} A_{i\alpha} - B^\dagger_{i\alpha} B_{i\alpha})$$

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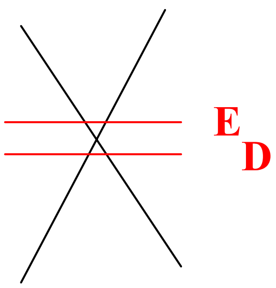

P & T

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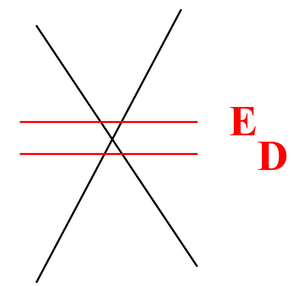

P & T

•Majorana mass terms (p-p/h-h):

$$\psi \sigma_0 \otimes \tau_1 \otimes s_0 \psi = \sum_{i=L,R} (A_{i\alpha} A_{i\beta} + B_{i\alpha} B_{i\beta}) s_2^{\alpha\beta}$$

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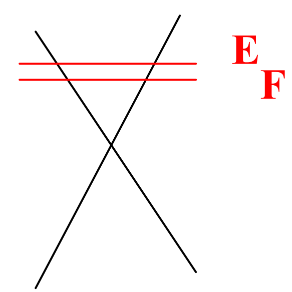

Massive Dirac fermions: Higgs-Yukawa model

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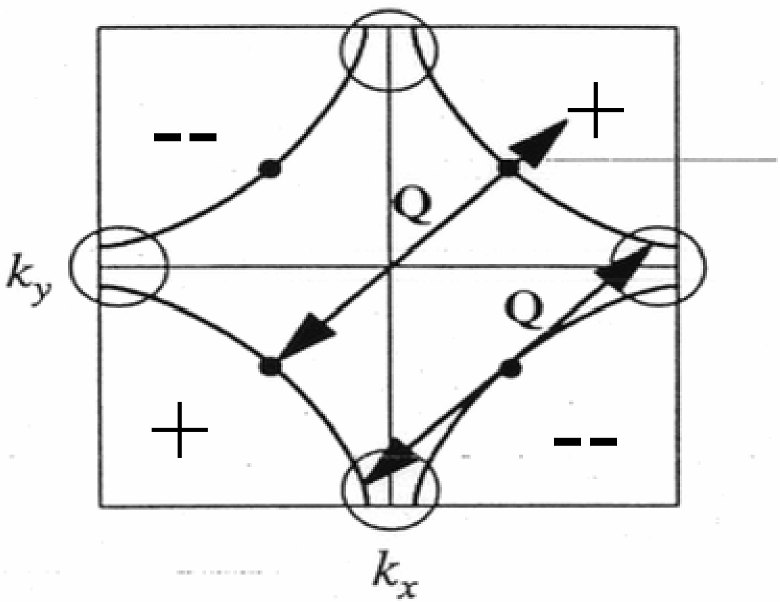
- **d-wave** superconductors (μ=0)

Emergent fermion mass = second superconducting pairing:

d → d +is (id) S. Sachdev et al '99; DVK and J. Paaske, '00



$\Delta_d \sim (\cos k_x - \cos k_y)$

$\varepsilon \sim (\cos k_x + \cos k_y)$

$E(k) = (\varepsilon^2 + \Delta^2_d + \Delta^2_{is/id})^{1/2} \sim$

$\sim (v_x^2 k_x^2 + v_y^2 k_y^2 + \Delta^2_{is/id})^{1/2}$

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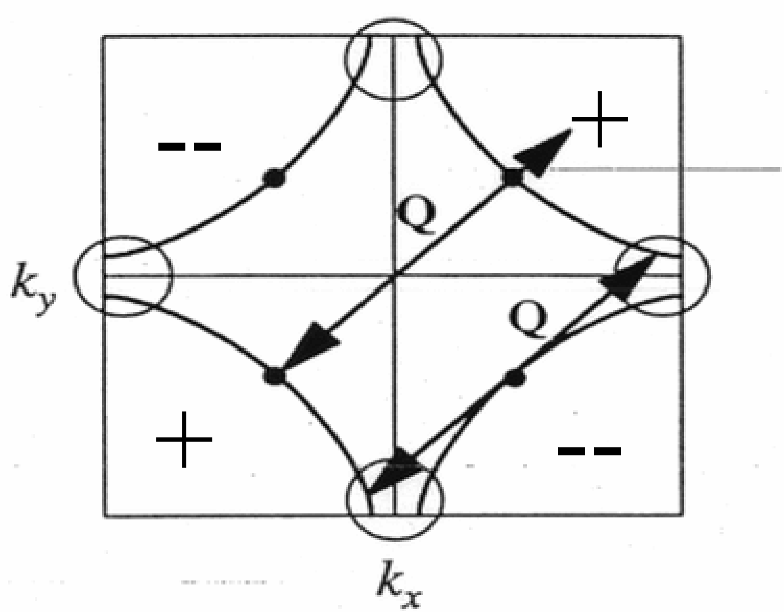

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•Dichalcogenides (2D **f-CDW ?** **A.Castro-Neto'02**);
•He3-A (3D), topological insulators…

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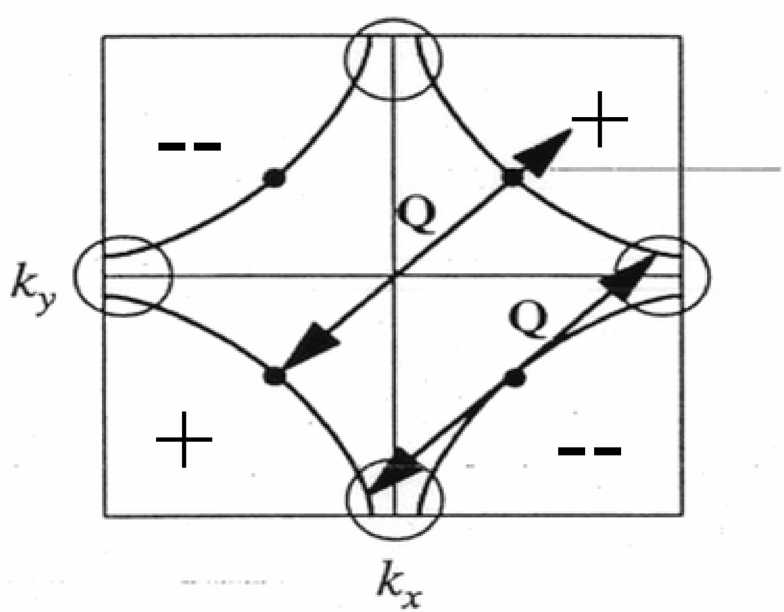

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Critical coupling: $g > g_c$

Massive Dirac fermions: Lorentz-invariant QED_3

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- CSB in QED_3 T.Appelquist et al, '88

$$L = i \sum^{N}_{f=1} \bar{\psi}_f \gamma(\partial + A)\psi_f + F^2/2g \qquad F = \partial \times A$$

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- Chiral rotation symmetry for **massless** fermions

$$\psi_f^{L,R} = (1 \pm \gamma_5)/2\, \psi_f^{L,R} \rightarrow \exp(i\gamma_5\varphi)\, \psi_f^{L,R}$$

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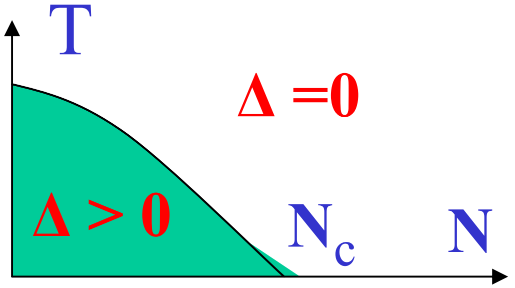
- CSB order parameter, U(2N)→U(N)×U(N)

$$\Delta \sim \sum_{f=1}^{N} \langle \bar{\psi}_f \psi_f \rangle$$

CSB phase transition

$$\Delta \neq 0, \quad N < N_c$$

$$\Delta = 0, \quad N > N_c \quad \text{(for arbitrary } g\text{)}$$



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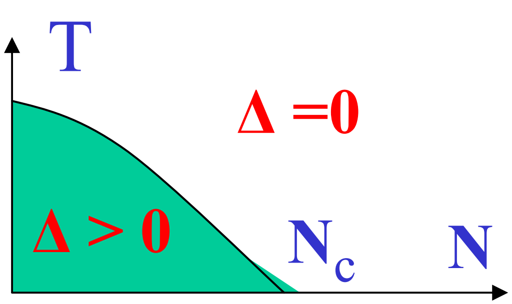

Critical number of species: N<Nc

Coulomb interacting Dirac fermions

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•Non-Lorentz-invariant Hamiltonian of graphene (no disorder):

$$H = iv_F \sum_{\alpha=1,2} \int_{\mathbf{r}} \Psi_\alpha^\dagger [\hat{\sigma}_x \nabla_x + (-1)^\alpha \hat{\sigma}_y \nabla_y] \Psi_\alpha$$

$$+ \frac{v_F}{4\pi} \sum_{\alpha,\beta=1,2} \int_{\mathbf{r}} \int_{\mathbf{r}'} \Psi^\dagger{}_\alpha(\mathbf{r}')\Psi_\alpha(\mathbf{r}') \frac{g}{|\mathbf{r}-\mathbf{r}'|} \Psi^\dagger{}_\beta(\mathbf{r})\Psi_\beta(\mathbf{r})$$

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•Dirac fermion propagator:

$$\hat{G}(\epsilon, \mathbf{p}) = [\epsilon - \hat{\rho}_3 \otimes (v\vec{\sigma}_{\|}\vec{p} - \mu + \vec{s}\vec{B}) + \hat{\Sigma}(p)]^{-1}$$

$$\omega = \mu + \sigma B \pm E(p)$$

$$E(p) = \sqrt{v^2(p)p^2 + \Delta^2(p)}$$

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- Effective interaction:

$$V_C(\omega, q) = \frac{2\pi g v}{q + 2\pi g v \Pi(\omega, q)} \qquad g = \frac{e^2}{\epsilon v} \approx \frac{2.16}{\epsilon}$$

$$\Pi(\omega, q) = \frac{Nq^2}{16\sqrt{v^2(q)q^2 - \omega^2}}$$

Coulomb interacting Dirac fermions

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NOT V(q)=const

NOT weak

Excitonic pairing between Dirac fermions: gap equation

- Dirac fermion self-energy: $$\hat{\Sigma}(p) = \sum_{\mathbf{q}} \int \frac{d\omega}{2\pi} V(\mathbf{p}-\mathbf{q}, \epsilon-\omega) \frac{\omega + v\vec{\sigma}_{\parallel}\vec{q} + \hat{\Sigma}(p)}{\omega^2 - E^2(p) + i0}$$

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- Velocity renormalization (diagonal term):

I.L.Aleiner et al, '07

D.T.Son, '07

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•Differential form: $\frac{d^2\Delta(p)}{dp^2} + \frac{2+\eta_N}{p}\frac{d\Delta(p)}{dp} + \frac{g_N(1+\eta_N)}{2p^{2-\delta\eta}}\frac{\Delta(p)}{\Lambda^{\delta\eta}} = 0$

•Boundary conditions: $\frac{d\Delta(p)}{dp}|_{p=\Delta/v} = 0 \qquad [(1+\eta_N)\Delta(p) + p\frac{d\Delta(p)}{dp}]|_{p=\Lambda/v} = 0$

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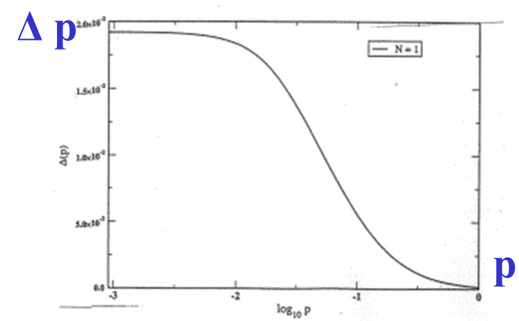
•WKB solution:

$$\Delta^{\pm}(p) = \frac{C_{\pm}}{p^{1-\delta\eta/2}P(p)^{1/2}} \exp(\pm i \int_{\kappa}^{p} P(p')dp')$$

$$P^2(p) = \frac{1}{p^2}[g\frac{1+\eta_N}{2}(\frac{p}{\Lambda})^{\delta\eta} - \frac{(1+\eta_N)^2}{4}]$$



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- Differential form: $$\frac{d^2\Delta(p)}{dp^2} + \frac{2+\eta_N}{p}\frac{d\Delta(p)}{dp} + \frac{g_N(1+\eta_N)}{2p^{2-\delta\eta}}\frac{\Delta(p)}{\Lambda^{\delta\eta}} = 0$$

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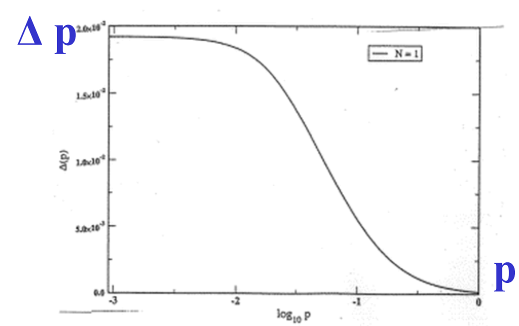
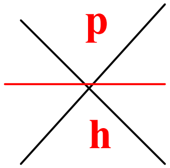
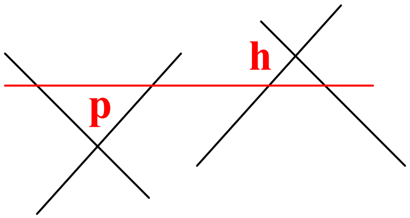
Strongly non-BCS



Excitonic pairing: undoped case

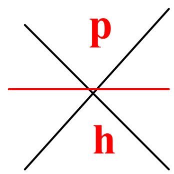


NOT

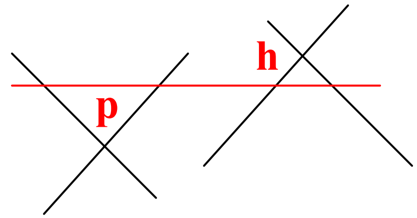


-finite in-plane field
-biased bi-layer

Excitonic pairing: undoped case



NOT



-finite in-plane field
-biased bi-layer

•Critical coupling: $\tilde{g} = \dfrac{g}{1 + \pi N g / 8\sqrt{2}}$ = 1/2, η=0

DVK,'01

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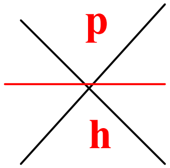

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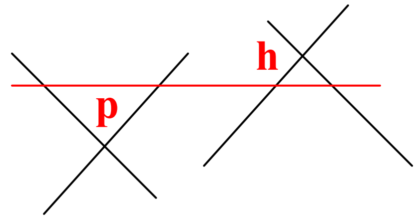

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Cf. Atomic collapse in the single-particle problem of a charged impurity in graphene

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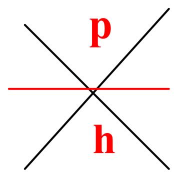

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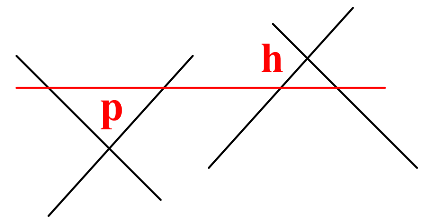

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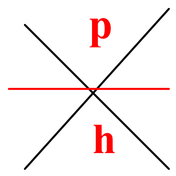

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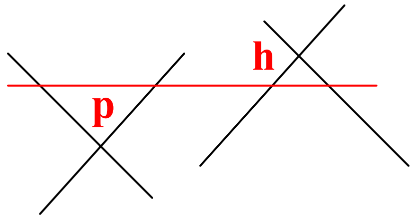

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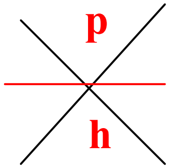

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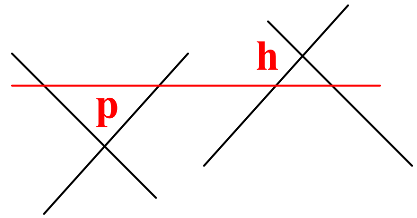

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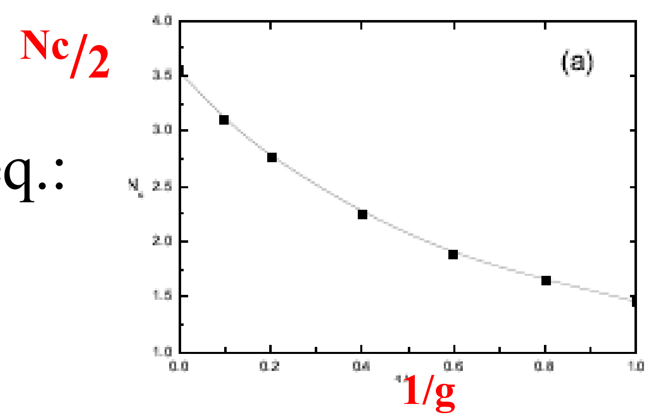
•Numerical solution of the gap eq.:

G.Liu et al, Phys. Rev. B 79, 205429 (2009)



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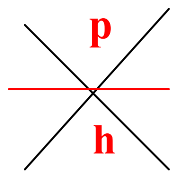

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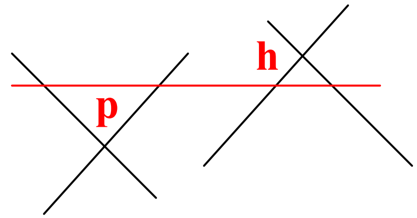

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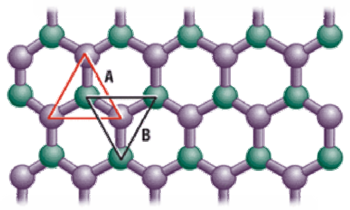
•Lifting of the sublattice (A/B) degeneracy:

CDW $\Delta \sim \rho_A - \rho_B$ $Q=(\sqrt{3}/2, 1/2, (1))\pi$



Excitonic insulator transition in undoped graphene

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- QED_3 : Gap equation: $N_c = 3.2$
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J.Kogut et al, 0808.2720

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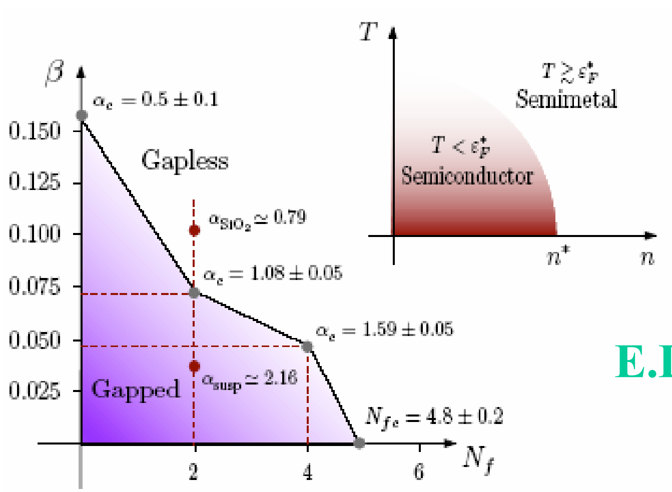

E.Drut and T.Lahde, 0905.1320

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- Free-standing graphene: $\Delta \sim$ 5-10 meV

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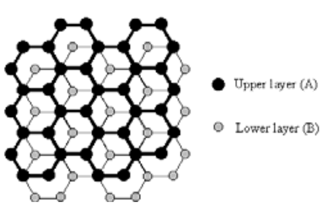
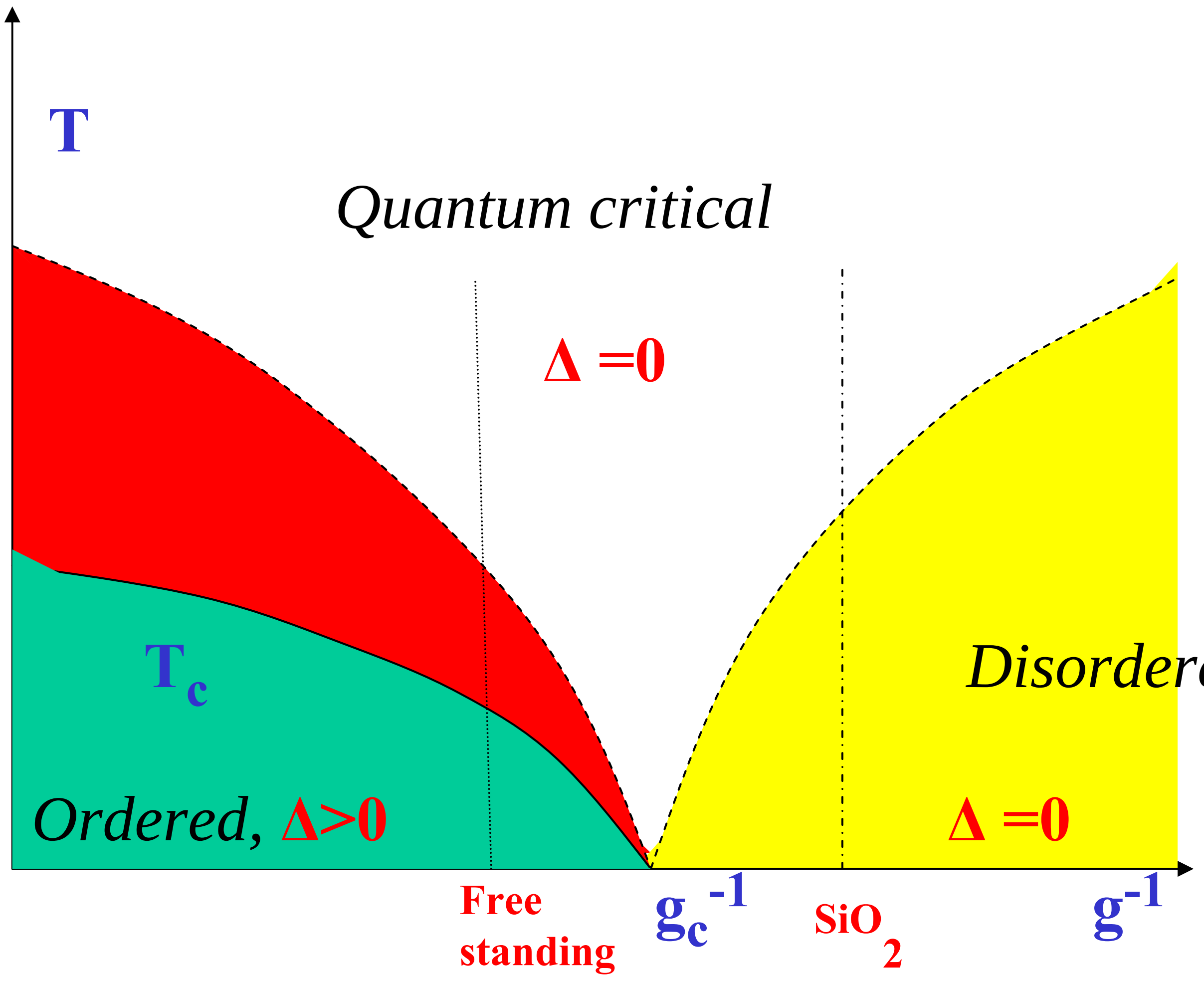
T.Hands et al, 0808.2714

•HOPG: EI is further stabilized by inter-layer Coulomb repulsion



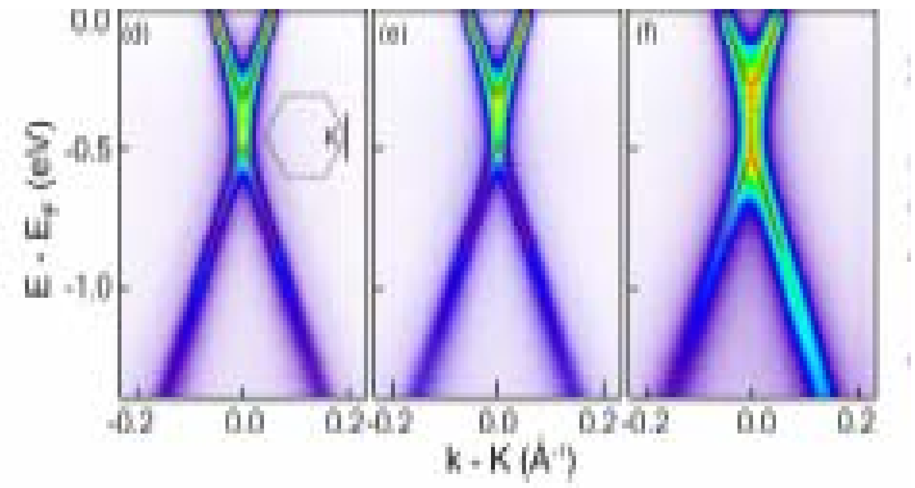
Quantum-critical behavior in undoped graphene?

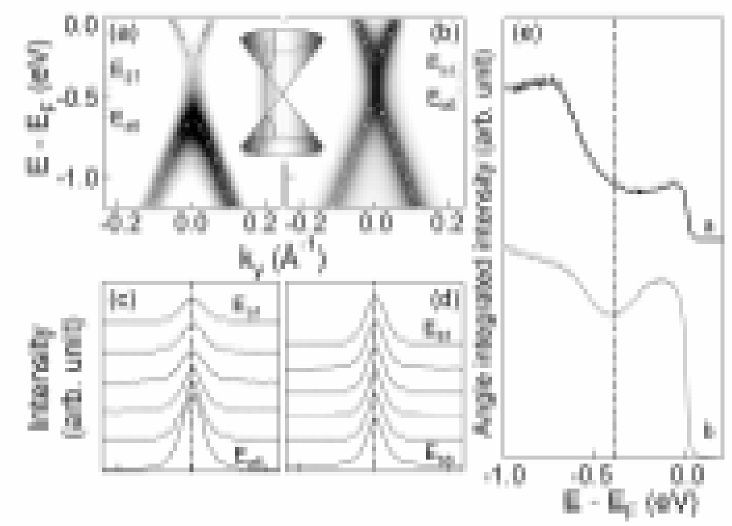
Coulomb coupling:
$g = e^2/\varepsilon v_F$



T=0 quantum critical point at $g=g_c$

Dirac fermion mass in epitaxial graphene? (ARPES)



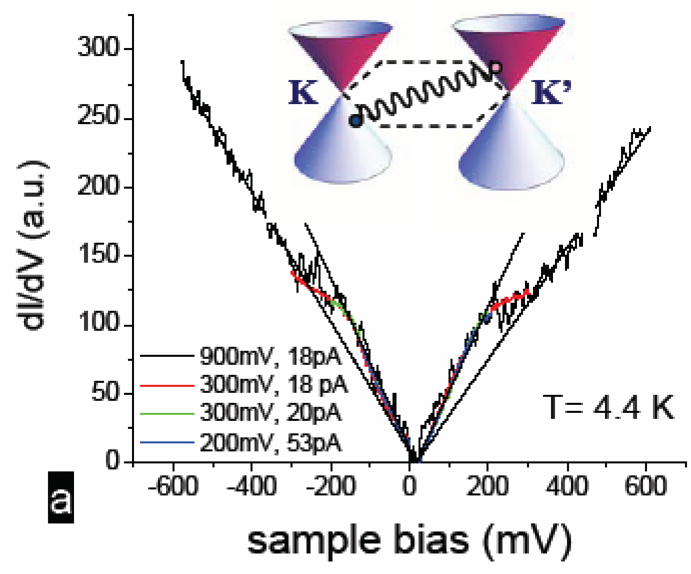


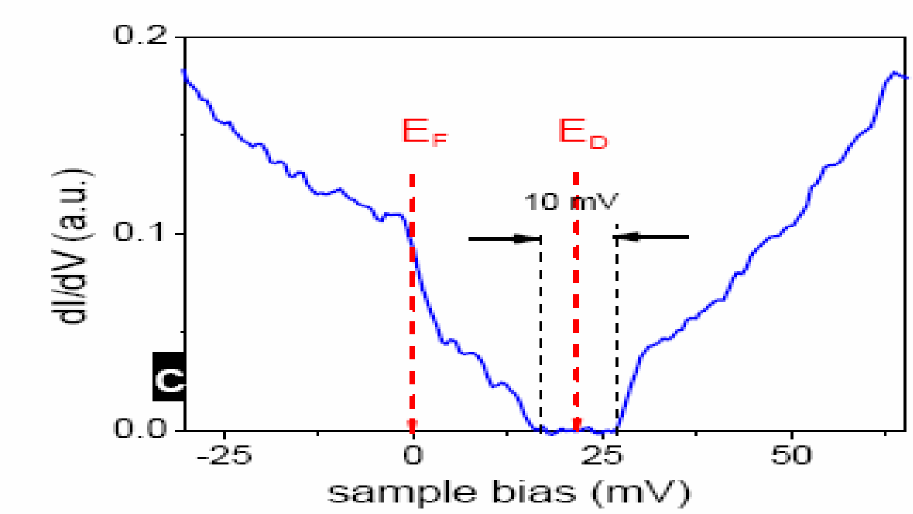
A. Lanzara et al '07

Strong substrate-related effects:

Large gap/mass ~130meV

Dirac fermion mass in suspended graphene? (STM)





E. Andrei et al '08

No substrate:

Small gap/mass~10meV

Moderately strong Coulomb interactions: photoemission

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- Electron spectral function:

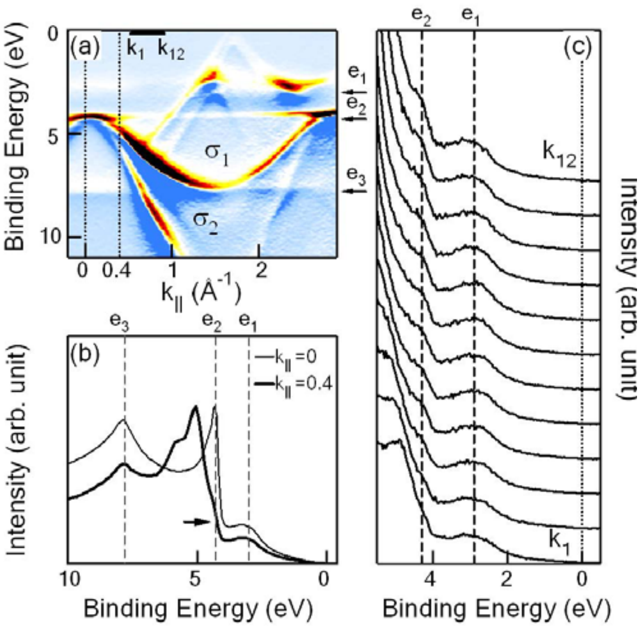
undoped, quantum-critical regime, **g<gc**

$$\Gamma(\epsilon,\mathbf{p}) \propto \theta(p_\mu^2)\frac{p_\mu^2}{max[\epsilon, v_F p]}\ln g, \quad max[\epsilon, v_F p] > T$$

$$\propto \theta(p_\mu^2)\left(\frac{p_\mu^2 T}{max[\epsilon, v_F p]}\right)^{1/2}, \quad max[\epsilon, v_F p] < T$$

$p^2_{\ \mu} = E^2 - v^2 \mathbf{p}^2$

- **NOT** just "~ E"



A. Lanzara et al, '05

Moderately strong Coulomb interactions: photoemission

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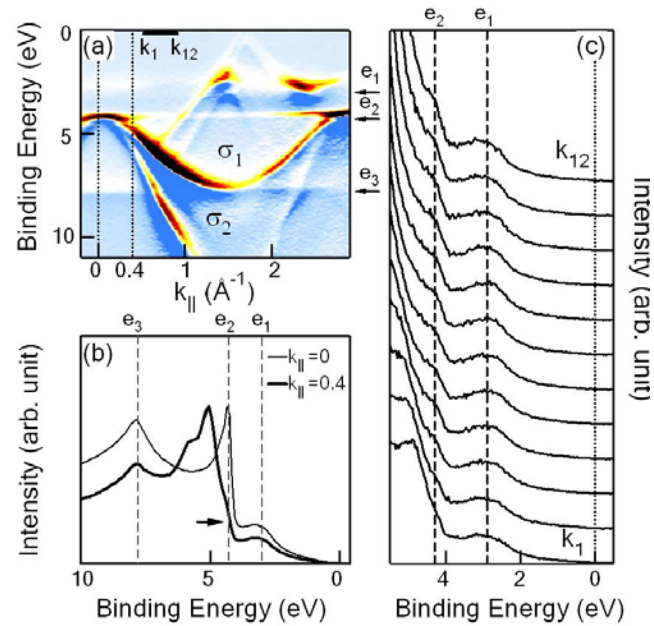
•Formally related problem:
normal quasiparticles in d-wave cuprates



A. Lanzara et al, '05

J. Paaske and DVK, '00;
A.Chubukov and A.Tsvelik, '05

Moderately strong Coulomb interactions: tunneling

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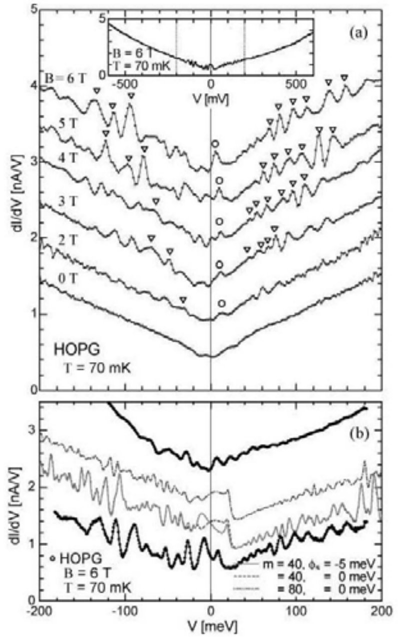
- Tunneling DOS:

$$\nu(\epsilon) \approx -\frac{1}{\pi} ImTr \int_{-\infty}^{\infty} \hat{G}_0^R(0,t) e^{-S(t)+i\epsilon t} dt$$

$$S(t) = \int_0^{\Lambda} \frac{d\omega}{4\pi} \sum_{\mathbf{q}} ImU(\omega, \mathbf{q}) \coth\frac{\omega}{2T} \int_0^t dt_1 \int_0^t dt_2 e^{-i\omega(t_1 - t_2)} \left\langle e^{i\mathbf{q}(\mathbf{r}(t_1)-\mathbf{r}(t_2))}\right\rangle$$



T.Matsui et al, '05

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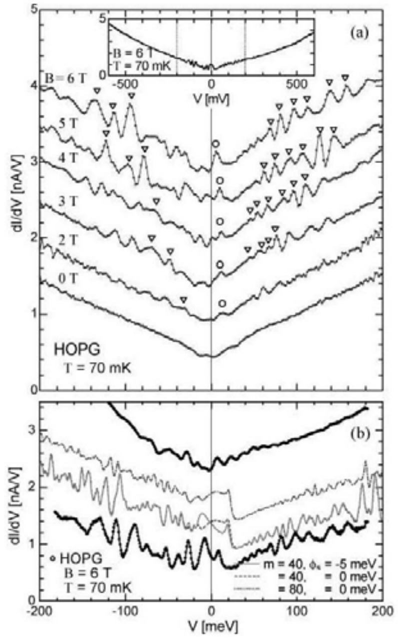
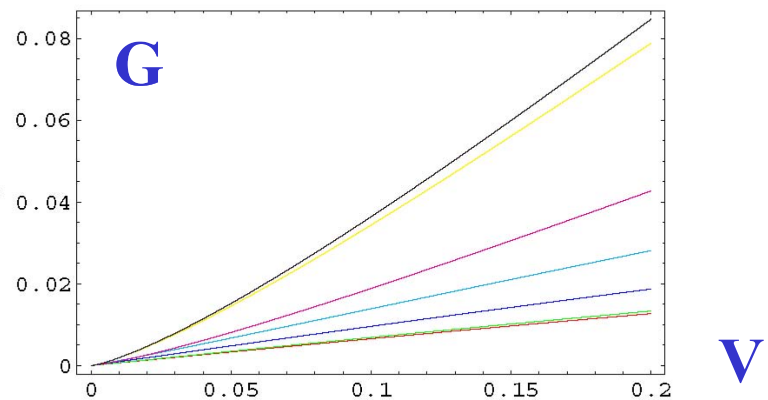


T.Matsui et al, '05

- Tunneling conductance:

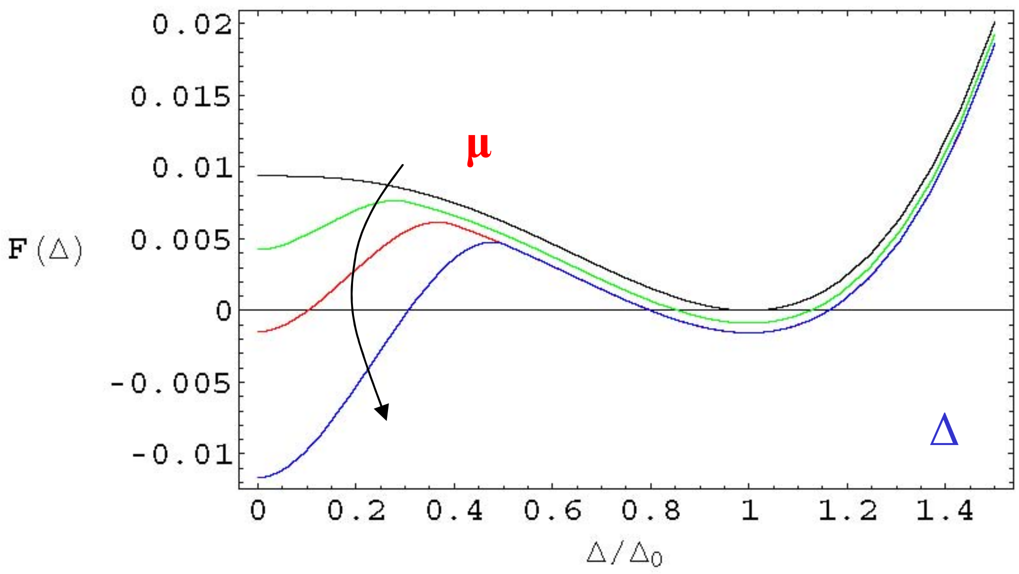
$$G(V) \propto \frac{d}{dV} \int_0^{\infty} \mathcal{G}^R(\mathbf{0},t)\, \mathcal{G}_0^R(\mathbf{0},t)\, e^{iVt} dt$$

$G(V,T) \sim \max[V, T]^{1+\eta(g,V)}$

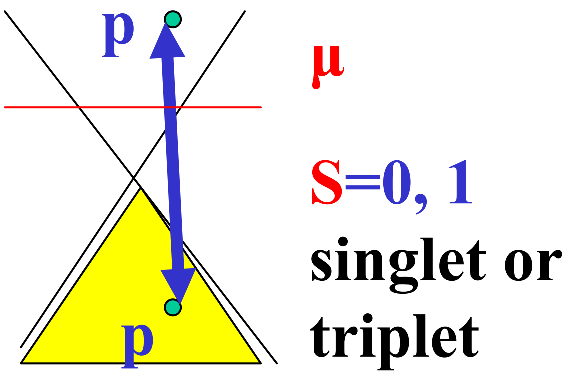


Excitonic pairing: finite doping

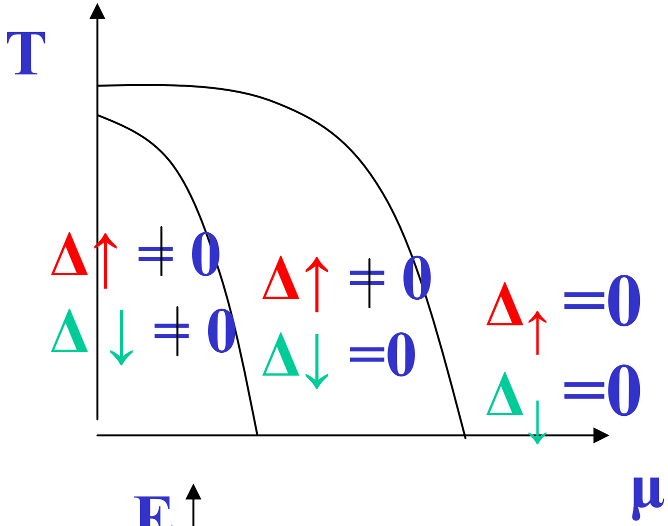
•Electron density dependence: first order transition

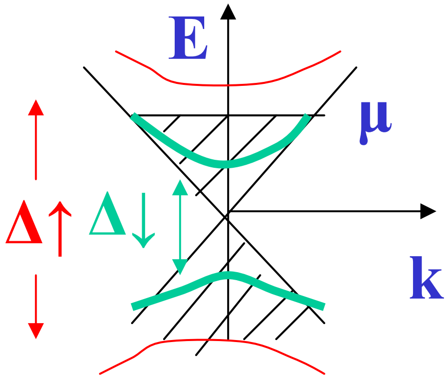


•Degeneracy between singlet and triplet pairing is lifted



Excitonic (weak?) ferromagnetism





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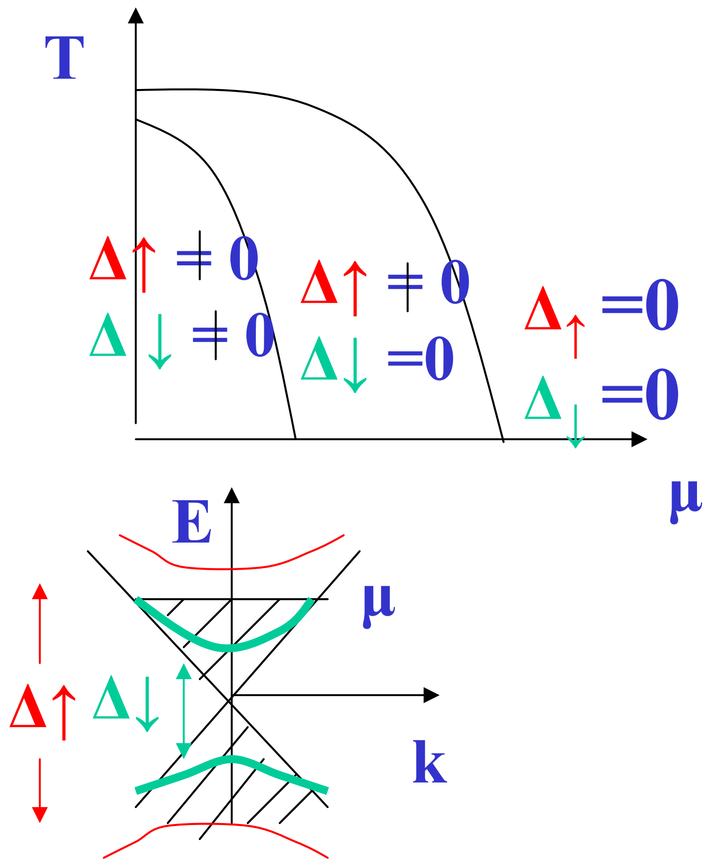
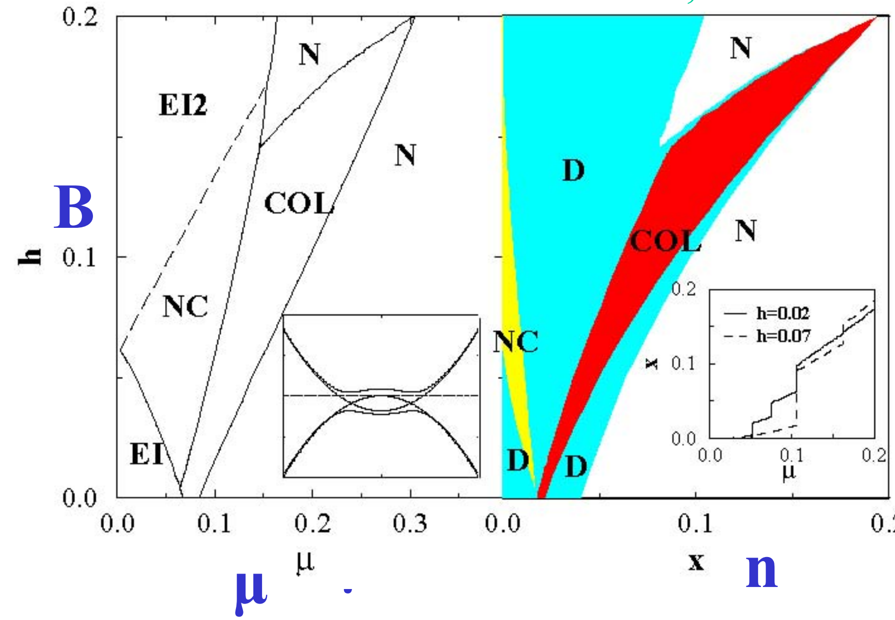


Example: hexaborides (BCS)

A. McDonald et al, ’00



N = paramagnetic (semi)metal;

EI = excitonic insulator;

COL/NC = (non-)collinear ferromagnet

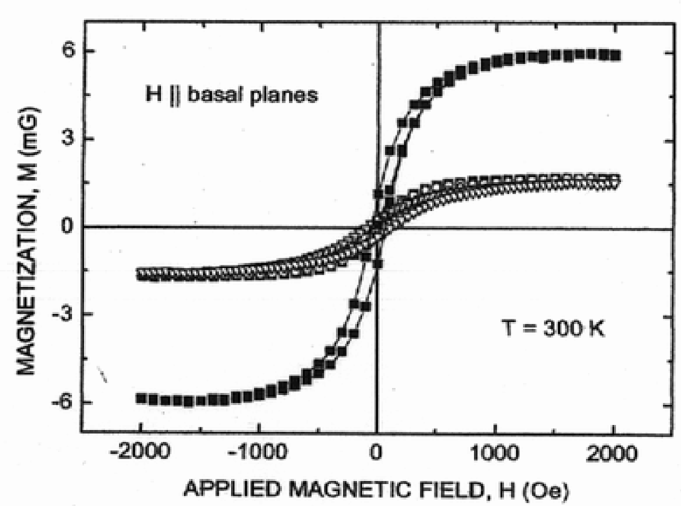
Weak ferromagnetism in HOPG

•Small, yet robust, magnetic moment:

M ~ 0.03-0.05 μ_B/carrier, Tc~500K

Y.Kopelevich et al ‘00

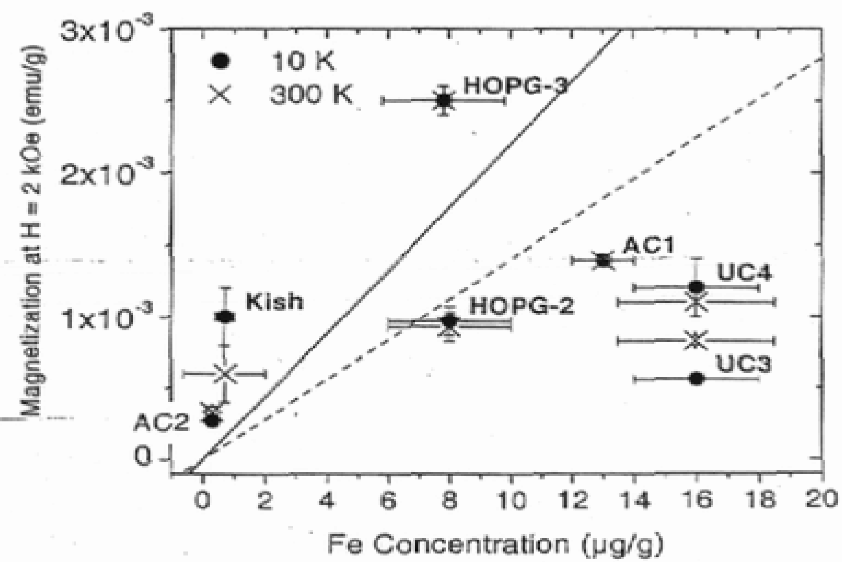
M



H

P.Esquinazi et al ‘02

M



n_{imp}

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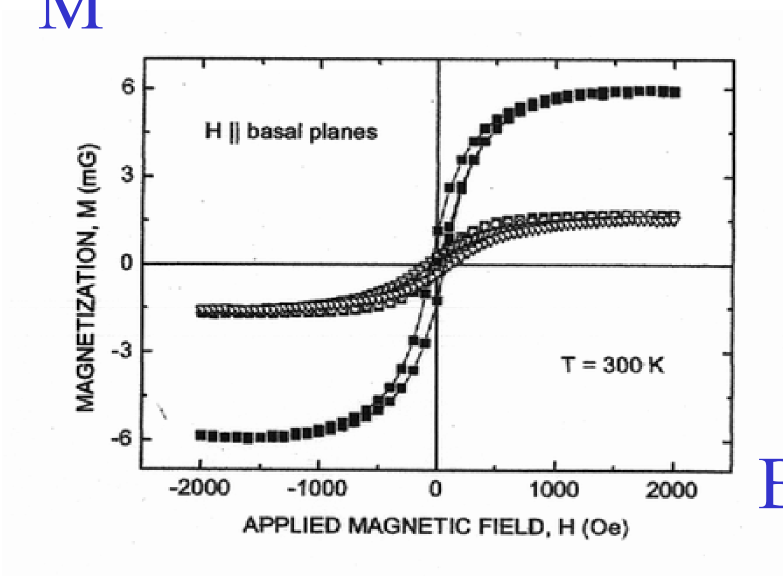
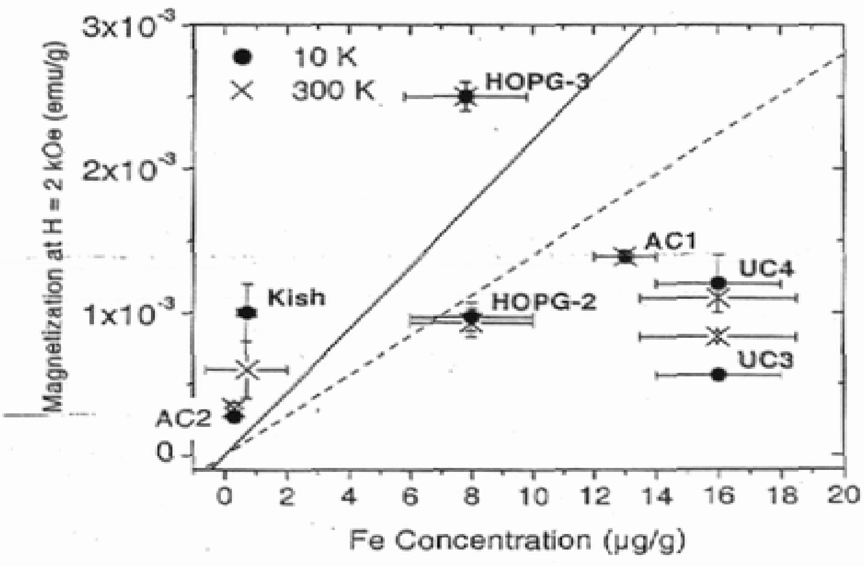


P.Esquinazi et al '02



• Possible mechanisms:

-Single-particle (magnetic impurities; structural defects, edges, H-bonds)

-Many-body (Coulomb interactions) ?

Dirac fermion-phonon coupling: Cooper pairing

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•Elastic energy:

$$F = \frac{\rho}{2}[(\partial_t \vec{u})^2 + (\partial_t h)^2 - \kappa^2(\partial_i^2 h)^2 - c^2(\partial_i u_j + \partial_j u_i + \partial_i h \partial_j h)^2]$$

$$D_a(\omega, q) = \frac{\Omega_a(q)}{\omega^2 - \Omega_a^2(q) + i0}$$

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- Effective e-e interaction:

$$V_{ph}(\omega, q) = \sum_{a=0,1,2} (D_a(\omega, q)|M_q^a|^2 + \sum_{q'} \int \frac{d\omega'}{2\pi} D_z(\omega'_+, q'_+)|M^a_{q'_-}|^2 D_z(\omega'_-, q'_-)|M^a_{q'_+}|^2)$$

$$V_{ph,\parallel}(q) = -\sum_{a=0,1} \frac{|M_q^a|^2}{\Omega_a} = -(V_0 + V_1)$$

$$V_{ph,z}(q) = -\int \frac{d\omega}{2\pi} \sum_k e|M_k|^2 D(\omega, k+q) D(\omega, k) = -V_2 \ln \frac{\Lambda}{q}$$

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D.Basko and I.Aleiner, '07

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$$\Delta_{SC} \approx \sum_{a=0,1,2} E_a e^{-1/\lambda_a} \qquad E_0 \sim min[\Omega_0, \mu], \quad E_1 \sim \frac{c\mu}{v}, \quad E_2 \sim \mu(\frac{\Lambda\kappa}{v})^{1/2}$$

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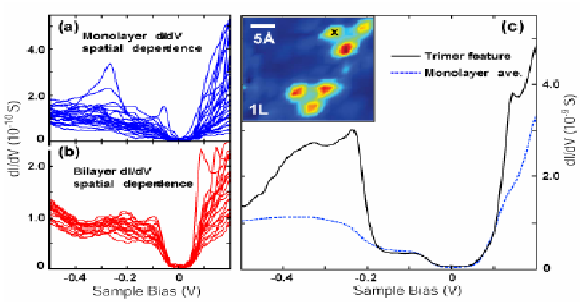
V.Brar et al, '07



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V.Brar et al, '07

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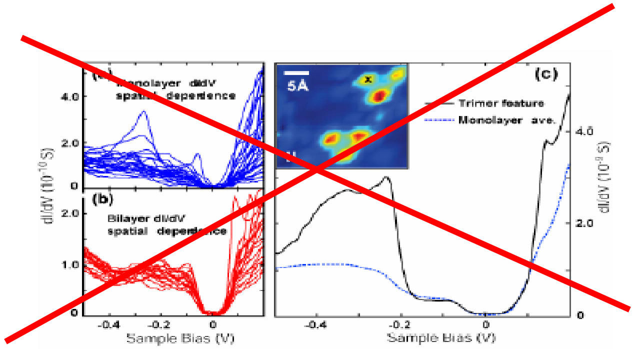

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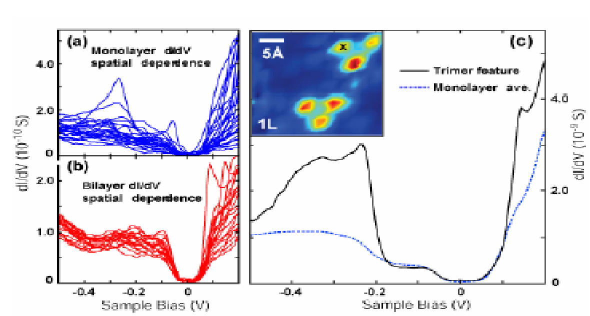
V.Brar et al, '07

Strong Coulomb repulsion →
ε>>1 substrate?



Excitonic and Cooper instabilities in real-life graphene

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would destroy the excitonic gap $\Delta \sim 5$ meV

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•Ripples: **$B_{eff} \sim 5T$**

would destroy the Cooper gap **$\Delta \sim 30$ meV**

II. Coulomb interacting Dirac fermions in magnetic field

Coulomb interacting Dirac fermions in magnetic field

- Relativistic analog of FQHE: **magnetic catalysis**

$$\Delta(p) = i\int \frac{d\omega d\mathbf{k}}{(2\pi)^3}\frac{\Delta(k+p)}{(\epsilon+\omega+i\delta)^2-\Delta^2(k+p)}$$
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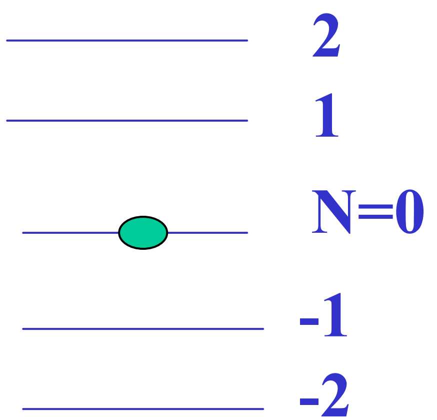
DVK, cond-mat/0106261
V.Gorbar et al, cond-mat/0202422



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screening is even weaker than at B=0
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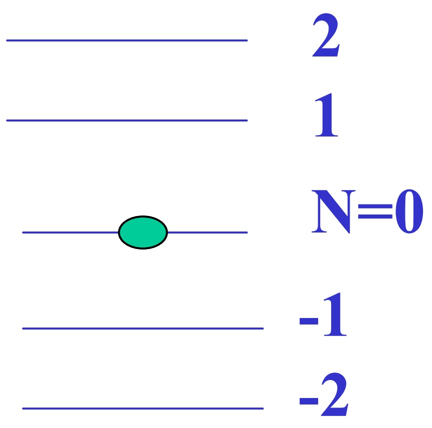

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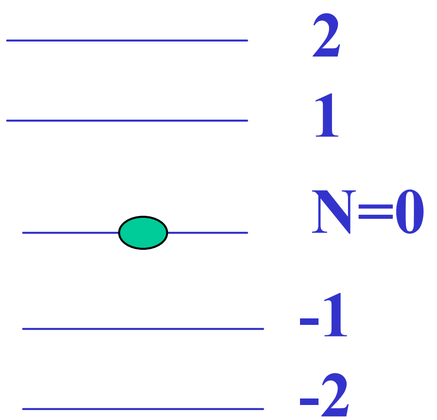

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- A magnetic field-induced fermion mass can provide a means of **spatially confining** the Dirac fermions (cf. electrostatic potential – Klein's tunneling).

Moderately strong fields: (Half)Integer Quantum Hall Effect

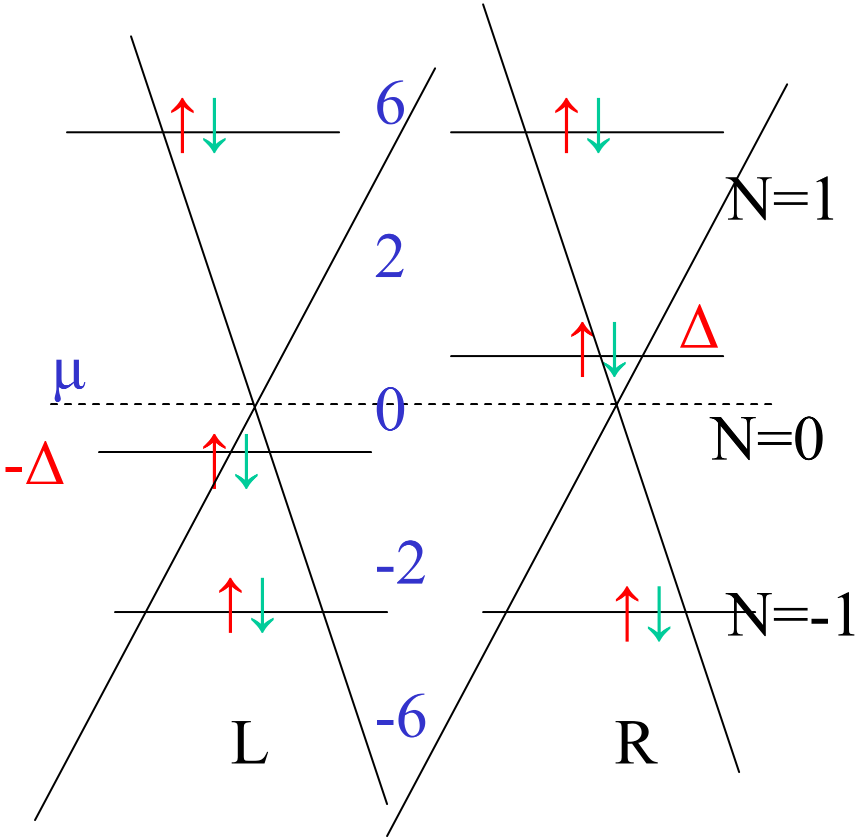
•Dirac fermions' Landau levels:

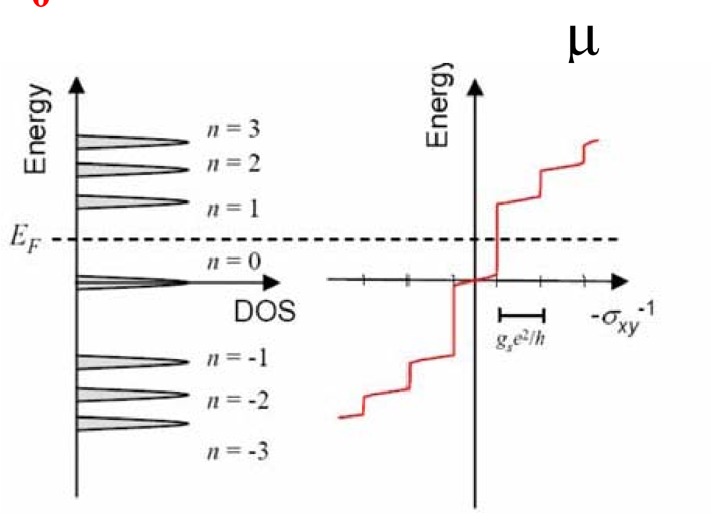
$$E_N = \pm (2v_F^2 NB + \Delta^2)^{1/2}$$

•"Anomalous" IQHE:

$$\sigma_{xy}(T)=4(e^2/h)(N+1/2)$$

B < B_0 ~ 10T: Δ=0





A.Geim et al '05
P. Kim, et al '05

Stronger fields: magnetic field-induced mass

•New plateaus: B > ~10T

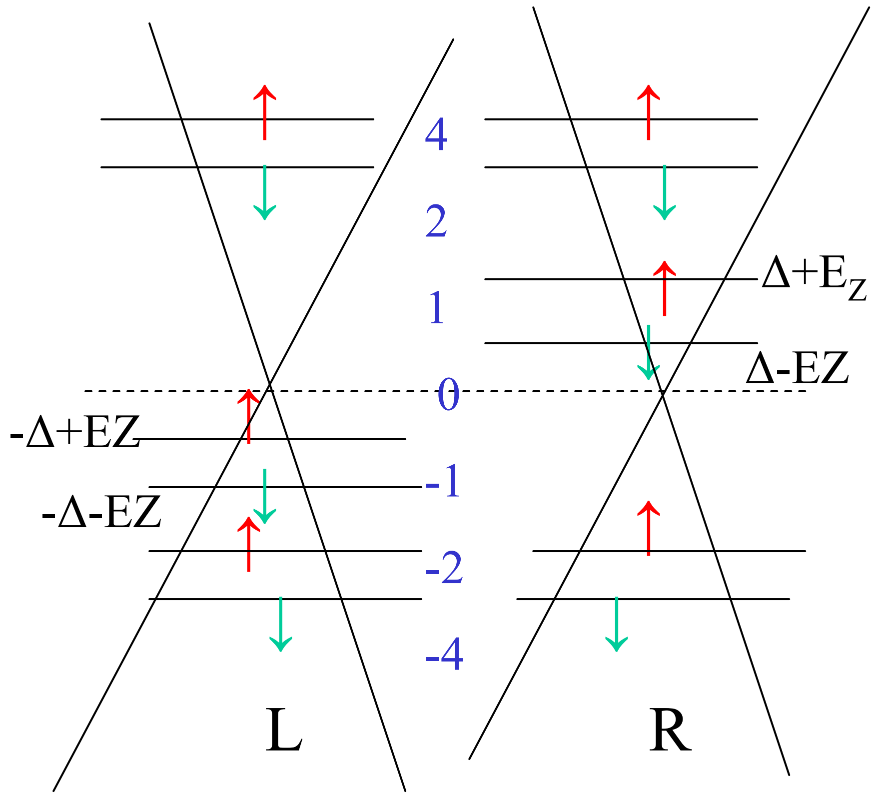
$\sigma_{xy}(T) = \pm(e^2/h)(0, 1, 4)$

Y.Zhang et al,'06

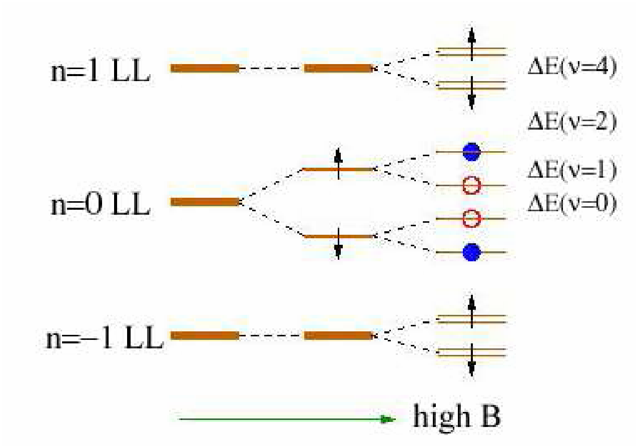
•Spin and valley splitting at LLL (N=0)

•Valley degeneracy remains intact for N≠0

•**NO** plateaus observed at ±3, ±5,..



Field dependence of spectral gaps



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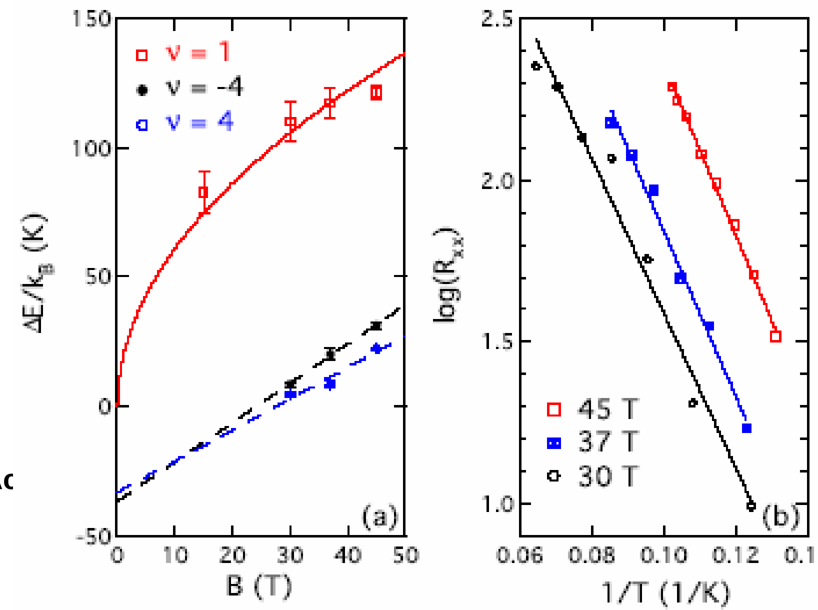
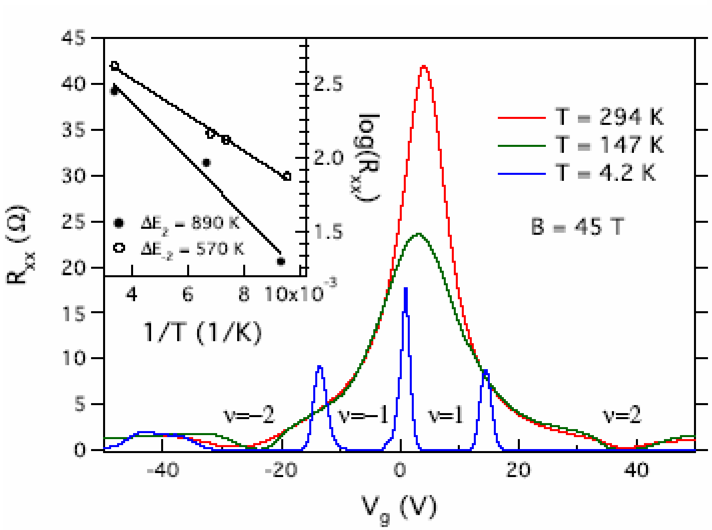




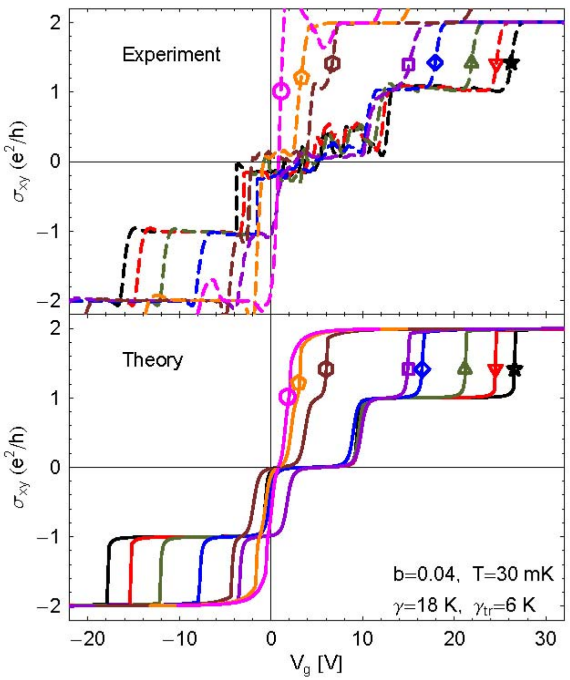
P.Kim et al, ‘07

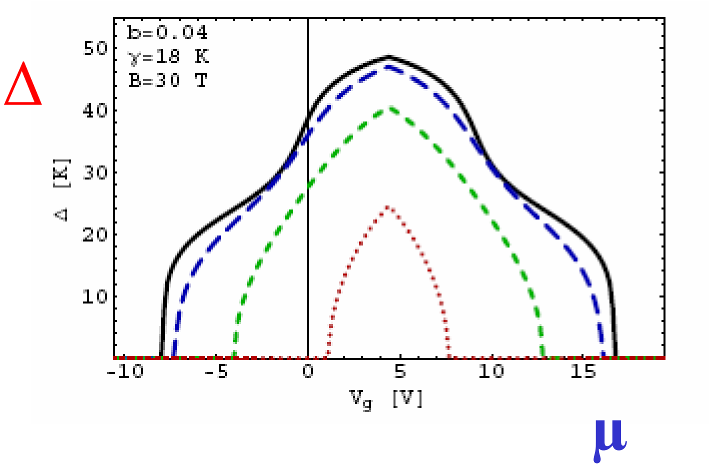
$\nu = \pm 4$ $\Delta \sim B$ Zeeman?

$\nu = \pm 1$ $\Delta \sim B^{1/2}$ Coulomb?

Magnetic catalysis scenario: data fitting

V. Gusynin et al, '06





Δ~50K
B=30T

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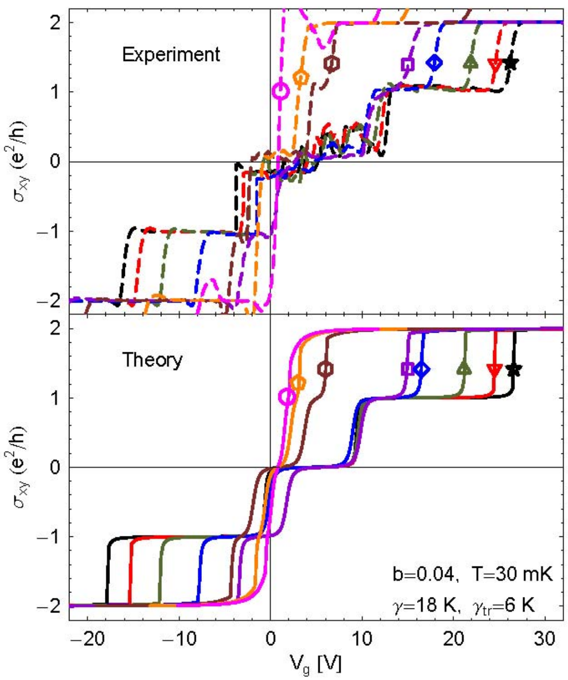
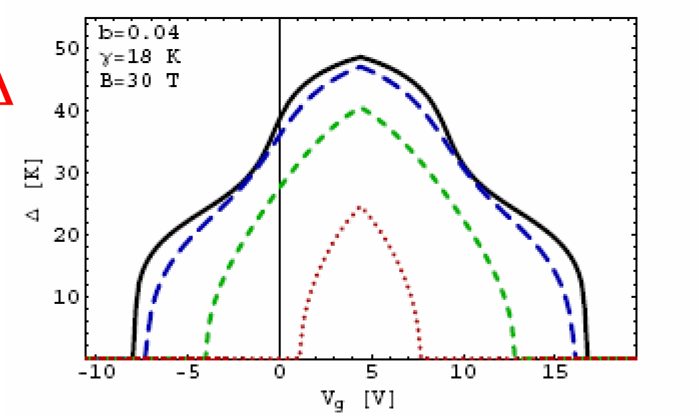


V. Gusynin et al, '06

Δ



μ

Δ~50K
B=30T

Alternative mechanisms:

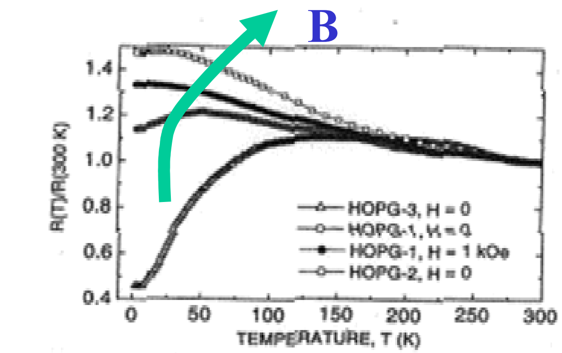
- Many-body: QH Ferromagnetism

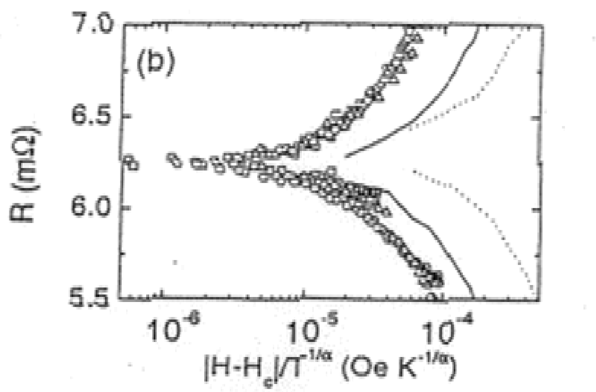
K.Nomura, A.McDonald, '06; J.Alicea, M.P.E. Fisher, '06; M.Goerbig et at, '06, K.Yang et al, '06.

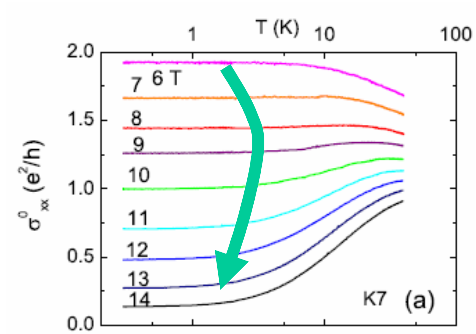
- Single-particle: Peierls distortion

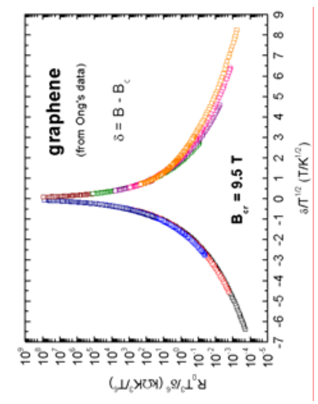
J. Fuchs and P. Lederer, '06

Field-induced MIT in HOPG and graphene?



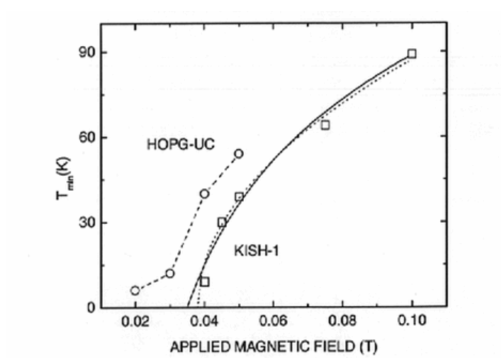






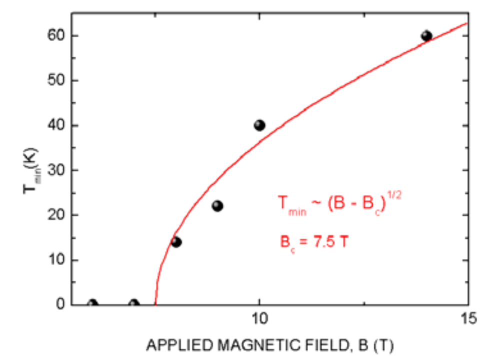
Y.Kopelevich et al, '00

Y.Kopelevich, '08



$$\Delta \sim (B - B_0)^{1/2}$$

B_0 (μ)



FQHE in graphene

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spin and valley polarized

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•Composite Dirac fermions: new fractions $\sigma_{xy} = (\pm)\frac{2m}{2m \pm 1}$

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DVK, '06

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spin and/or valley singlets

$$\sigma_{xy} = (\pm)\frac{2}{2m \pm 1}$$

DVK, '06

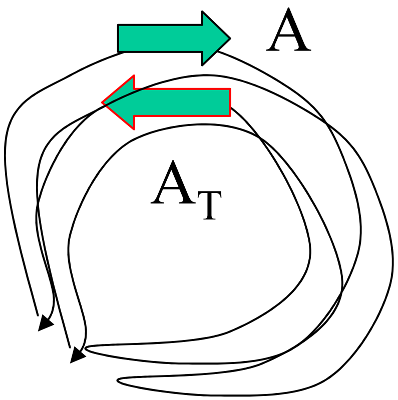
•Also found numerically:

V.Apalkov and T.Chakraborty, '06; C.Toke et al, '06

III Disordered Dirac fermions

Disordered Dirac fermions

Negative interference → WAL→

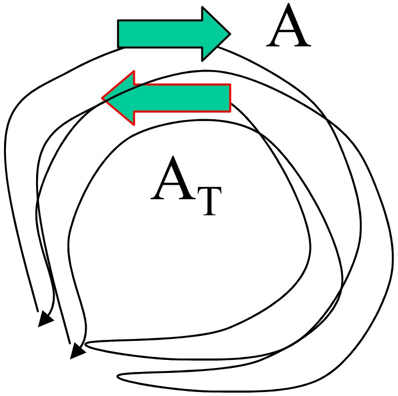


Intrinsic Berry phase π

$A_T = - A$

Disordered Dirac fermions

Negative interference → WAL→ Positive MR



Intrinsic Berry phase π

$A_T = - A$

No inter-valley scattering:
WAL

T. Ando and H. Suzuura, '02

$\Delta\sigma_{WL}(H) < 0$

Disordered Dirac fermions

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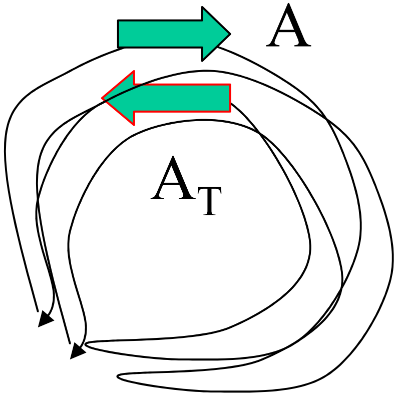

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Intra- and inter-valley scattering:
crossover between WL and WAL

DVK, PRL 97, 036802,'06 (0602398)

E. McCann et al, PRL 97, 146805,'06 (0604015)

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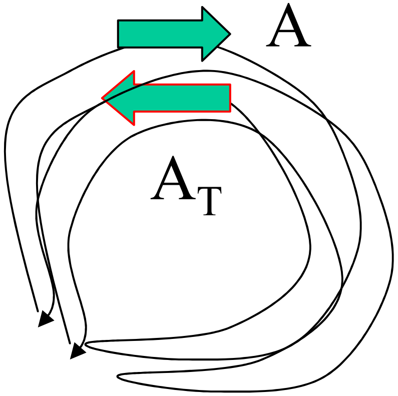

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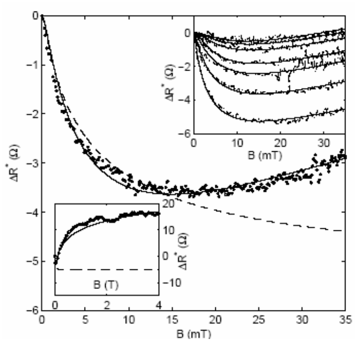
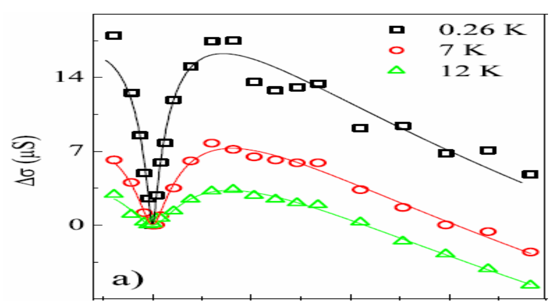


X.Wu et al, '07



V.Tikhonenko et al '07

Disordered Dirac fermions

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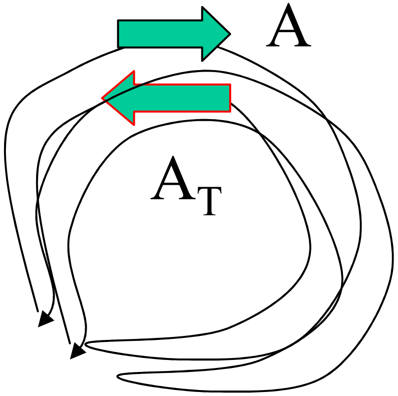
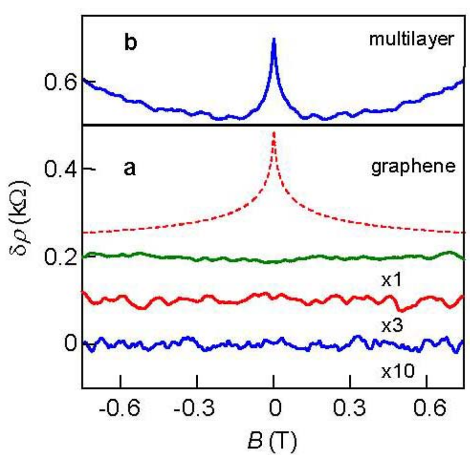


Intrinsic Berry phase π
$A_T = - A$

Special disorder models:
-commensurate substrate potential (Umklapp permitted),
-chiral disorder,…

Morozov et al ‘06



$\Delta\sigma_{WL}(H) = 0$

DVK,’06,
P.Ostrovsky et al, ‘07

Disordered Dirac fermions

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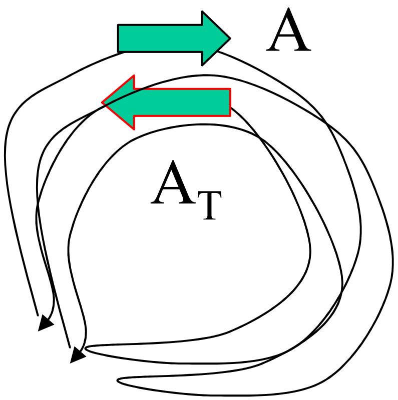

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Theory: momentum-independent,
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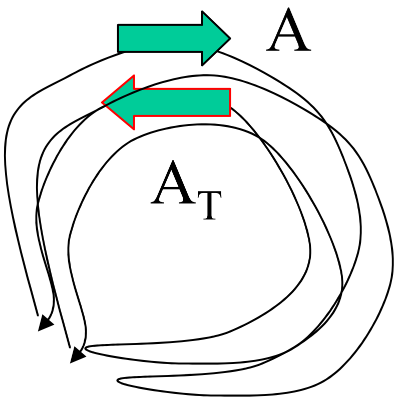

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No such scattering mechanism in **undoped** graphene?

Experimentally relevant disorder

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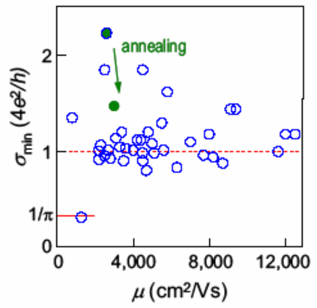
- (Non)universal minimal conductivity:



A.Geim et al, ’06

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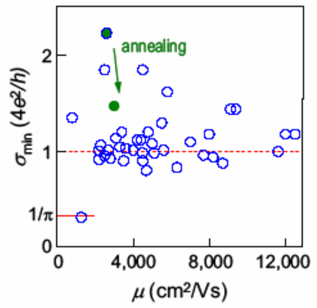
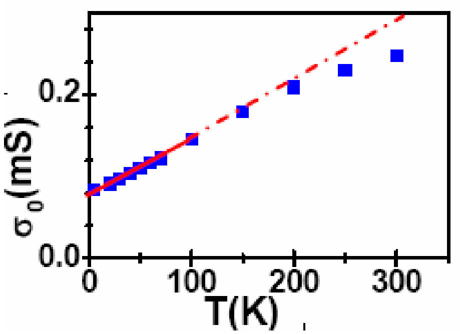


A.Geim et al, ’06

- Linear T-dependence:



G.Li et al, ‘08

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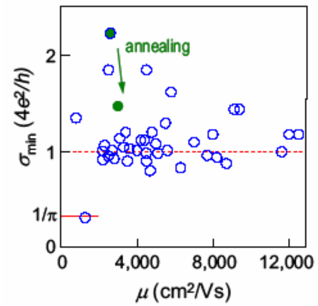

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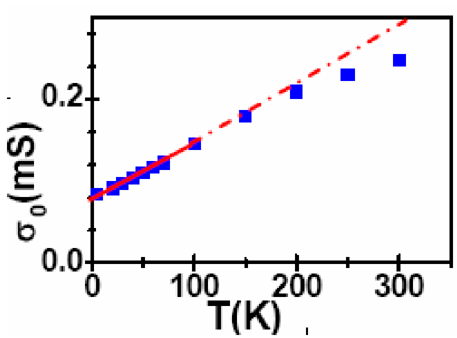
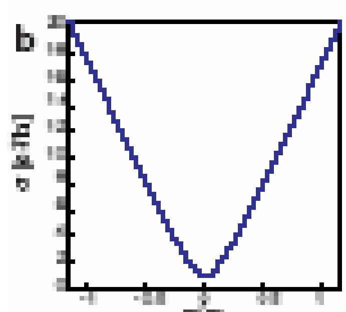


G.Li et al, ‘08

- Linear (?) density dependence:

→ long-range-correlated disorder?



Long-range-correlated disorder

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- Scalar vs vector disorder: $< V_q V_{-q} >= \Gamma_s / q^{2\eta}$ intra-valley

  $< A_q A_{-q} >= \Gamma_v / q^{2\eta}$

"T-reversal": even (V) vs odd (A)

$$H = \sigma_2 H^T \sigma_2$$

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"T-reversal": even (V) vs odd (A)

$H = \sigma_2 H^T \sigma_2$

NOT <VV>=const

<AA>=const

η =0

Long-range-correlated disorder

- Scalar vs vector disorder: $$< V_q V_{-q} >= \Gamma_s / q^{2\eta}$$ $$< A_q A_{-q} >= \Gamma_v / q^{2\eta}$$
- Experiment: linear conductivity of graphene **(σ~n)** → η=1

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-Coulomb impurities: η=1

A.McDonald and K. Nomura,'06; S. Das Sarma et al, '06

-Cf. short-range potential disorder: η=0

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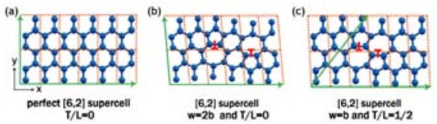

F. Guinea et al,'93

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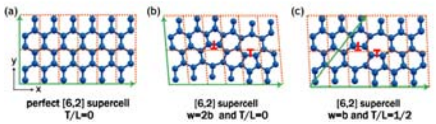
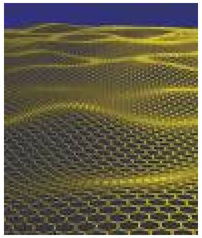


F. Guinea et al,'93

-Cf. Dislocations (pentagon/heptagon pairs): η=0

- Ripples (asymptotic regime): η=0.2



M.Katsnelson et al, '07; N.Abedpour et al, '07

Non-linear conductivity at n→0 ?

Long-range-correlated disorder

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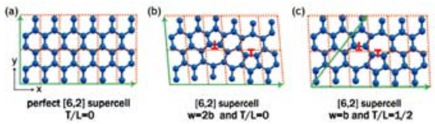

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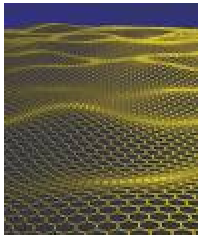

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M.Katsnelson et al, ’07; N.Abedpour et al, ‘07

Non-linear conductivity at n→0 ?

- Random strain ?

Scalar vs vector disorder with η=1: perturbation theory

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- Self-consistent Born approximation, **doped** case:

$$\hat{\Sigma}^R_\alpha(\epsilon,\mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^2} \frac{w_\alpha(\mathbf{q})}{\hat{G}^R(\epsilon,\mathbf{p}+\mathbf{q})^{-1} + \hat{\Sigma}^R_\alpha(\epsilon,\mathbf{p}+\mathbf{q})}$$

$$\hat{G}_R(\omega,\mathbf{p}) = [(\epsilon + i0)\hat{\gamma}_0 - p_\mu\hat{\gamma}_\mu]^{-1}$$

$$w(q) = g/(lq)^{2\eta}$$

RP gets screened at ε>0 but RMF doesn't →
strongly non-Lorentzian qp spectral function in the RMF case

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- Fermion lifetimes:

$$\gamma_s = ImTr\hat{\gamma}_0\hat{\Sigma}^R_s(\epsilon,\epsilon/v) \sim \frac{v^2\Gamma_s}{\epsilon} min[\frac{1}{g},\frac{1}{g^2}]$$

$$\gamma_v = ImTr\hat{\gamma}_0\hat{\Sigma}^R_v(\epsilon,\epsilon/v) \sim v\Gamma_v^{1/2}\sqrt{\ln L}$$

- Failure of perturbation theory (genuine IR divergence due to a gauge non-invariant nature of G)

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- Transport times: $(\varepsilon_F >> \Gamma_{s,v}^{1/2})$

$$\gamma^{tr}_\alpha = \int \frac{d\mathbf{q}}{(2\pi)^2}\delta(\epsilon_F - v|\mathbf{p}+\mathbf{q}|)w_\alpha(\mathbf{q})\sin^2\theta \qquad \gamma^{tr}_s \sim \frac{v^2\Gamma_s}{\epsilon_F} min[1,\frac{1}{g^2}], \quad \gamma^{tr}_v \sim \frac{v^2\Gamma_v}{\epsilon_F}$$

- Can't discriminate between RP and RMF: $\sigma \sim \varepsilon_F / \gamma \sim \varepsilon^2{}_F \sim n$

DVK, 0607174
A.Geim and M.Katsnelson, 0706.2490

Scalar vs vector disorder: characteristic cyclotron rates

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- Envelope function of the SdH/dHvA oscillations:

$$\nu(\epsilon|B) = \nu(\epsilon|0) \sum_{n=-\infty}^{\infty} e^{2\pi i n A(\epsilon) - n^2 \delta S_1(\epsilon)}$$

$$\delta S_1(\epsilon) = -\sum_{\alpha=s,v} \ln W_\alpha = \pi[\frac{\Gamma_s}{B} + \frac{\Gamma_v \epsilon^2}{v^2 B^2}]$$

$$\nu(\epsilon|B) \propto \sum_{n=0}^{\infty} \exp[-\pi \frac{(\epsilon^2 - \omega_n^2)^2}{v^2 B (\gamma_s^{cycl})^2 + (\gamma_v^{cycl})^4}]$$

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- Characteristic cyclotron times ($\varepsilon \gg \Gamma^{1/2}$):

$$\gamma_s^{cycl} \sim v\Gamma_s^{1/2}, \quad \gamma_v^{cycl} \sim (\epsilon^2 v^2 \Gamma_v)^{1/4}$$

- Scalar vs vector disorder: different energy (=density) dependences

Scalar vs vector disorder: decay of Friedel oscillations

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- Wave functions' correlation function:

$$L^4 < |\psi^2(\mathbf{r})\psi^2(0)| > -1 = \frac{< Im\hat{G}^R(\epsilon,\mathbf{r}) Im\hat{G}^R(\epsilon,-\mathbf{r}) >}{(\pi\nu(\epsilon))^2}$$

$$\sim \left(\frac{\gamma_\alpha^{FO}}{\epsilon^2 r}\right)^{1/2} \cos(2\epsilon r) e^{-r\gamma_\alpha^{FO}}$$

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- Characteristic rates of the Friedel oscillations' spatial decay:

($\varepsilon >> \Gamma^{1/2}$)

$$\gamma_s^{FO} \sim v\Gamma_s^{1/2}, \quad \gamma_v^{FO} \sim v^{4/3}\frac{\Gamma_v^{2/3}}{\epsilon^{1/3}} \qquad \delta\rho(r) \propto (\frac{\gamma_\alpha^{FO}}{r^5})^{1/2} \cos(2\epsilon r) e^{-r\gamma_\alpha^{FO}}$$

- STM probe could distinguish between RP and RMF, too.

Long- vs short-range correlated RMF: density of states

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- Chiral order parameter:

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$$\nu_0(\epsilon) \propto \epsilon^{2/z-1}$$

$z = 1 + g$ for $g<2$

$z = (8g)^{1/2} - 1$ for $g>2$

A.Ludwig et al'94

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Can η be measured directly (STM)?

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Quenched Schwinger model (η=1): A. Smilga, '92

Conclusions

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•Dirac fermions in graphene exhibit novel **disorder effects,** potentially relevant disorder being **of long-range-correlated** nature;
-**evidence**: experiment
-**relevance**: probes for ascertaining the **nature of disorder**