Analog of Graphene using a Microwave Photonic Crystal

New Frontiers in Graphene Physics
ECT* 2010

- Dirac spectrum in a photonic crystal
    - Experimental setup
    - Transmission and reflection spectra
- Extremal transmission
- Photonic crystal in a box

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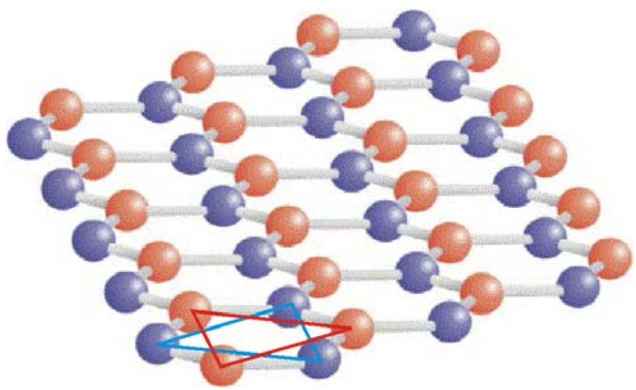
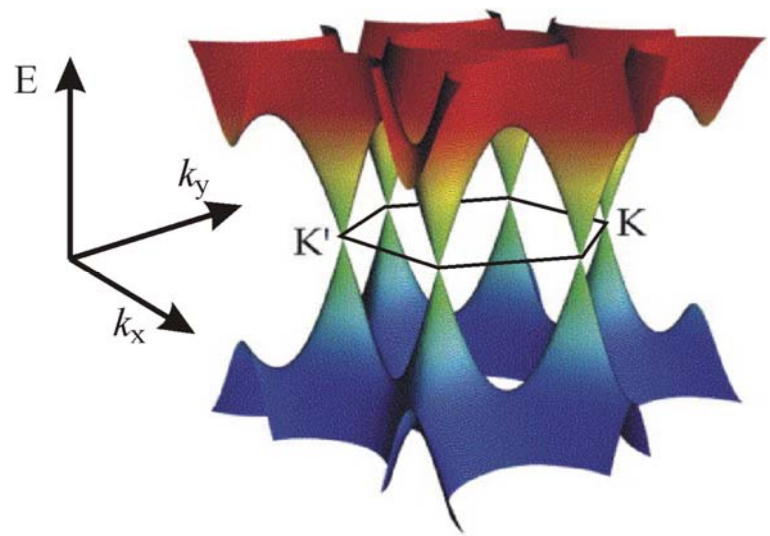
Graphene

- "What makes graphene so attractive for research is that the spectrum closely resembles the Dirac spectrum for massless fermions."
  M. Katsnelson, Materials Today, 2007

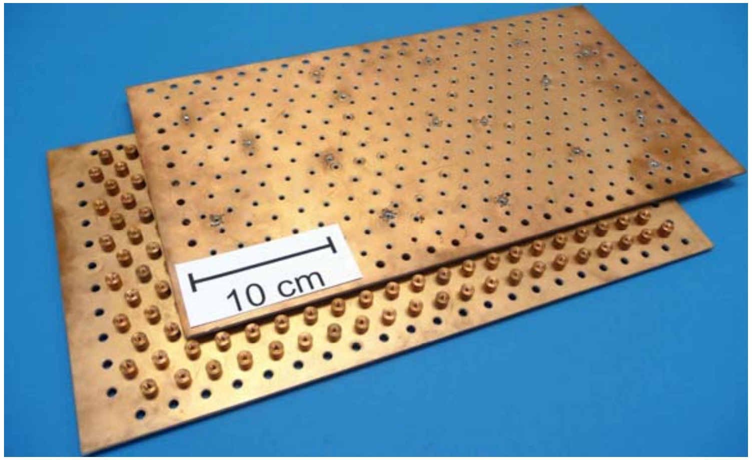




- Two triangular sublattices of carbon atoms
- Near each corner of the first hexagonal Brillouin zone the electron energy E has a conical dependence on the quasimomentum

Photonic Crystal

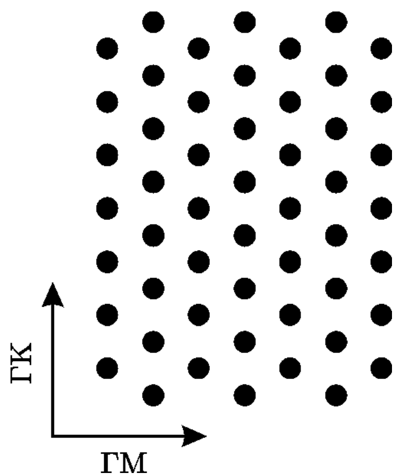
- A photonic crystal is a structure, whose electromagnetic properties vary periodically in space, e.g. an array of metallic cylinders
  → open microwave resonator

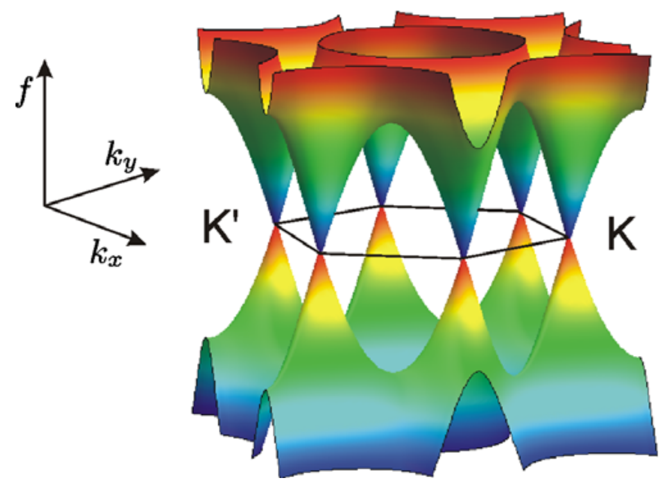


- Flat "crystal" (resonator) → E-field is perpendicular to the plates (TM_0 mode)
- Propagating modes are solutions of the scalar Helmholtz equation
  → Schrödinger equation for a quantum multiple-scattering problem

Calculated Photonic Band Structure

- Dispersion relation of a photonic crystal exhibits a band structure analogous to the electronic band structure in a solid





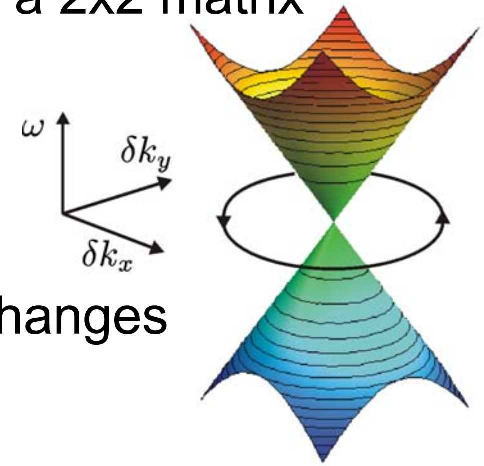
- The triangular photonic crystal possesses a conical dispersion relation
  → Dirac Spectrum

Hamiltonian around Dirac Point

- Close to Dirac point the Hamiltonian can be written as a 2x2 matrix

$$\hat{H} = \omega_D \mathbb{1} + v_D \left(\delta k_x \hat{\sigma}_x + \delta k_y \hat{\sigma}_y \right)$$

- Hamiltonian of this type possesses a diabolic point



- Encircling of a diabolic point in the parameter space changes the sign of eigenvectors (Berry phase)

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \circlearrowleft - \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

- Encircling in $(\delta k_x, \delta k_y)$ space corresponds to a 2π rotation of the coordinate frame $\rightarrow$ spinor property

Dirac Equation

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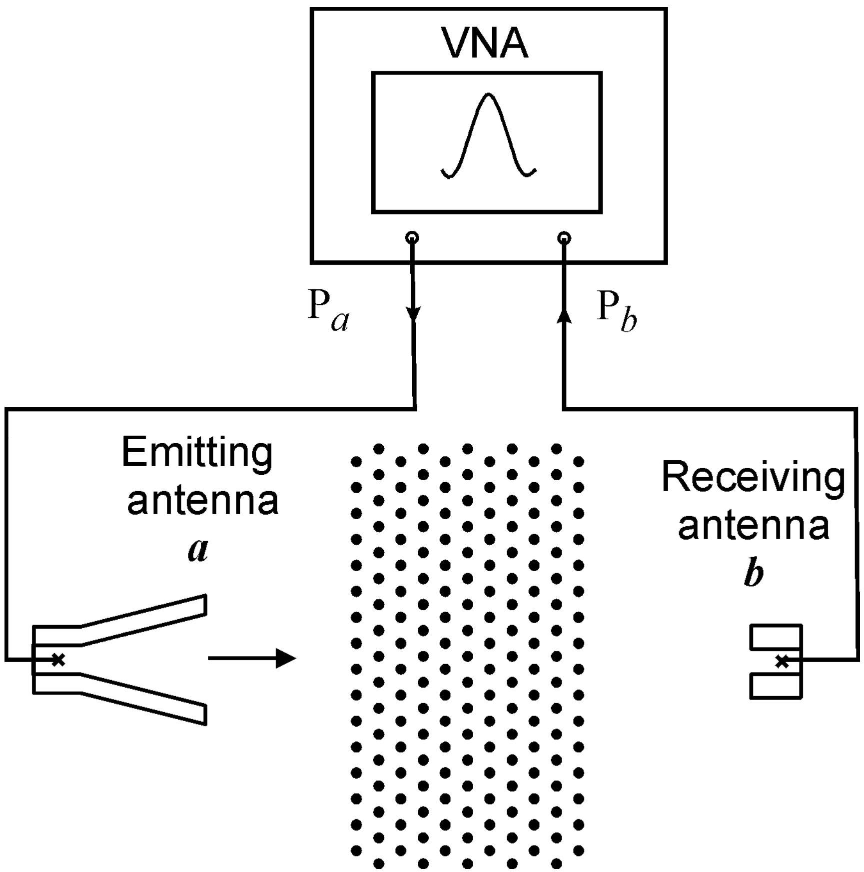
- Substitution $\delta k_x \rightarrow -i\partial_x$ and $\delta k_y \rightarrow -i\partial_y$ leads to the Dirac equation

$$\begin{pmatrix} 0 & \partial_x - i\partial_y \\ \partial_x + i\partial_y & 0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = i\,\frac{\omega - \omega_D}{v_D} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

- The components ψ_1, ψ_2 of the spinor are the amplitudes of two degenerate Bloch states at the Dirac frequency
- The same equation holds for the spinor around the K' corner in the 1st Brillouin zone
- In the vicinity of the Dirac frequency ω_D, waves in a photonic crystal can be effectively described by the Dirac equation

Scattering Experiment



- Scattering matrix $|S_{ba}|^2 = \frac{P_b}{P_a}$
- VNA measures the complete scattering matrix
- Transmission: S_{ba}, S_{ab}
- Reflection: S_{aa}, S_{bb}
- Horn antenna emits approximately plane waves

Experimental Realization of 2D Photonic Crystal

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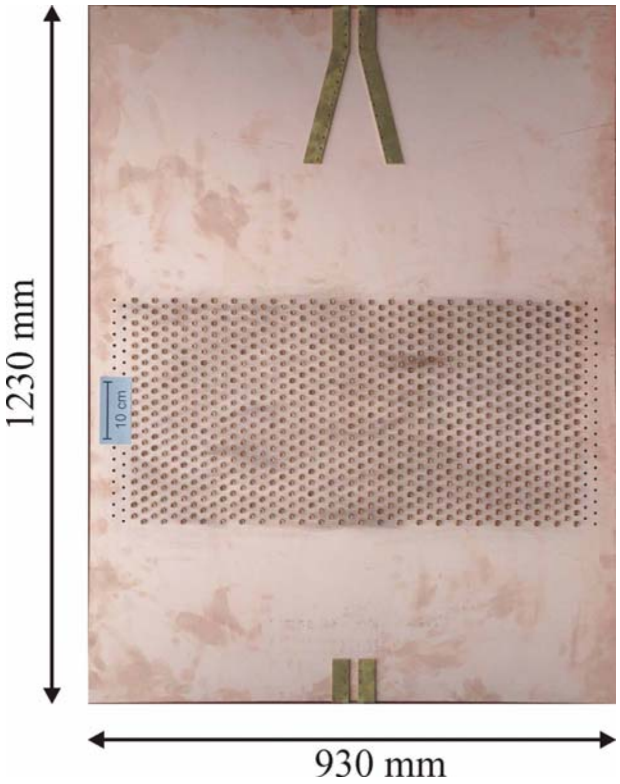
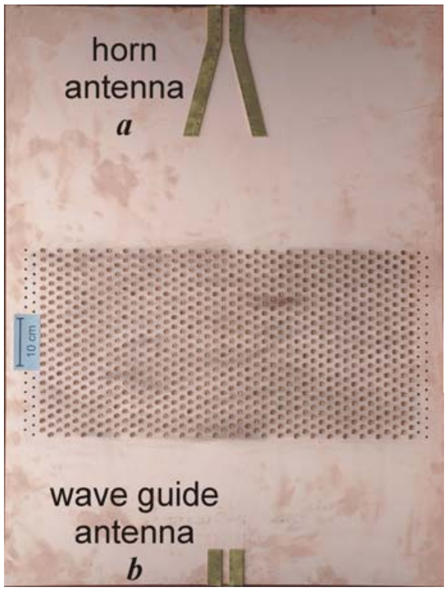
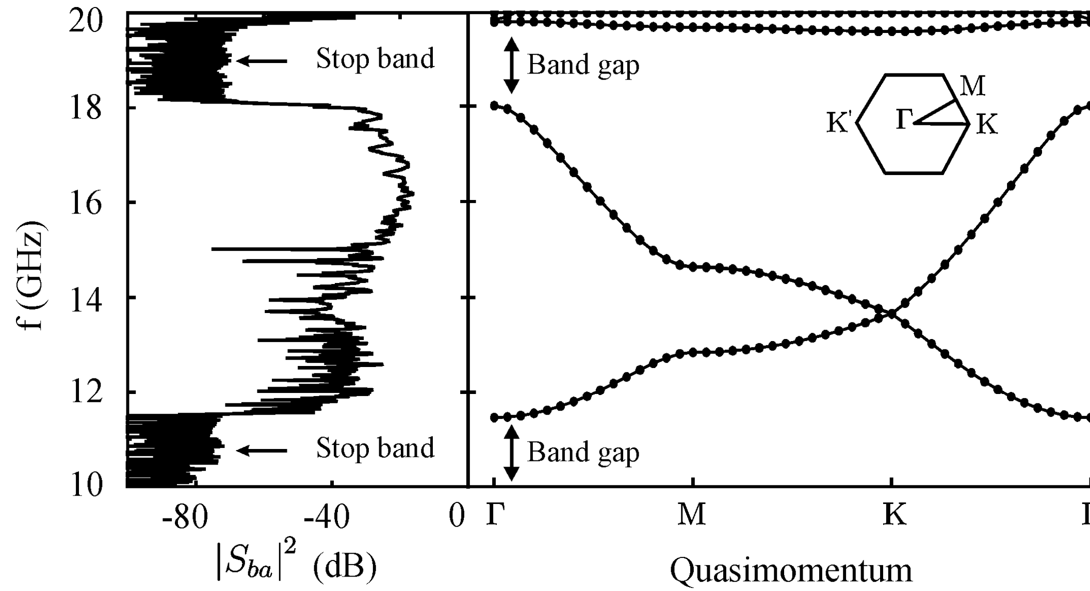


- Lattice constant: a = 20 mm
- Cylinder radius: R = 5 mm
- Number of cylinders: 23 x 38 = 874
- Crystal size: 400 x 900 x 8 mm
- Maximal frequency: 19 GHz

- First step: experimental observation of the band structure

Transmission through the Photonic Crystal

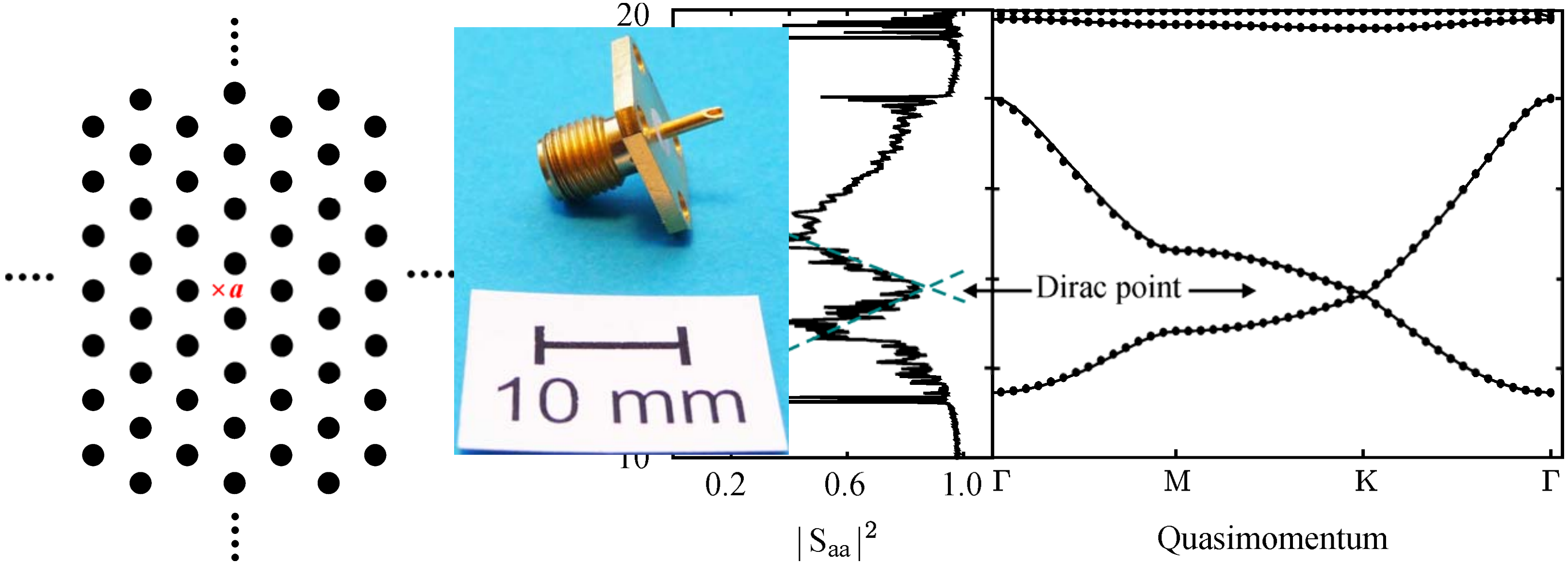




- The positions of measured stop bands coincide with the calculated ones
  → lattice parameters chosen correctly
- Dirac point is not pronounced in the transmission spectra
  → single antenna measurement

Single Antenna Reflection Spectrum

- Measurement with a wire antenna ***a*** put through a drilling in the top plate
  → point like field probe



- Characteristic cusp structure around the Dirac frequency
- Next: analysis of the measured spectrum

Local Density of States and Reflection Spectrum

- The scattering matrix formalism relates the reflection spectra to the local density of states (LDOS)

$$1 - |S_{aa}(f)|^2 \propto L(\vec{r}_a, f)$$

- LDOS

$$L(\vec{r}, f) \propto \int_{BZ} |\psi_n(\vec{k}, \vec{r})|^2 \, \delta(f - f_n(\vec{k})) d^2k$$

- LDOS around the Dirac point (Wallace,1947)

$$L(\vec{r}_a, f) \sim \frac{\langle|\psi(\vec{r_a})|^2\rangle}{v_D^2} |f - f_D|$$

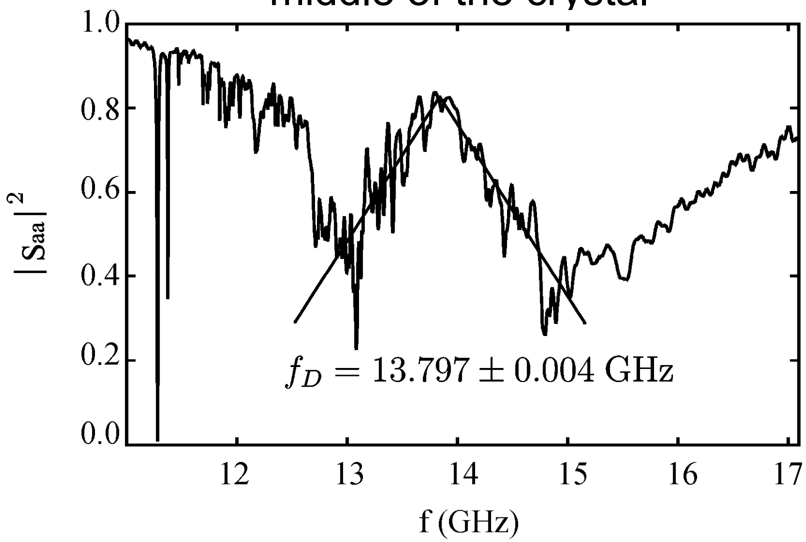
- Three parameter fit formula $|S_{aa}(f)|^2 = D - C\,|f - f_D|$

fit parameters (D, C, f_D)

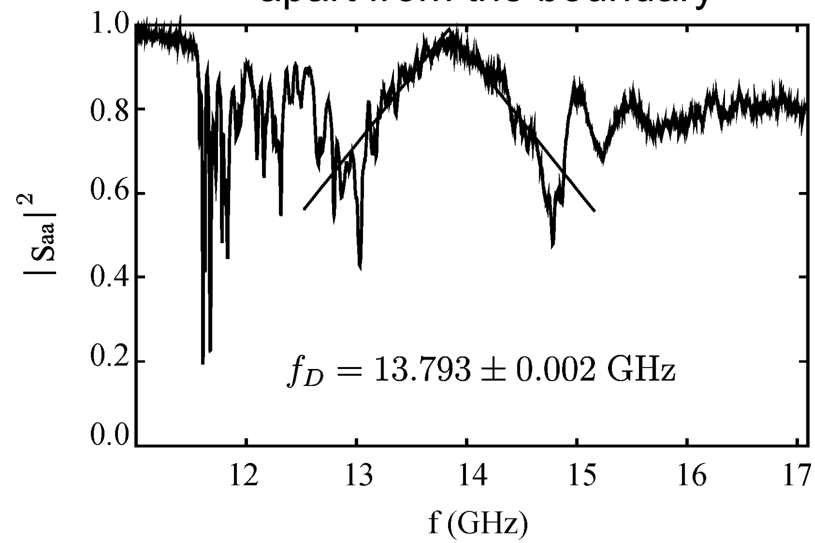
Reflection Spectra

- Description of experimental reflection spectra $|S_{aa}(f)|^2 = D - C\,|f - f_D|$

antenna ***a*** in the middle of the crystal

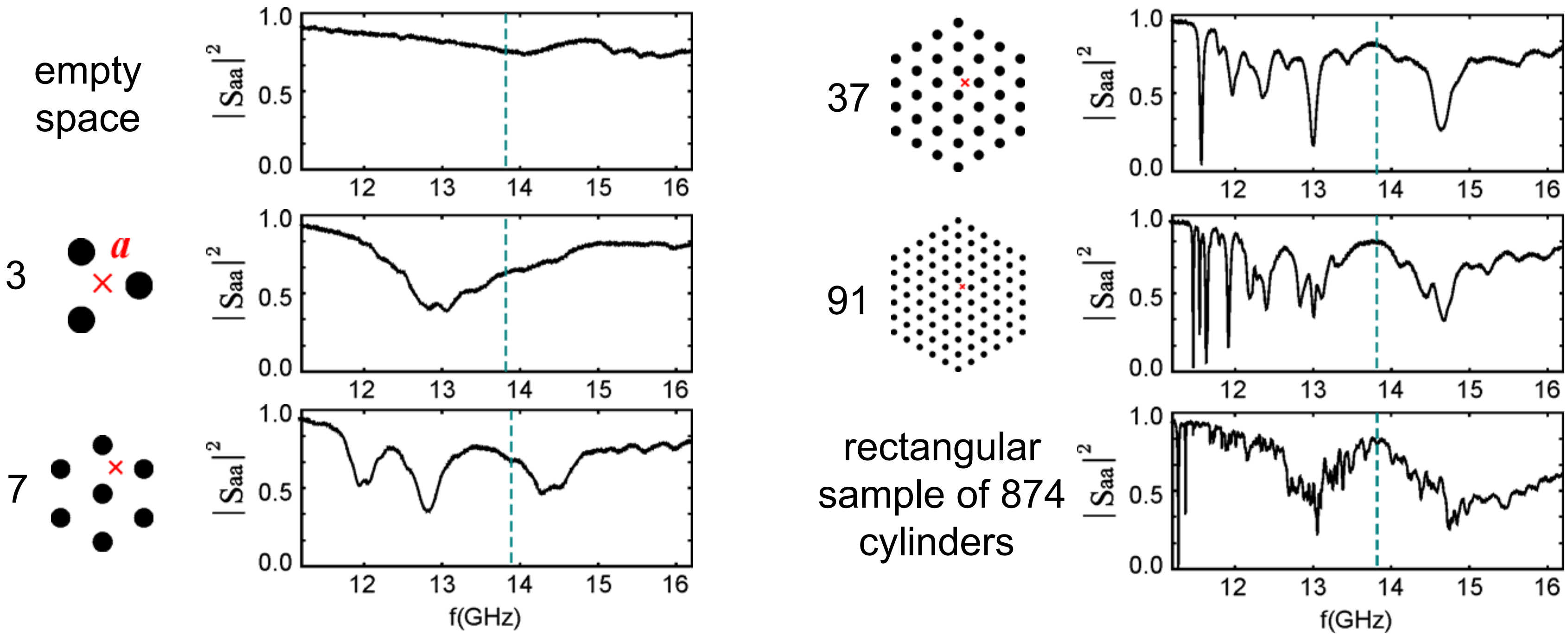


antenna ***a*** 6 rows apart from the boundary



- Experimental Dirac frequencies agree with calculated one, f_D=13.81 GHz, within the standard error of the fit
- Oscillations around the mean behavior

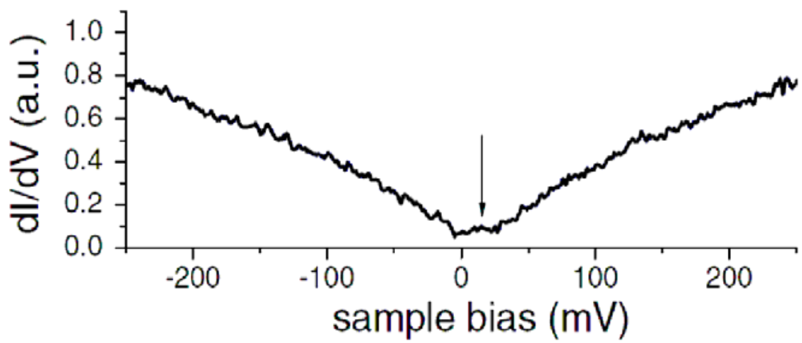
Dependence of Oscillations on Crystal Size



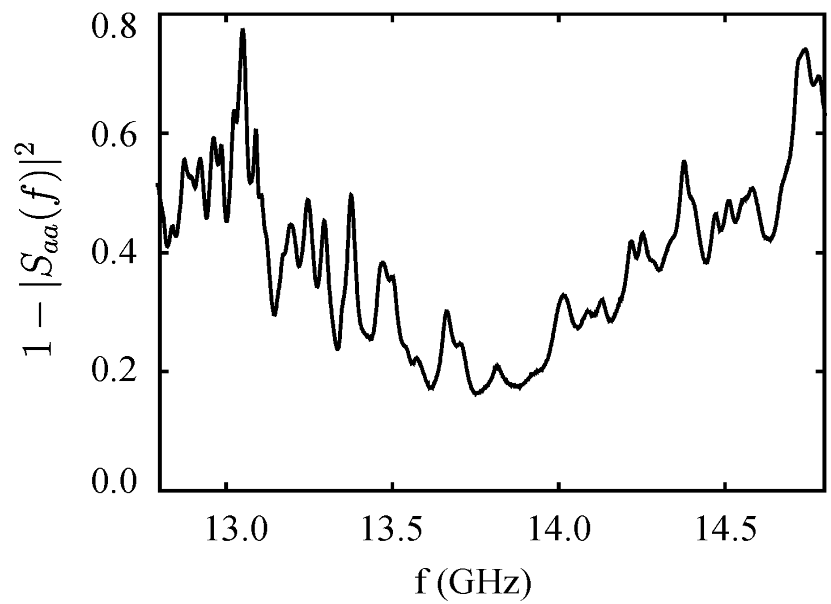
- Nature of the oscillations is a finite size effect
- Period of the oscillations is thus related to the photonic crystal size, i.e. to the density of states

Comparison with STM Measurements

graphene flake, Li *et al.* (2009)



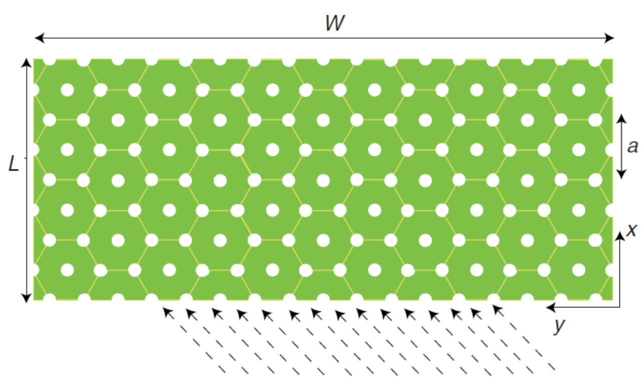
photonic crystal

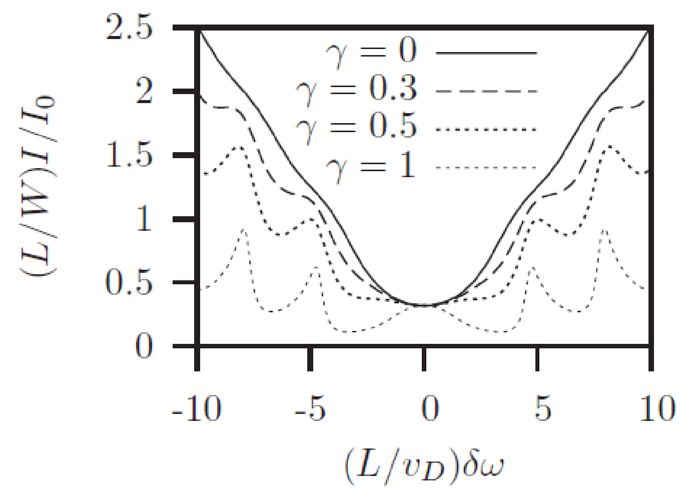


- Tunneling conductance is proportional to LDOS
- Similarity with measured reflection spectrum of the photonic crystal
- Oscillations in STM are not as pronounced due to the large sample size

Extremal Transmission through a Photonic Crystal

R.A. Sepkhanov, Ya.B. Bazalij and C.W.J. Beenakker (2007)

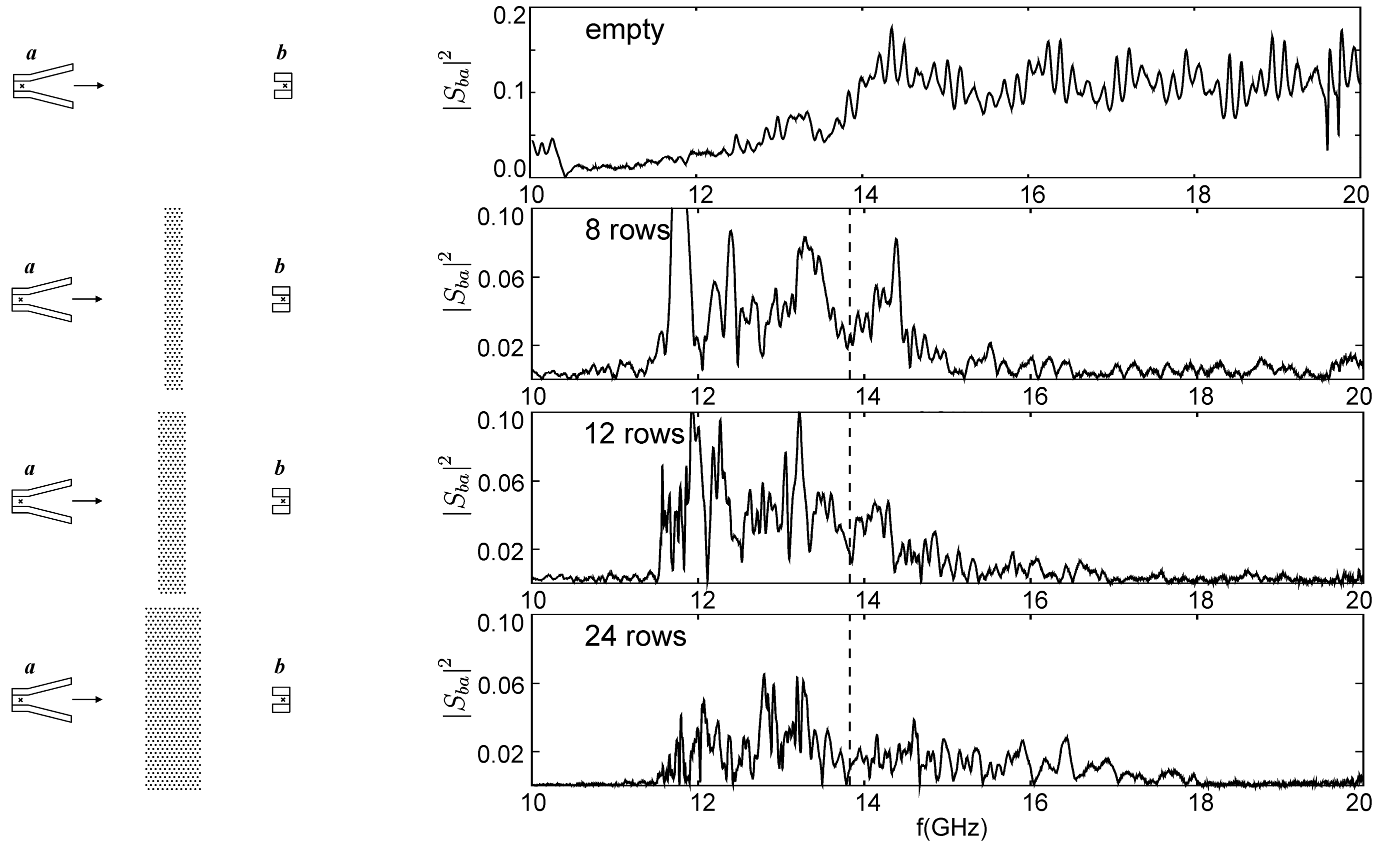




γ describes the interface

- Characteristic transmission has a minimum at the Dirac frequency
- Similar effect is seen for the conductance in graphene
- Strong dependence on the interface of the photonic crystal
- The 1/L scaling for the transmitted power is independent of the details of the interface

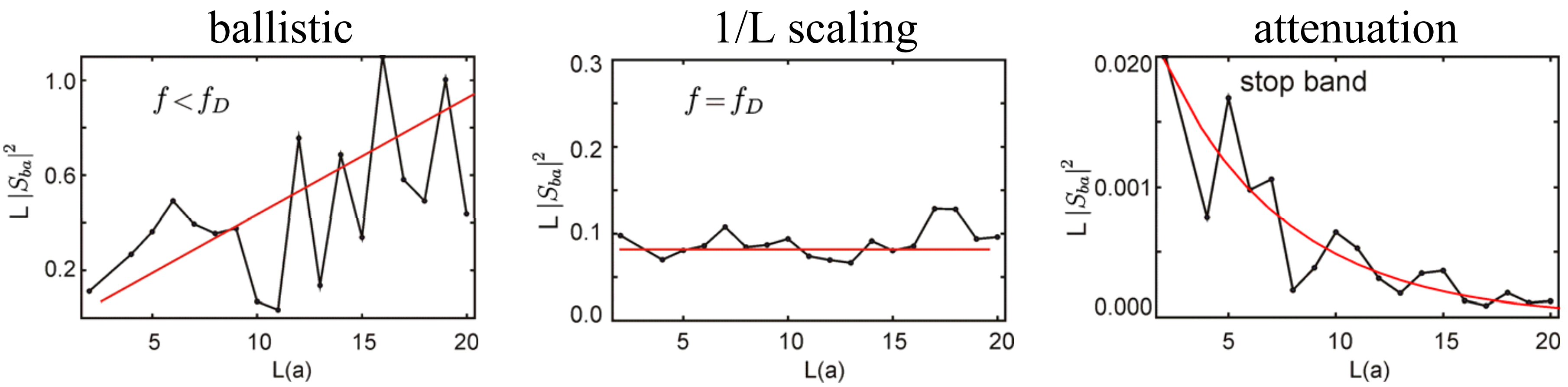
Transmission Spectra through Photonic Crystals in ΓK Direction



- Transmission minimum at the Dirac frequency

Extremal Transmission at the Dirac Point

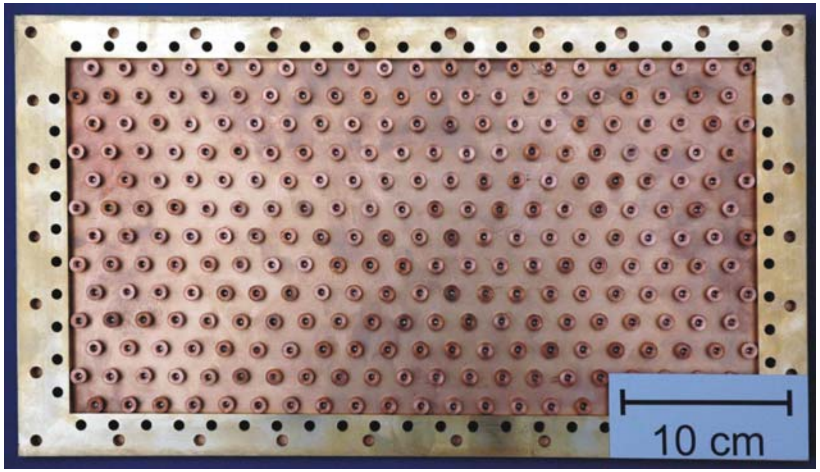
- Thickness L of the photonic crystal varies from 4 to 40 layers



- Ballistic transport at the transmission bands
- Extremal transmission at the Dirac frequency
- Exponential attenuation at the stop band

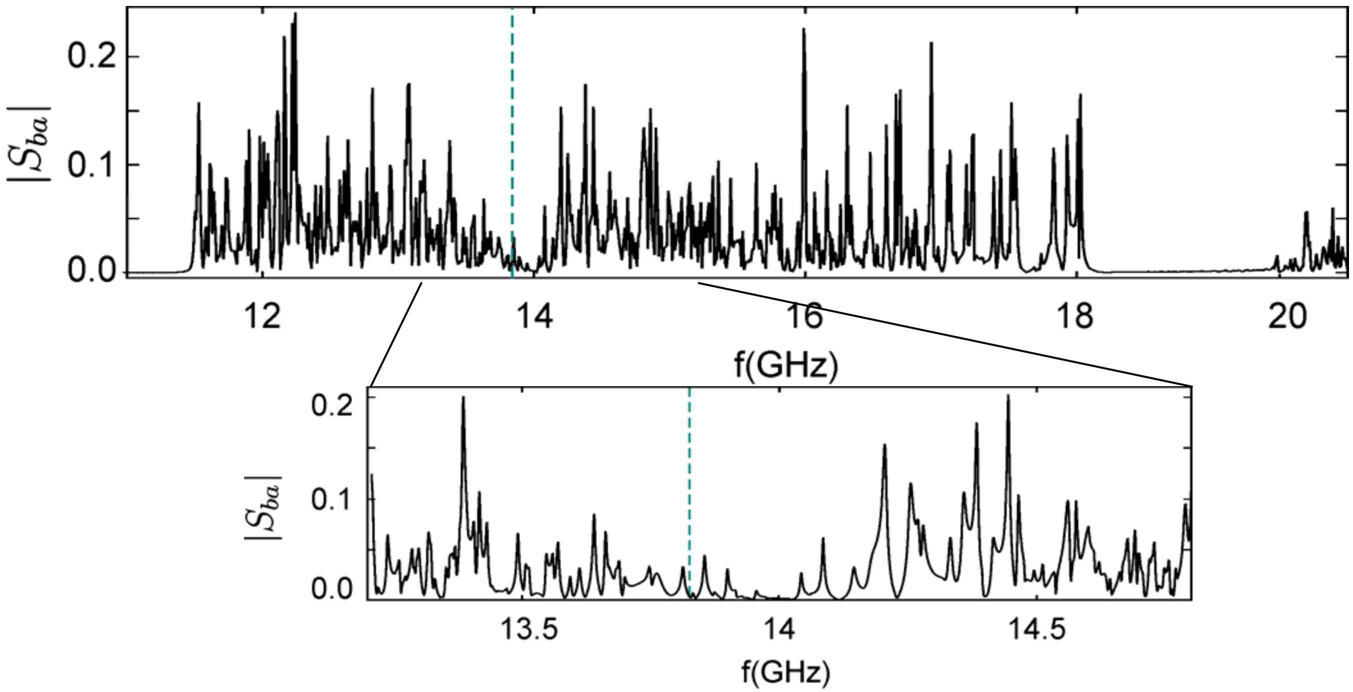
Neutrino Billiard

- Relativistic massless spin-half particles in a billiard
  M.V. Berry and R.J. Mondragon (1987)
- Energy spectrum is symmetric with respect to zero (Dirac) energy
- GUE statistics predicted for the energy levels



- Can we simulate the neutrino billiard with a photonic crystal in a metal box?

Billiard Spectrum



- Transmission between two wire antennae inside the billiard
- Spectrum is not symmetric with respect to the Dirac frequency
- Overlapping resonances causes missing levels
  → superconducting measurements needed

Measurement of Electric Field Intensity

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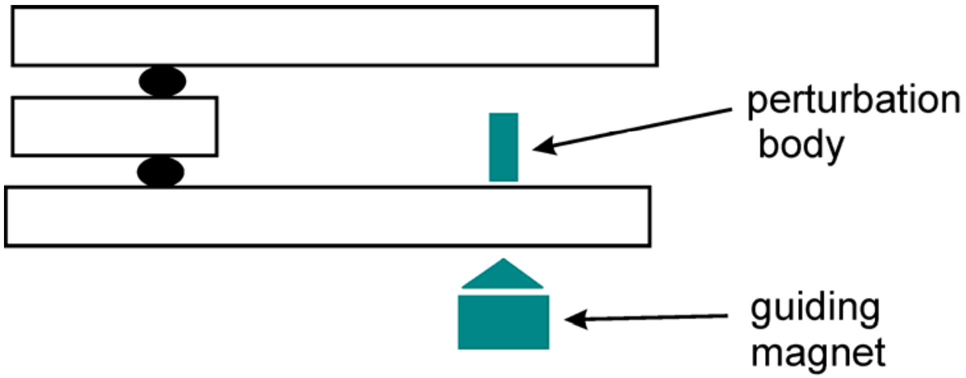
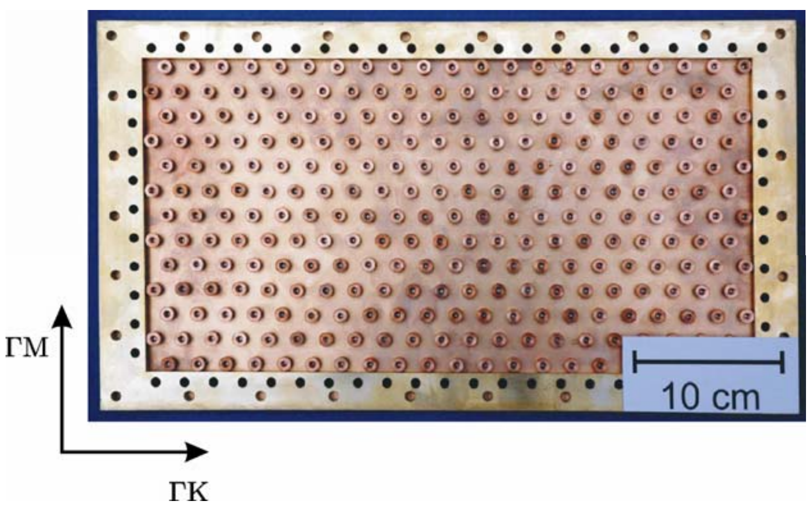


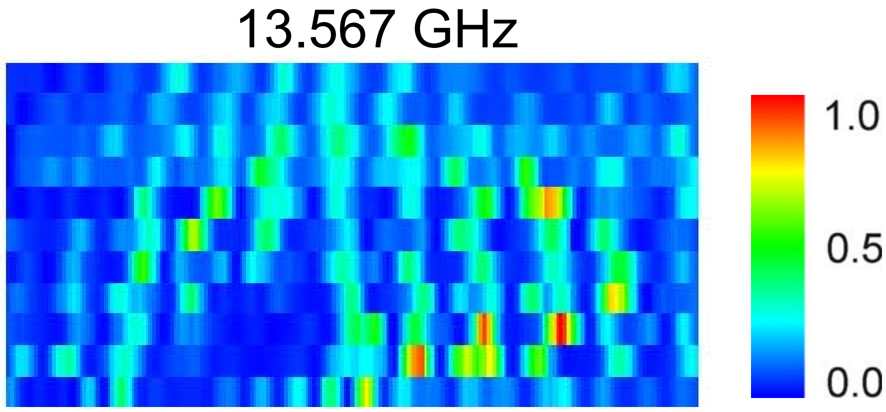
- Resonance frequency shift is related to the electric field strength

  Maier and Slater(1952)

$$\delta f(x,y) = c_1 \cdot E^2(x,y)$$

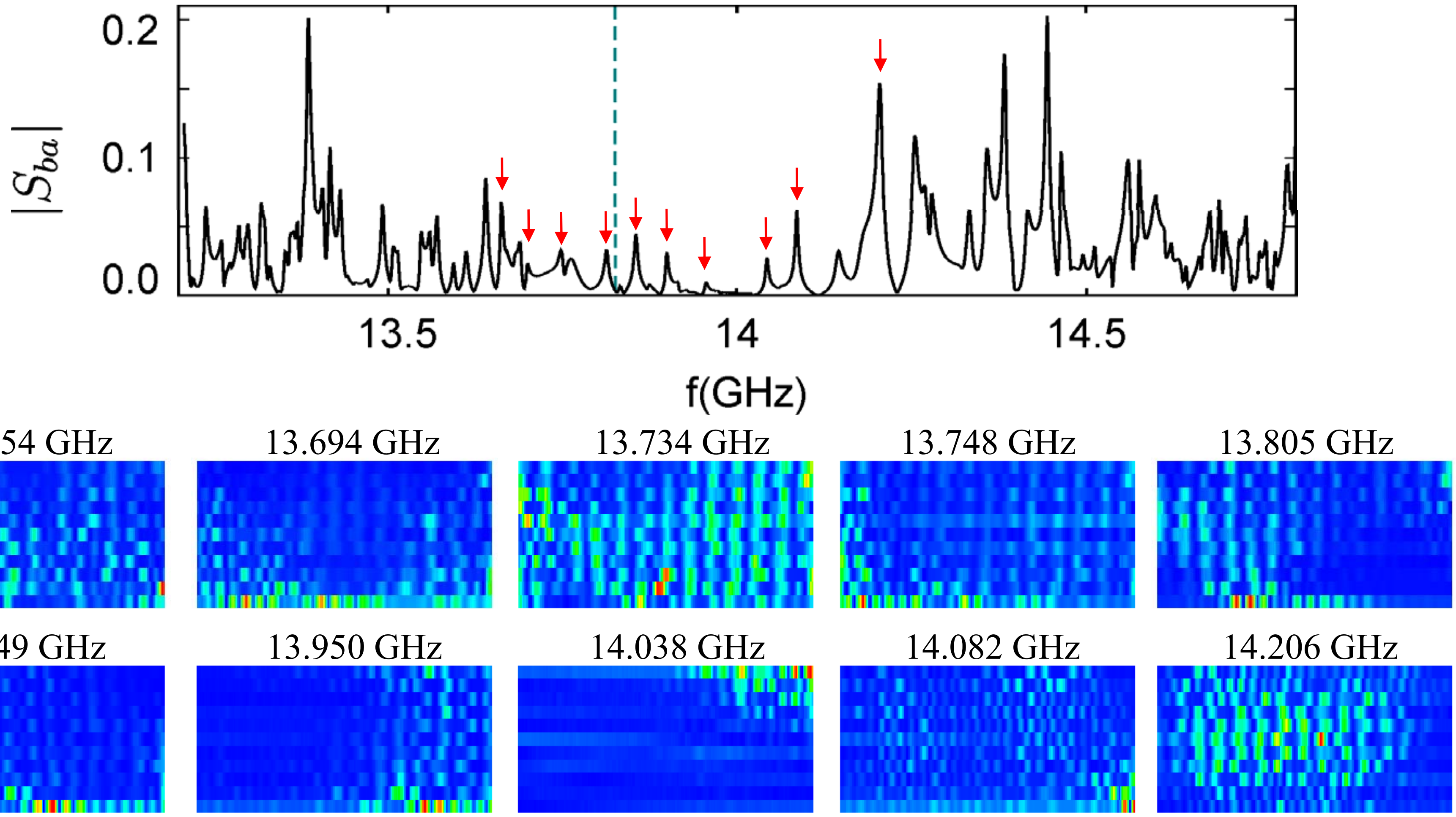
Measured Intensity Distribution



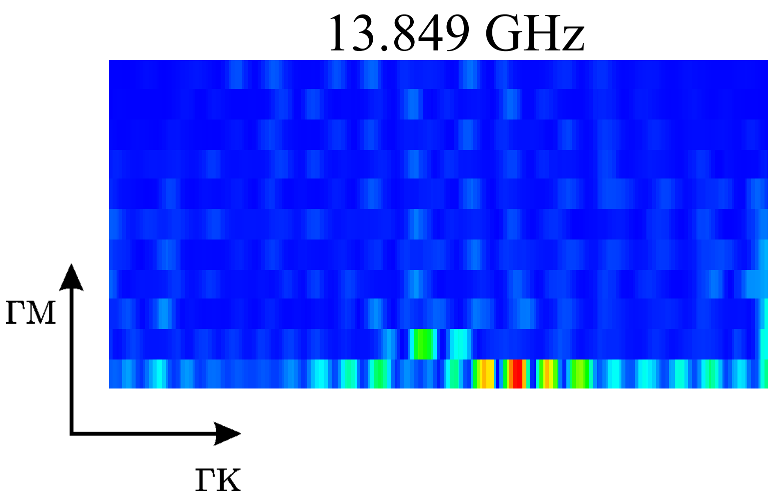


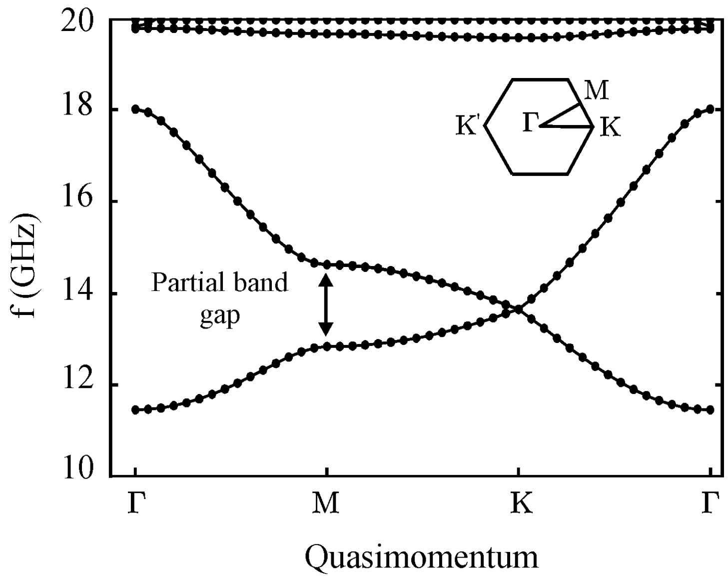
- Perturbation body moved between cylinder rows
- 5 hours for 180 intensity distributions

Measured Electric Field Intensity Distribution



Edge States





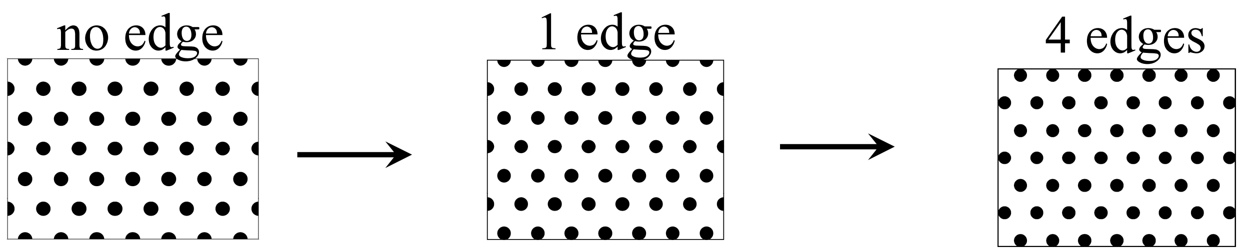
- No propagation in ΓM direction due to partial band gap
- Edge along ΓK direction corresponds to zigzag edge in graphene
- Edge along ΓM direction corresponds to armchair edge in graphene
- Analog to edge states in graphene nanoribbons

Summary

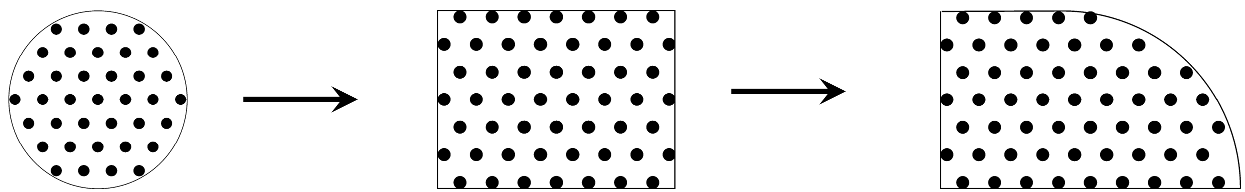
- Connection between the reflection spectra and LDOS is established
- Cusp structure in the reflection spectra is identified with the Dirac point
- Photonic crystal simulates one particle properties of graphene
- Experimental observation of extremal transmission
- Observation of edge states in the neutrino billiard

Outlook

- Billiards family for the study of the edge states



- Superconducting billiards for study of the spectral properties



- Simulation of many body effects with RF nonlinear materials