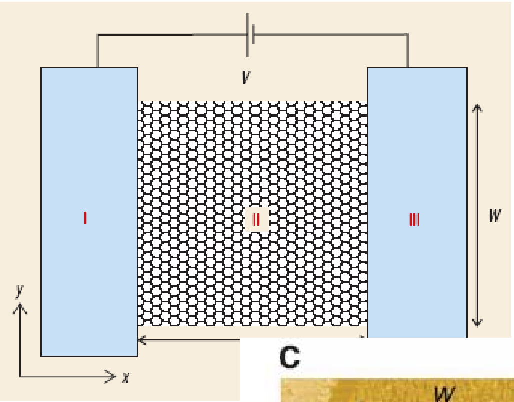
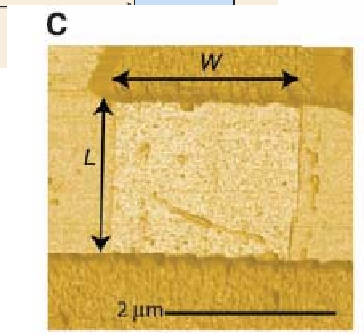
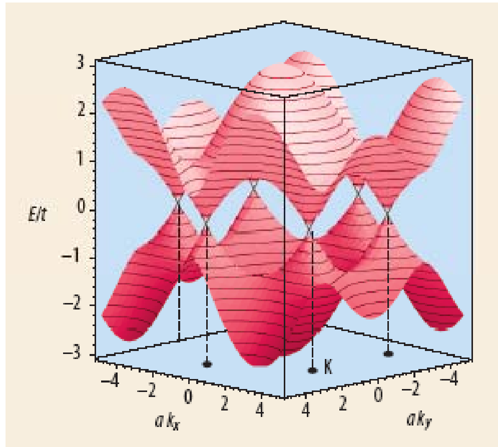
Peculiarities of ballistic transport in graphene





ECT* workshop
New frontiers in graphene physics

Trento, Italy
April 2010



Björn Trauzettel

CWJ Beenakker (Leiden)
YM Blanter (Delft)
M Bräuninger (Würzburg)
D Bohr (Basel)
AF Morpurgo (Geneva)
M Müller (Trieste)
P Recher (Würzburg)
A Rycerz (Regensburg)
J Schelter (Würzburg)
M Titov (Edinburgh)
J Tworzydlo (Warsaw)

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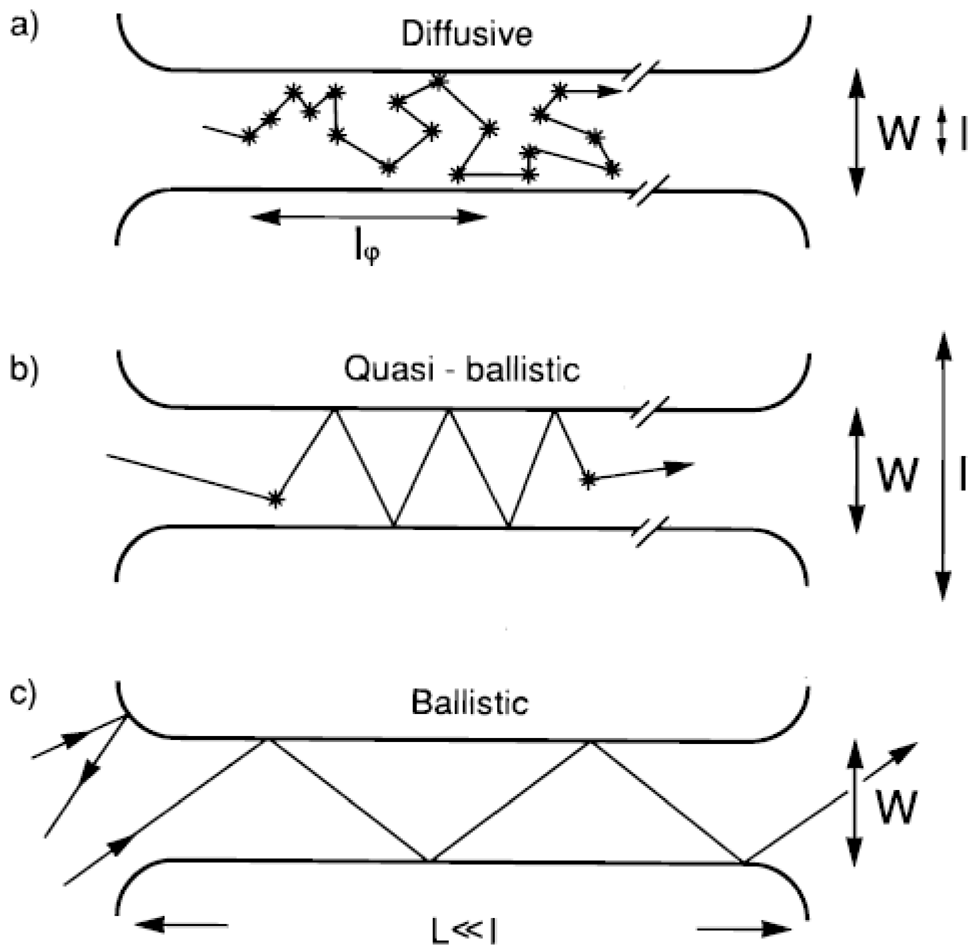
Ballistic transport in a nutshell I



Beenakker & van Houten Solid State Physics 1991

Ballistic transport in a nutshell I

total current

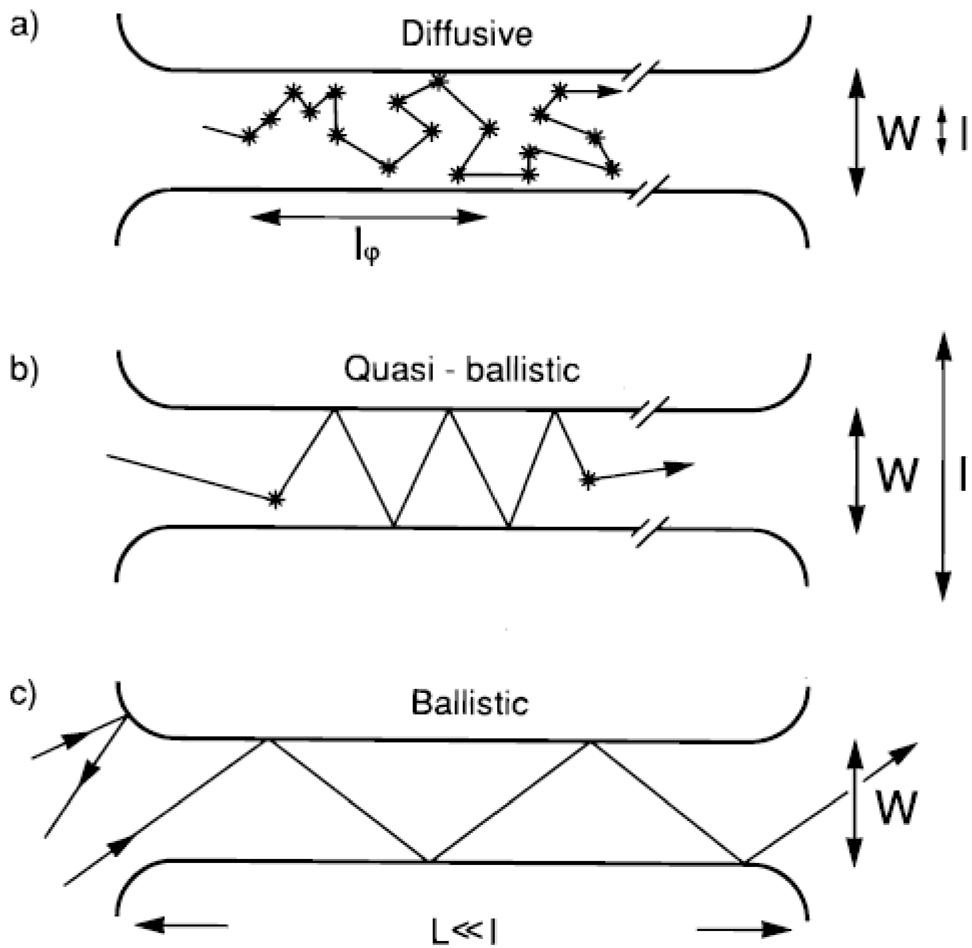
$$I = GV$$

For large homogeneous conductor:

$$G = \sigma \frac{W}{L}$$

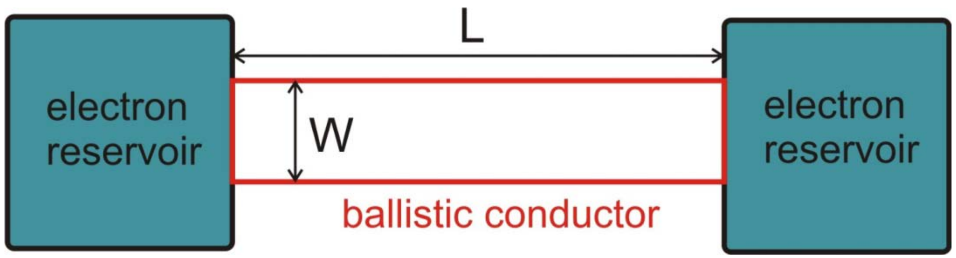
$$j = \sigma E$$

local current density



Beenakker & van Houten Solid State Physics 1991

Ballistic transport in a nutshell II

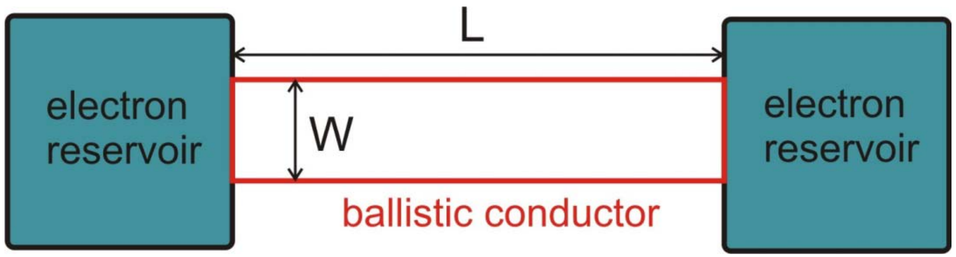


$$G = \sigma \frac{W}{L}$$

What happens in the limit $W/L \to \infty$?
In experiments G saturates to G_c.

$1/G_c$ = contact resistance

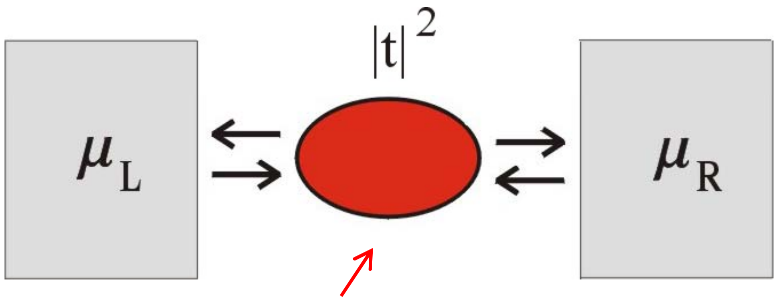
Ballistic transport in a nutshell II



$$G = \sigma \frac{W}{L}$$

What happens in the limit $W/L \to \infty$?
In experiments G saturates to G_c.

How do we describe transport?
Landauer transport theory
→ transport as transmission



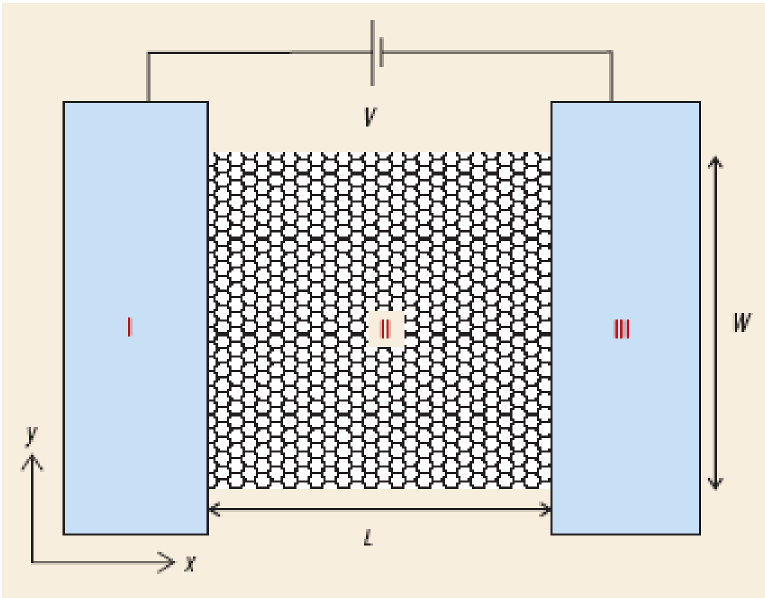
phase-coherent sample ⟶ scattering region
(described by scattering matrix)

Outline

- Transport in graphene as scattering problem
- Conductance/conductivity and shot noise
- Temperature dependence of ballistic transport
- Interplay of Aharonov-Bohm effect and Klein tunneling in graphene ring structures
- Summary and outlook

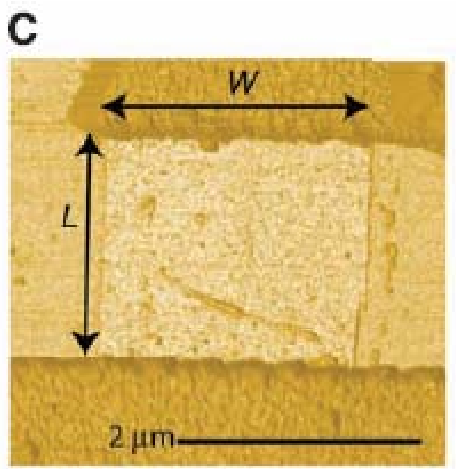
Setup for ballistic transport

theoretical point of view:



Tworzydlo, BT, Titov, Rycerz & Beenakker PRL 2006

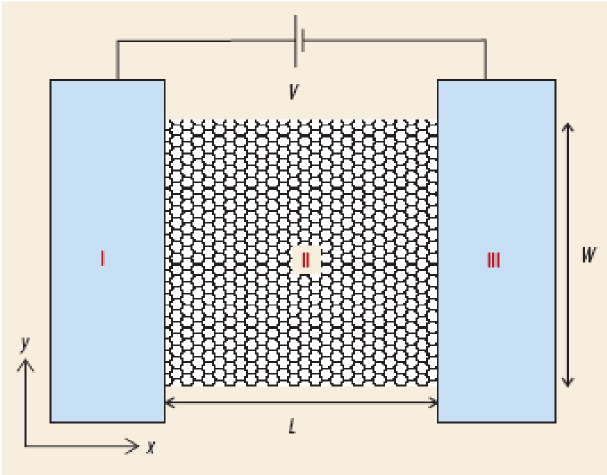
experimental point of view:

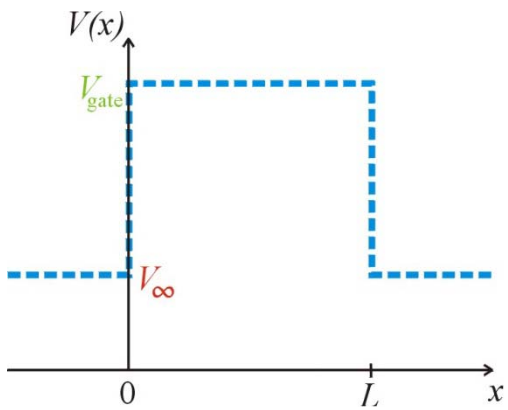


Miao et al Science 2007

related (early) theoretical work: *Peres, Castro Neto & Guinea* PRB 2006; *Katsnelson* EPJB 2006

How do we model it?





$\longrightarrow$ different boundary conditions in y-direction

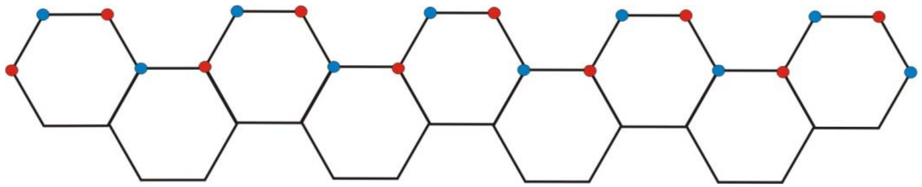
$\longrightarrow$ voltage source drives current through strip

$\longrightarrow$ gate electrode changes carrier concentration

Boundary conditions

(i) armchair edge

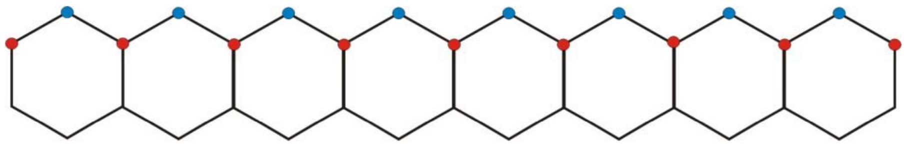
(mixes the two valleys;
metallic or semi-conducting)



Brey & Fertig PRB 2006

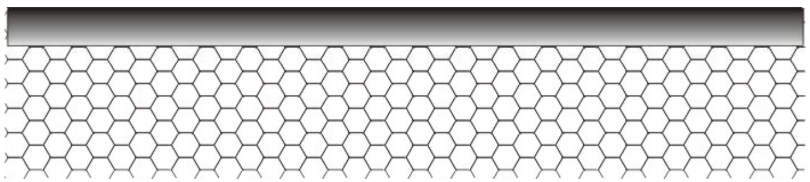
(ii) zigzag edge

(one valley physics;
couples k_x and k_y)



(iii) infinite mass confinement

(one valley physics;
smooth on scale of lattice spacing)



Berry & Mondragon Proc. R. Soc. Lond. 1987

Underlying wave equation

$$\left[vp_x\sigma_x + vp_y\sigma_y + v^2 M(y)\sigma_z + \mu(x) \right]\Psi = \varepsilon\Psi$$

kinetic term

gate voltage term

boundary term (infinite mass confinement)

Underlying wave equation

$$\left[v p_x \sigma_x + v p_y \sigma_y + v^2 M(y) \sigma_z + \mu(x) \right] \Psi = \varepsilon \Psi$$

kinetic term

gate voltage term

boundary term (infinite mass confinement)

<u>Ansatz:</u>

$$\Psi_{n,k}(\vec{r}) = \left(a_n \begin{pmatrix} 1 \\ z_{n,k} \end{pmatrix} e^{iq_n y} + b_n \begin{pmatrix} z_{n,k} \\ 1 \end{pmatrix} e^{-iq_n y} \right) e^{ikx}$$

$$z_{n,k} = \frac{k + iq_n}{\sqrt{k^2 + q_n^2}}$$

in leads:

$$\varepsilon = \mu_{lead} + \hbar v \sqrt{k^2 + q_n^2}$$

in graphene:

$$\varepsilon = \mu_{gate} + \hbar v \sqrt{\tilde{k}^2 + q_n^2}$$

Scattering state ansatz

$$\Psi = \begin{cases} X_{n,k}(y)e^{ikx} + r_n X_{n,-k}(y)e^{-ikx}; x < 0 \\ \alpha_n X_{n,\tilde{k}}(y)e^{i\tilde{k}x} + \beta_n X_{n,\tilde{k}}(y)e^{-i\tilde{k}x}; 0 < x < L \\ t_n X_{n,k}(y)e^{ik(x-L)}; x > L \end{cases}$$

Dirac equation (first order differential equation)

$\Longrightarrow$ continuity of wave function at x=0 and x=L

$\Longrightarrow$ determines t_n and r_n

$\Longrightarrow$ transmission $T_n=|t_n|^2$

Solution of transport problem

In the limit N≫1 (propagating modes in leads):

$$T_n(E) = \frac{E^2 - (\hbar v q_n)^2}{E^2 - (\hbar v q_n)^2 \cos^2(k_n L)}$$

$$\hbar v k_n \equiv \sqrt{E^2 - (\hbar v q_n)^2}$$

Transmission coefficient (at Dirac point E=0):

$$T_n = \frac{1}{\cosh^2[(n+\alpha)\pi L/W]}$$

phase α depends on boundary conditions

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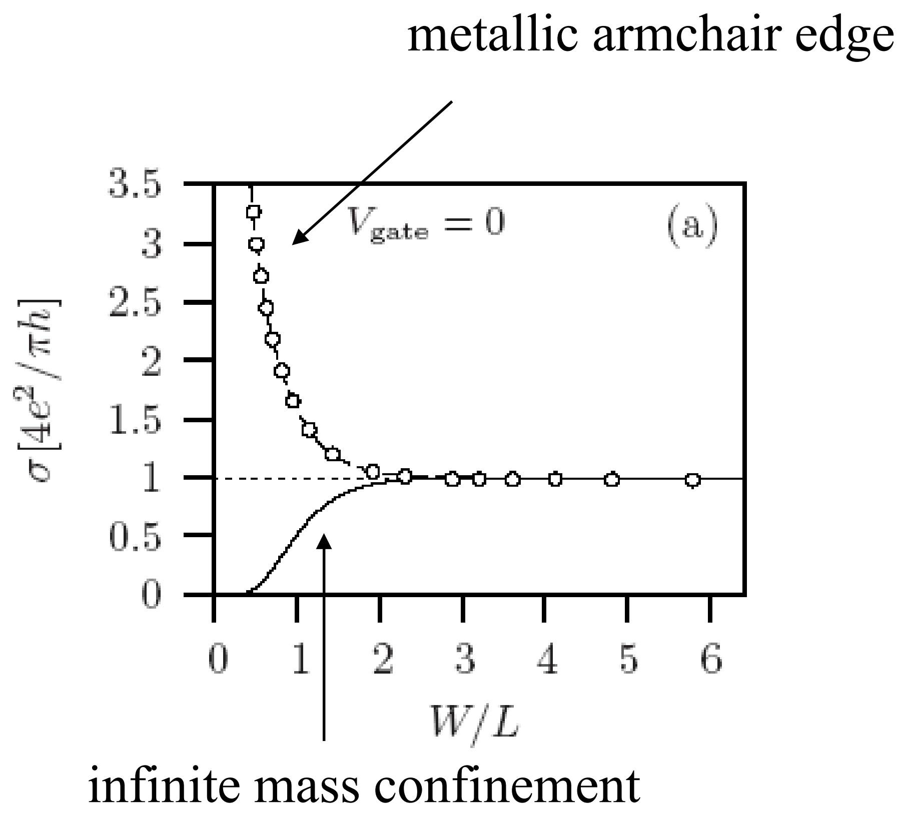
Conductivity: influence of b.c.

Landauer formula:

$$G = \frac{4e^2}{h} \sum_{n=0}^{N-1} T_n$$

conductivity:

$$\sigma = \frac{L}{W} \frac{4e^2}{h} \sum_{n=0}^{N-1} T_n$$



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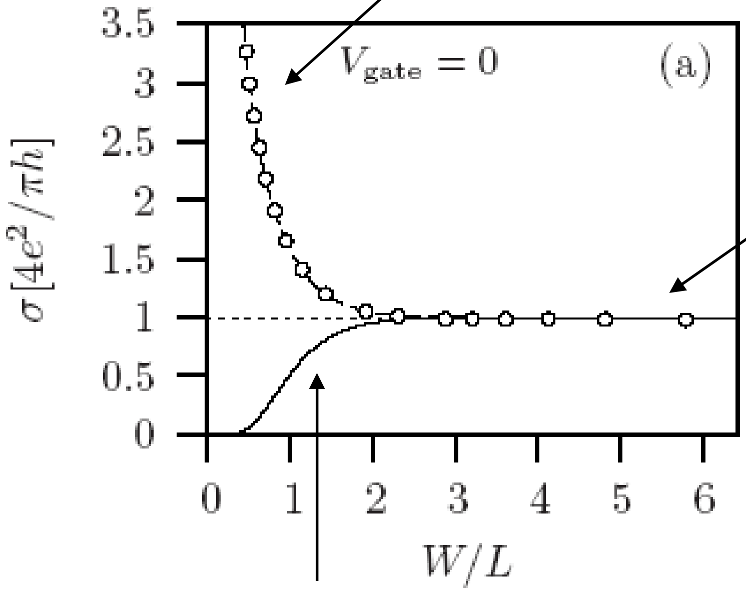
conductivity:

$$\sigma = \frac{L}{W} \frac{4e^2}{h} \sum_{n=0}^{N-1} T_n$$

metallic armchair edge



universal limit:
W/L ≫ 1

infinite mass confinement

at Dirac point (in universal regime): conductance proportional to 1/L

Transport through evanescent modes

$$\varepsilon = \hbar v_F \sqrt{k_x^2 + k_y^2}$$

$$k_y = q_n \in \mathbb{R}$$

at the Dirac point: $$\varepsilon = 0 \Rightarrow k_x = k = iq_n$$

Transport through evanescent modes

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$$k_y = q_n \in \mathbb{R}$$

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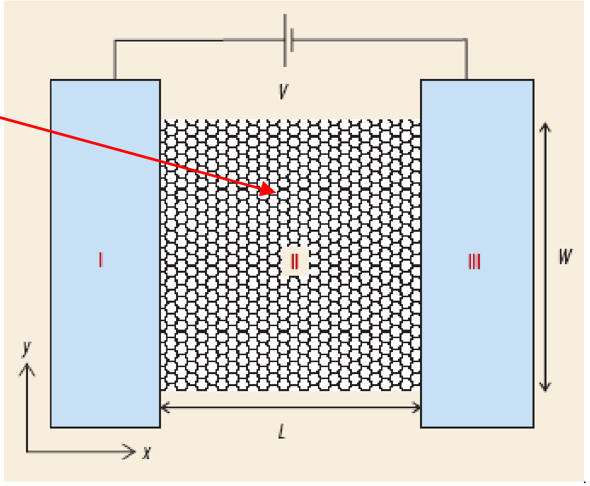
structure of wave function in II

$$\alpha_n X_{n,k}(y) e^{ikx} + \beta_n X_{n,-k}(y) e^{-ikx}$$



Transport through evanescent modes

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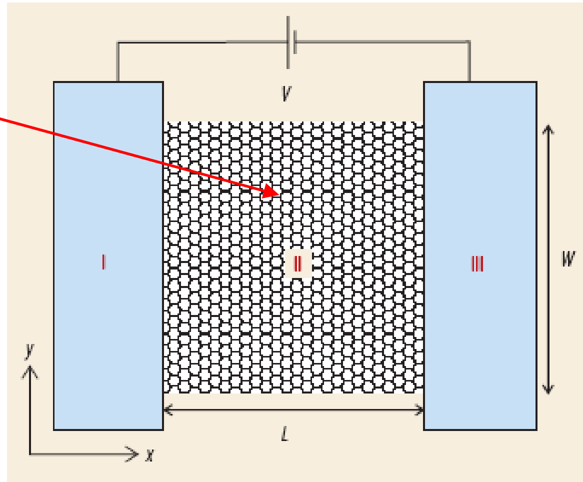
$$\alpha_n \mathrm{X}_{n,k}(y) e^{ikx} + \beta_n \mathrm{X}_{n,-k}(y) e^{-ikx}$$

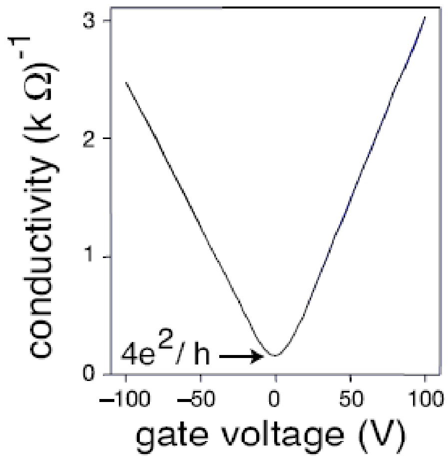


$\Longrightarrow$ Transport through evanescent modes:

$$\alpha_n \mathrm{X}_{n,iq_n}(y) e^{-q_n x} + \beta_n \mathrm{X}_{n,-iq_n}(y) e^{q_n x}$$

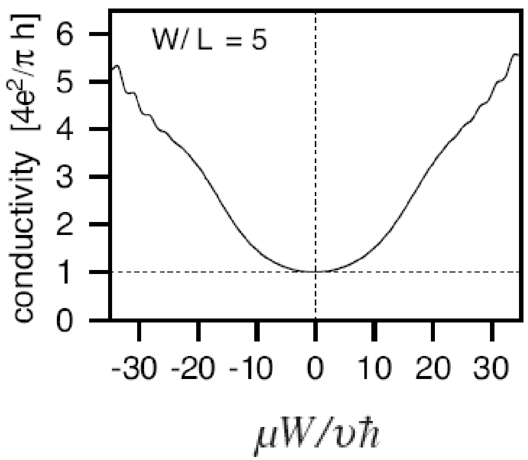
Conductivity: V_{gate} dependence

Experiment:



Novoselov et al Nature 2005

Our theory:



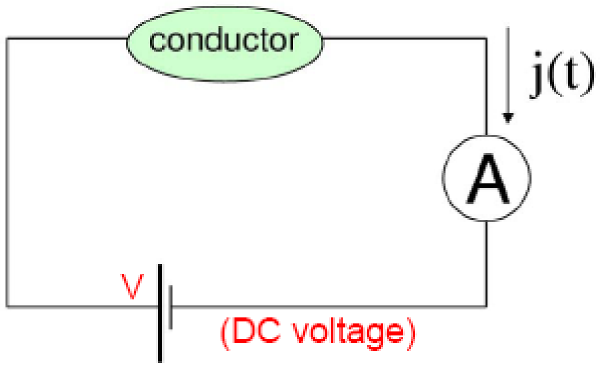
Tworzydlo et al PRL 2006

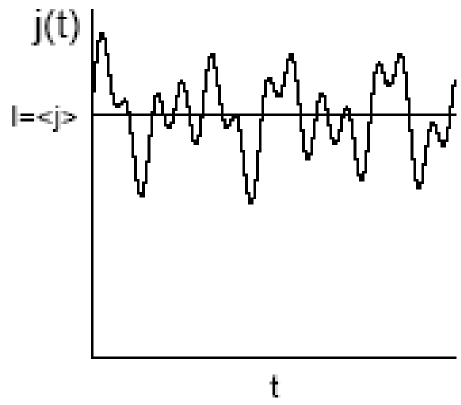
Possible explanations:

charged Coulomb impurities *Nomura & MacDonald* PRL 2007
strong (unitary) scatterers *Ostrovsky, Gornyi & Mirlin* PRB 2006

$$\nu(E) \propto E \;\Rightarrow\; n_e \propto \mu^2$$
$$en_e = CV_{gate} \;\Rightarrow\; \mu \propto \sqrt{V_{gate}}$$

Current noise





Average current: $I = \langle j(t) \rangle$

Current fluctuations: $S(\omega) = \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \{ \Delta j(t), \Delta j(0) \}_+ \rangle$

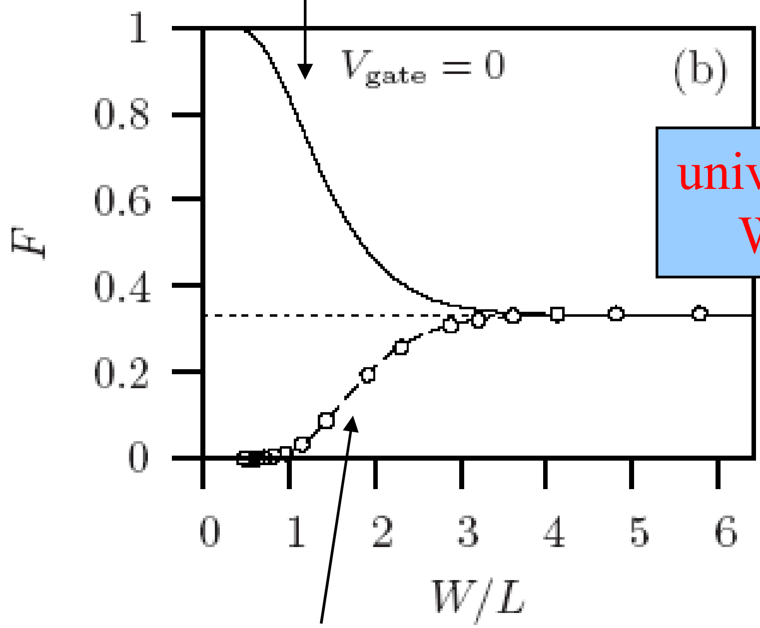
We are interested in the zero frequency and zero temperature limit. $\longrightarrow$ **shot noise**

Shot noise: effect of b.c.

Fano factor:

$$F = \frac{S}{2eI} = \frac{\sum_{n=0}^{N-1} T_n (1 - T_n)}{\sum_{n=0}^{N-1} T_n}$$

infinite mass confinement

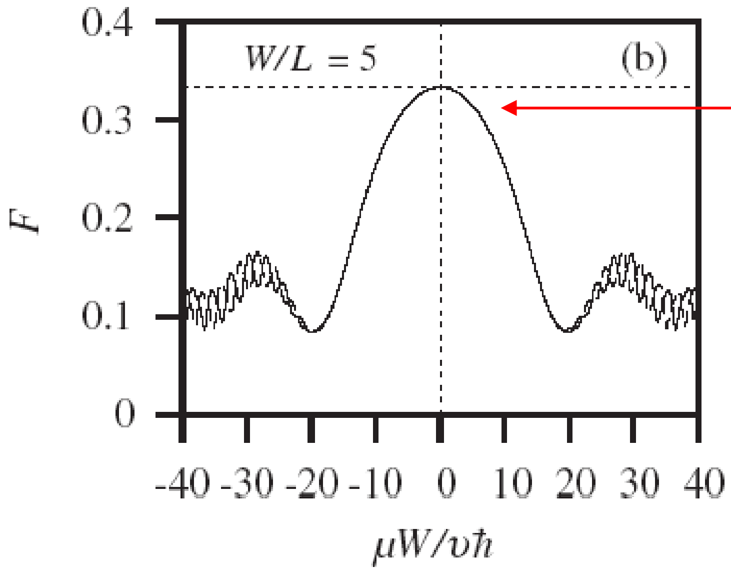


universal limit: W/L ≫ 1

metallic armchair edge

Tworzydlo, BT, Titov, Rycerz & Beenakker, PRL 2006

Maximum Fano factor



unaffected by different boundary conditions & scaling system size to infinity

$\Longrightarrow$ sub-Poissonian noise $\longrightarrow$ particles anti-bunch

$\Longrightarrow$ universal Fano factor 1/3 for W/L $\gg$ 1

same Fano factor as for disordered quantum wire
Beenakker & Büttiker, PRB 1992; *Nagaev*, Phys. Lett. A 1992

PRL 100, 196802 (2008) PHYSICAL REVIEW LETTERS week ending 16 MAY 2008

Shot Noise in Ballistic Graphene

R. Danneau,[1,*] F. Wu,[1] M. F. Craciun,[2] S. Russo,[2] M. Y. Tomi,[1] J. Salmilehto,[1] A. F. Morpurgo,[2] and P. J. Hakonen[1]
[1]*Low Temperature Laboratory, Helsinki University of Technology, Espoo, Finland*
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(Received 27 November 2007; published 13 May 2008)

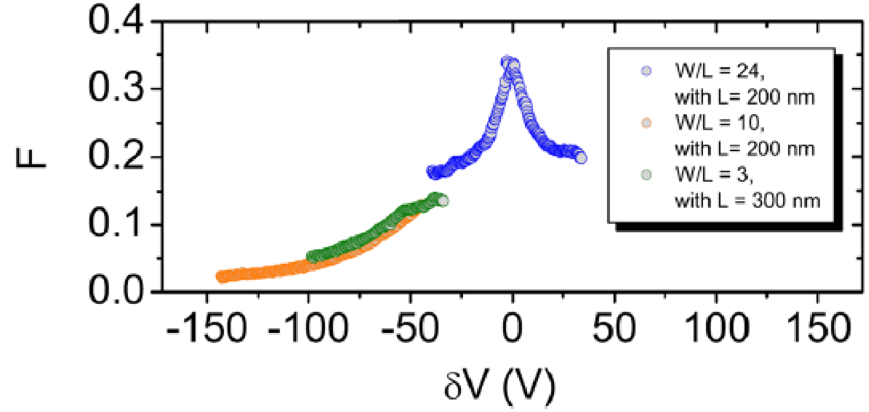

FIG. 3 (color online). F extracted at $V_{bias} = 40$ mV for three different samples, all having $W/L \geq 3$, as a function of $\delta V = V_{gate} - V_{Dirac}$. For the two unintentionally highly p-doped samples (orange and green dots), the Dirac point was estimated via extrapolation of the minimum conductivity at $\frac{4e^2}{\pi h}$.

Outline

- Transport in graphene as scattering problem
- Conductance/conductivity and shot noise
- Temperature dependence of ballistic transport
- Interplay of Aharonov-Bohm effect and Klein tunneling in graphene ring structures
- Summary and outlook

Motivation

Selected for a Viewpoint in *Physics*
PRL **101,** 096802 (2008) PHYSICAL REVIEW LETTERS week ending 29 AUGUST 2008

Temperature-Dependent Transport in Suspended Graphene

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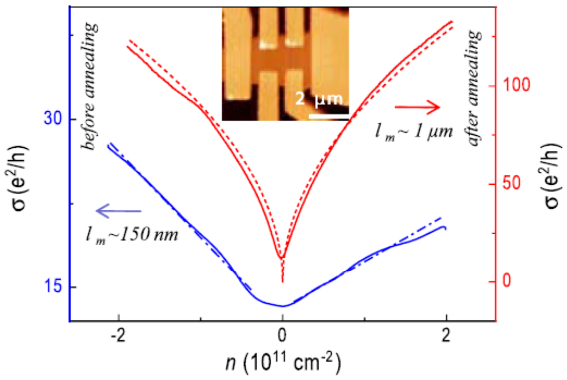
(Received 23 April 2008; published 25 August 2008)



LETTERS

Approaching ballistic transport in suspended graphene

XU DU, IVAN SKACHKO, ANTHONY BARKER AND EVA Y. ANDREI*
Department of Physics and Astronomy, Rutgers the State University of New Jersey, 136 Frelinghuysen Rd, Piscataway, New Jersey 08854, USA
*e-mail: eandrei@physics.rutgers.edu

Finite temperature calculation

Landauer formula:

$$I = \frac{4e^2}{h} \sum_{n=0}^{N-1} \int T_n(E)\left(f_L(E) - f_R(E) \right)$$

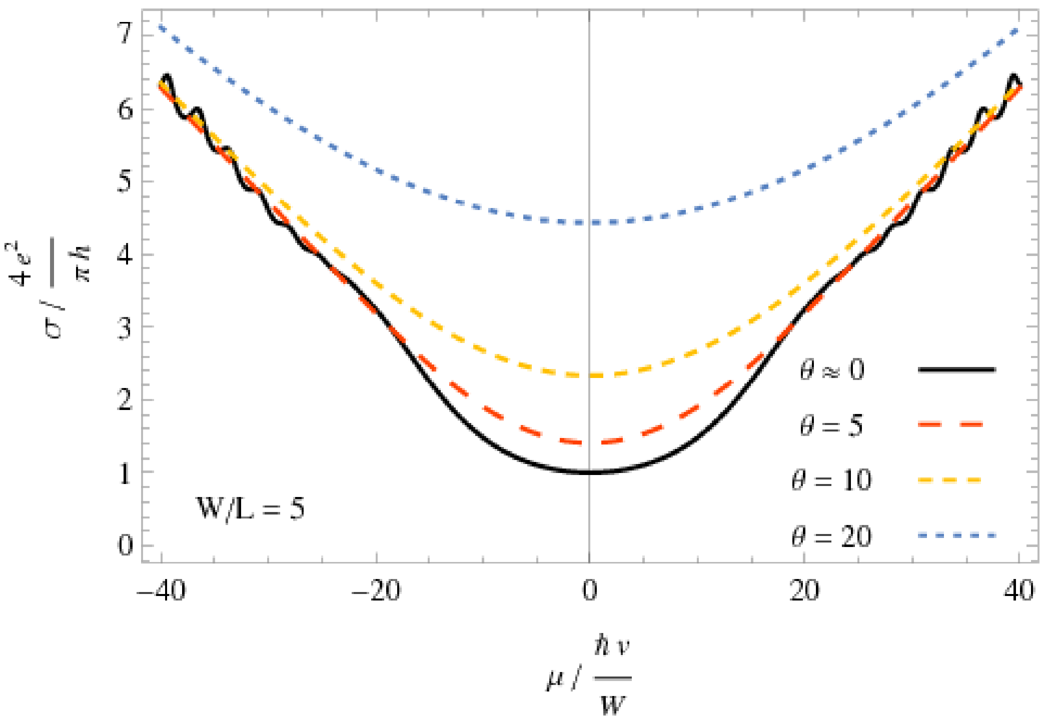
$$f_{L/R}(E) = \frac{1}{1 + \exp\left(\beta \left(E - \mu_{L/R} \right) \right)} \qquad \mu_{L/R} = \mu \pm \frac{eV}{2}$$

Conductivity:

$$\sigma(\mu, T) = \int \sigma_0(E) \frac{f(E)\left(1 - f(E)\right)}{T}$$

$$\sigma_0(E) = \frac{L}{W} G(E) = \frac{L}{W} \frac{4e^2}{h} \sum_n T_n(E) \qquad f(E) = \frac{1}{1 + \exp\left(\beta \left(E - \mu \right) \right)}$$

Temperature dependence: numerics



Approximation scheme
→ analytical results

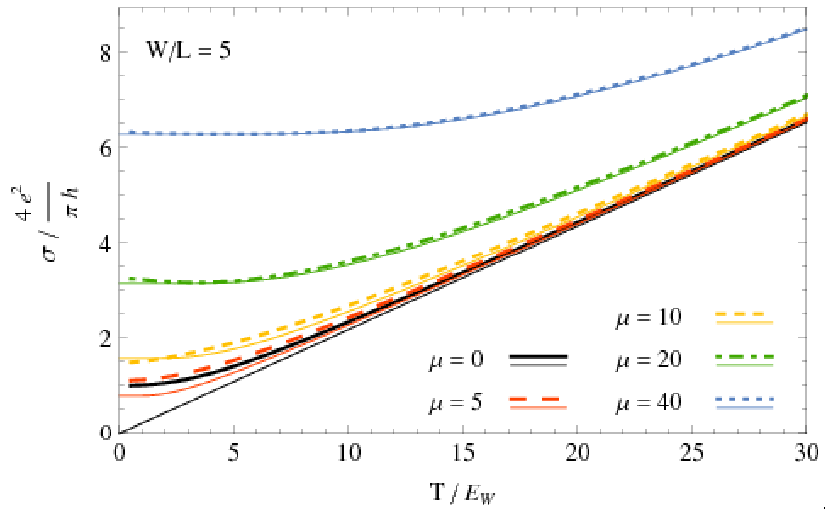
restriction to propagating modes of sample

$$\sigma_0(E) \approx \frac{4L}{\pi}\frac{e^2}{h}\int_0^{E/\hbar v} dq \int_0^{2\pi} \frac{d\varphi}{2\pi} \frac{E^2 - (\hbar v q)^2}{E^2 - (\hbar v q)^2 \cos^2\varphi} = \frac{e^2}{h}\frac{LE}{\hbar v}$$

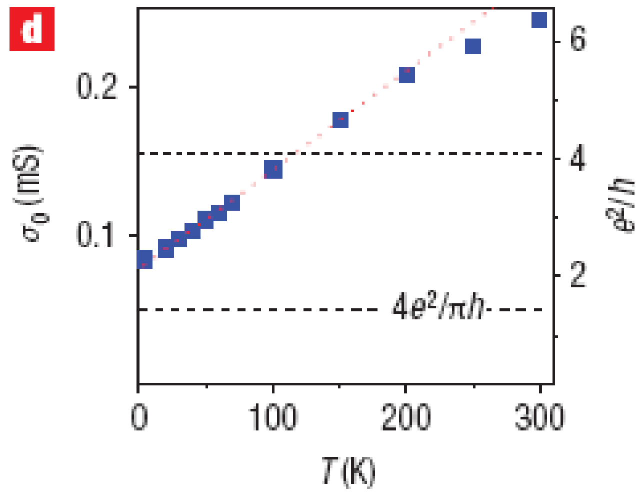
average over discreteness effects
(due to finite size quantization)

$$\Rightarrow \quad \sigma(\mu, T) \approx \frac{e^2}{h}\frac{LT}{\hbar v}\left[\frac{|\mu|}{T} + 2\log\left(1 + e^{-|\mu|/T}\right)\right]$$

Temperature dependence: minimum conductivity



Müller, Bräuninger & BT PRL 2009

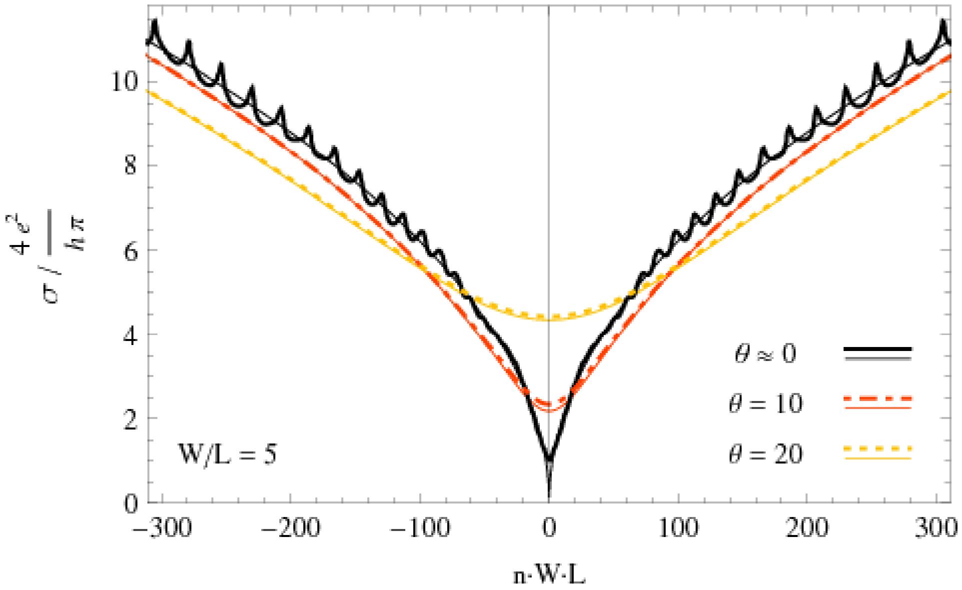


Du, Skachko, Barker & Andrei Nature Nano. 2008

alternative explanation: scattering from charged impurities with T-dependent screening

Hwang & DasSarma PRB 2009

Density dependence: numerics



$$n = n_e - n_h$$

$$n_e = \frac{1}{WL}\sum_{n=0}^{N-1}\int_0^\infty dE\,\nu_{1D,e}^{(n)}(E)\,f(E)$$

$$\nu_{1D,e}^{(n)}(E) = \frac{4L}{\hbar v_F \pi}\frac{E}{\sqrt{E^2-(\hbar v_F q_n)^2}}\Theta(E-\hbar v_F q_n)$$

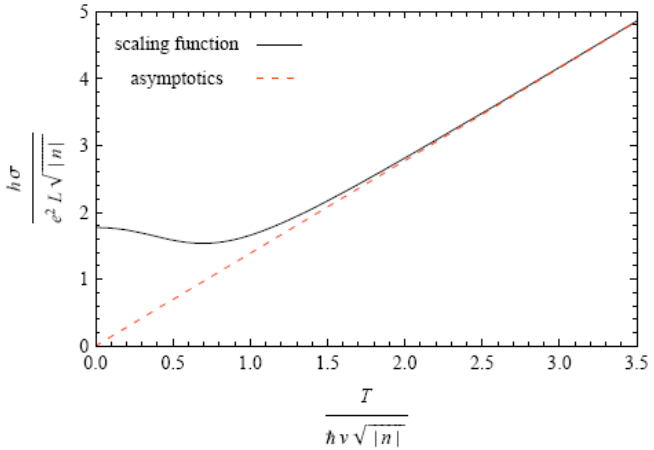
Scaling function

scaling function for chemical potential $\mu(T,n) \equiv \tilde{\mu} T$

$$\frac{\pi}{2}\frac{(\hbar v)^2 n}{T^2} = \int_0^\infty dx\, x\left[f(x,\tilde{\mu}) - f(x,-\tilde{\mu})\right] \quad \Rightarrow \quad \tilde{\mu} = \zeta\left(\frac{T}{\hbar v |n|^{1/2}}\right)$$

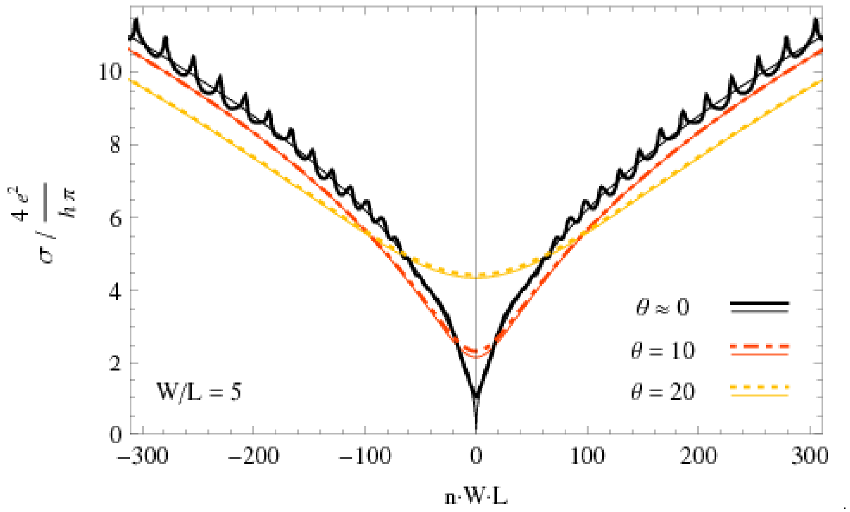
$$f(x,\tilde{\mu}) = \frac{1}{1+e^{x-\tilde{\mu}}}$$

$$\frac{h}{e^2}\frac{\sigma(n,T)}{L|n|^{1/2}} \approx \frac{T}{\hbar v |n|^{1/2}}\left(\tilde{\mu} + 2\log\left[1+e^{-\tilde{\mu}}\right]\right)$$

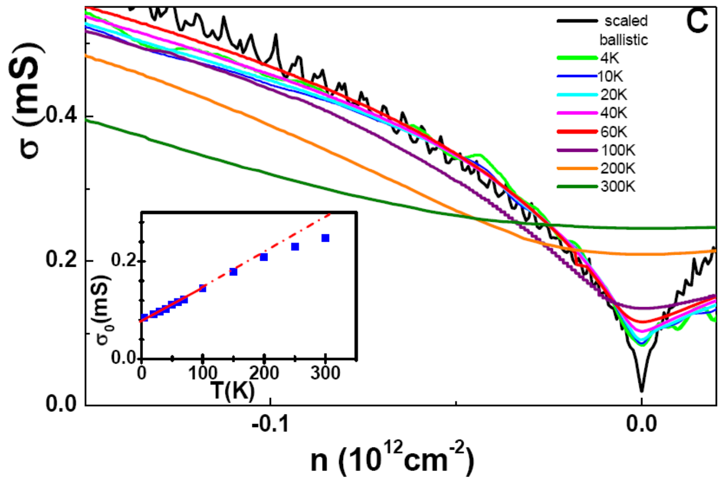


Conductivity: non-monotonic function of T and n; minimum at $T \approx 0.7\hbar v |n|^{1/2}$

Comparison with experiment



Müller, Bräuninger & BT PRL 2009



Du, Skachko, Barker & Andrei Nature Nano. 2008

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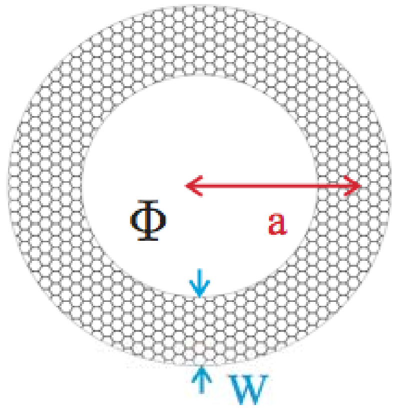
Closed graphene ring

breaks TRS

$$H_{\tau=\pm} = v\left(\mathbf{p} + e\mathbf{A}\right)\cdot\boldsymbol{\sigma} + \tau V\left(r\right)\sigma_z$$

breaks symplectic symmetry
(TRS within a single valley)



Recher, BT, et al PRB 2007

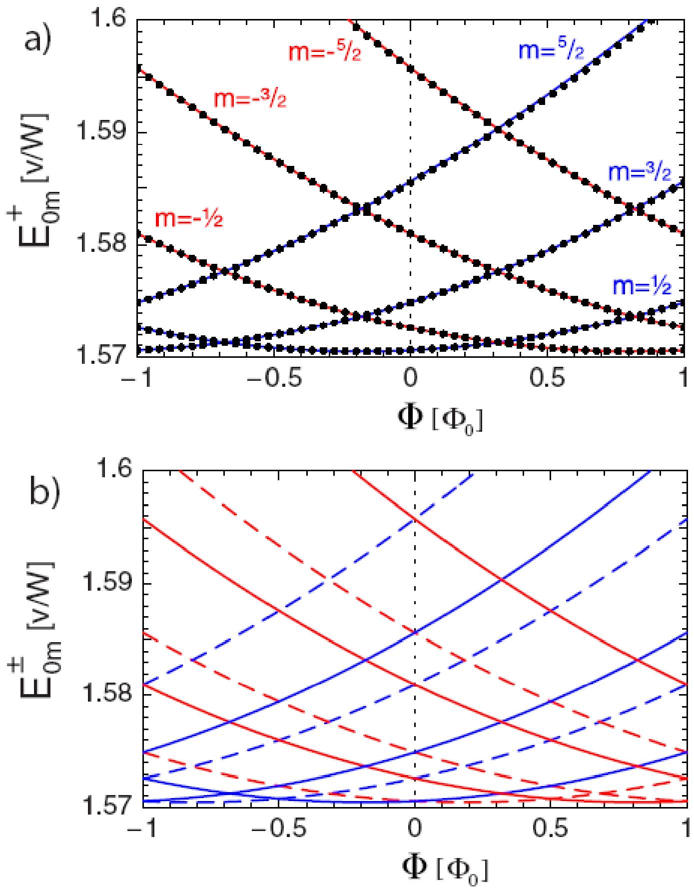
Infinite mass confinement: V(r)→∞ outside the ring $\psi_B^\tau = \mp i\tau e^{i\varphi}\psi_A^\tau \text{ at } r = a \mp \frac{W}{2}$

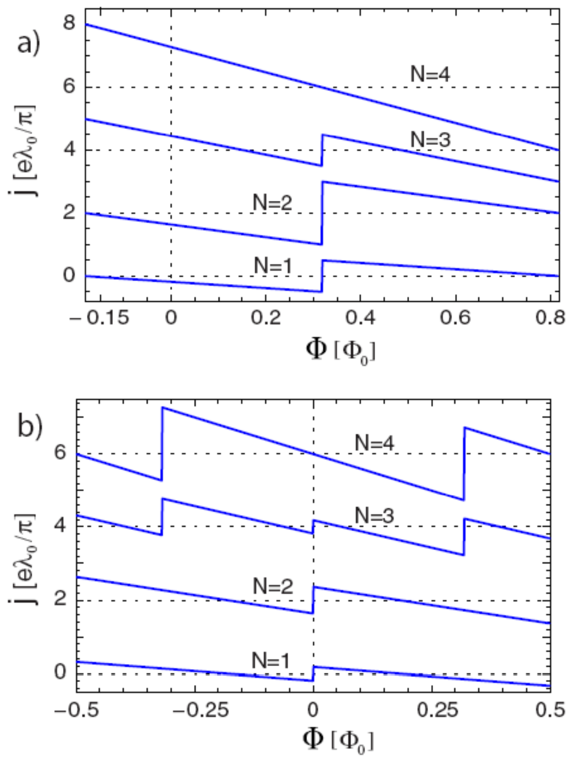
For W/a≪1: $$E_{nm}^{\tau} = \pm\varepsilon_n \pm \frac{v^2}{2a^2\varepsilon_n}\bar{m}\left(\bar{m} \mp \frac{\tau}{\left(n+\frac{1}{2}\right)\pi}\right)$$

$$\varepsilon_n = v\left(n+\frac{1}{2}\right)\frac{\pi}{W},\ n = 0,1,2,\ldots\ ,\mathrm{m}=\pm\frac{1}{2},\pm\frac{3}{2},\ldots\ ,\bar{\mathrm{m}}=\mathrm{m}+\frac{\Phi}{\Phi_0}$$

closed bilayer ring: Zarenia *et al* Nano Lett. 2009

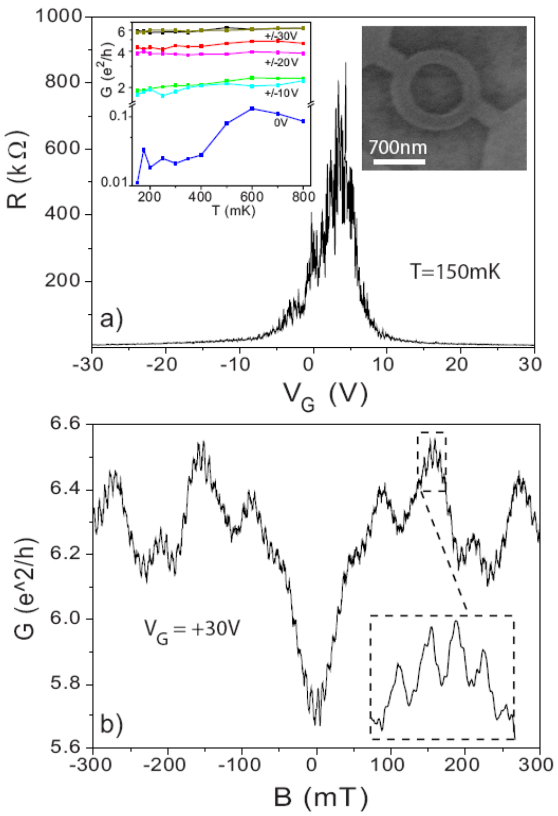
Persistent current at B=0



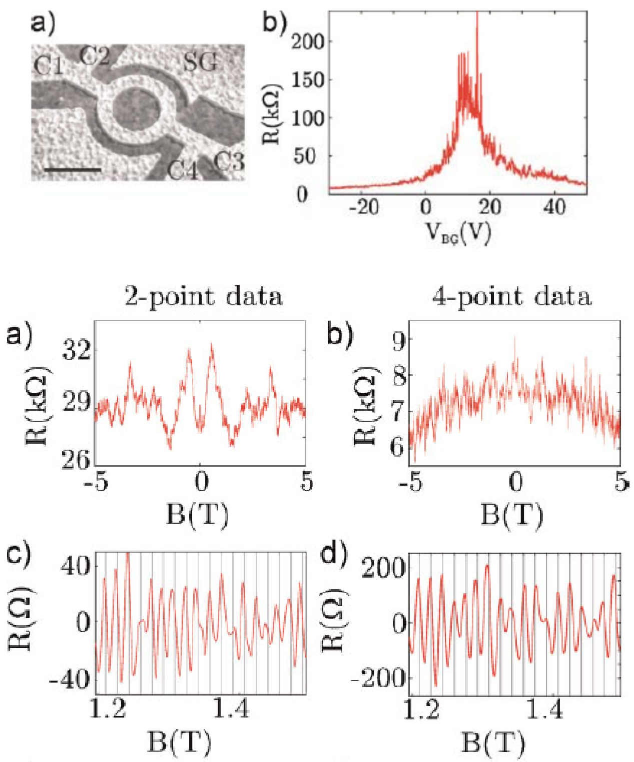


$$j = -\sum_{\tau}\sum_{n,m}\frac{\partial E_{nm}^{\tau}}{\partial \Phi}$$

Experiments on graphene rings



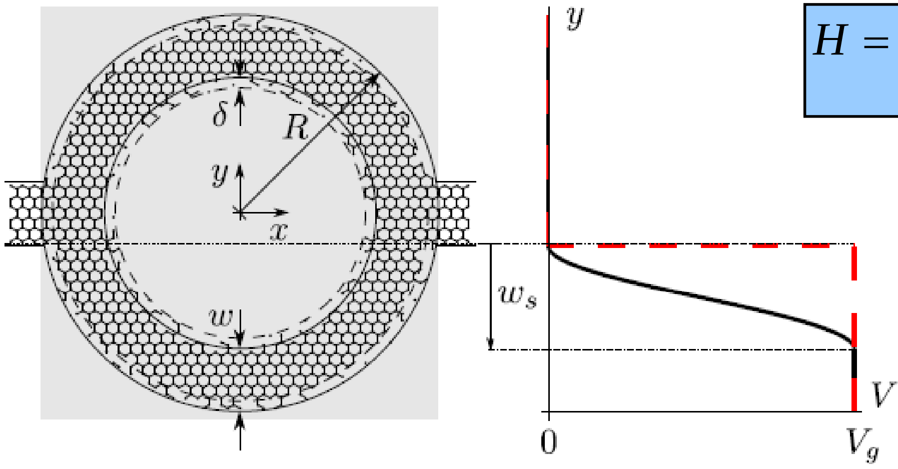
Russo et al PRB 2008



Huefner et al Phys. Stat. Sol. 2009

Conclusion: no graphene-specific experimental results

Open graphene ring

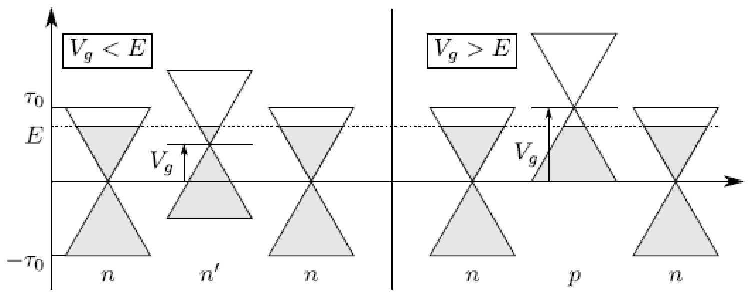


$$H = \sum_i V_i |i\rangle\langle i| + \sum_{\langle i,j\rangle} \tau_{ij} |i\rangle\langle j|$$

$$\tau_{ij} = -\tau_0 \exp\left(\frac{2\pi i}{\Phi_0}\int_{\mathbf{r}_i}^{\mathbf{r}_j} \mathbf{A}(\mathbf{r})\, d\mathbf{r}\right)$$

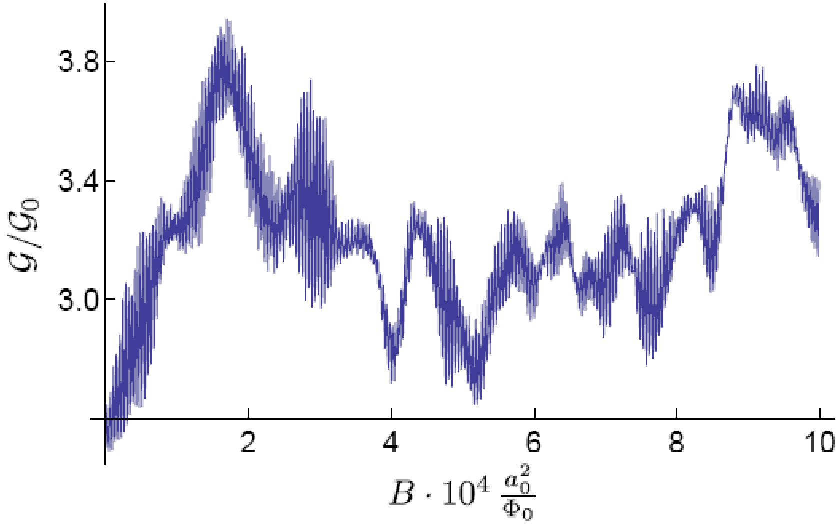
nn'n junctions vs. npn junctions
in lower arm of the ring ⇒

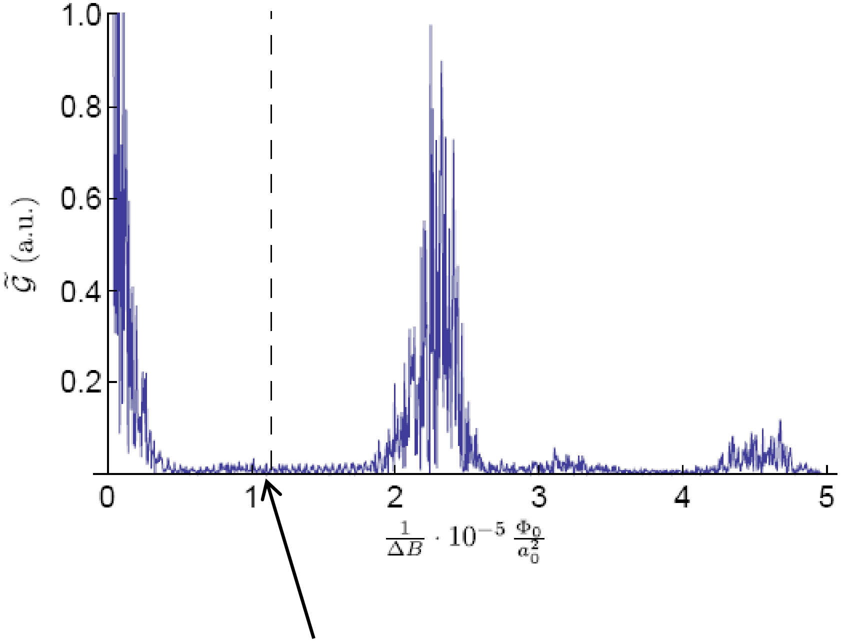
Schelter, Bohr & BT arXiv 2010



<u>related work:</u> *Rycerz* Act. Phys. Pol. A 2009; *Wurm et al* Semicond. Sci. Technol. 2010; *Katsnelson* EPL 2010

Magneto transport





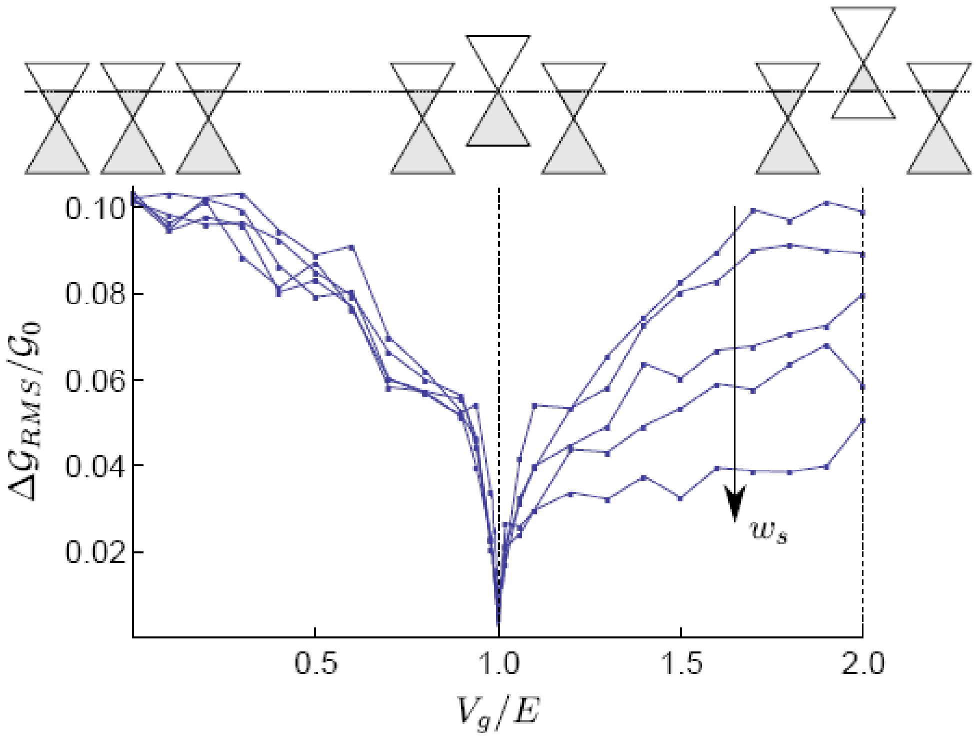
based on Fisher-Lee relation:

$$\left(S_{ij}\right)_{hl} = \bar{\phi}_h^{\dagger}\left(-\delta_{ij} \cdot \mathbf{1} + G_{ij}\mathbf{V}\right)\phi_l \sqrt{\frac{v_h}{v_l}}$$

Fisher & Lee PRB 1981

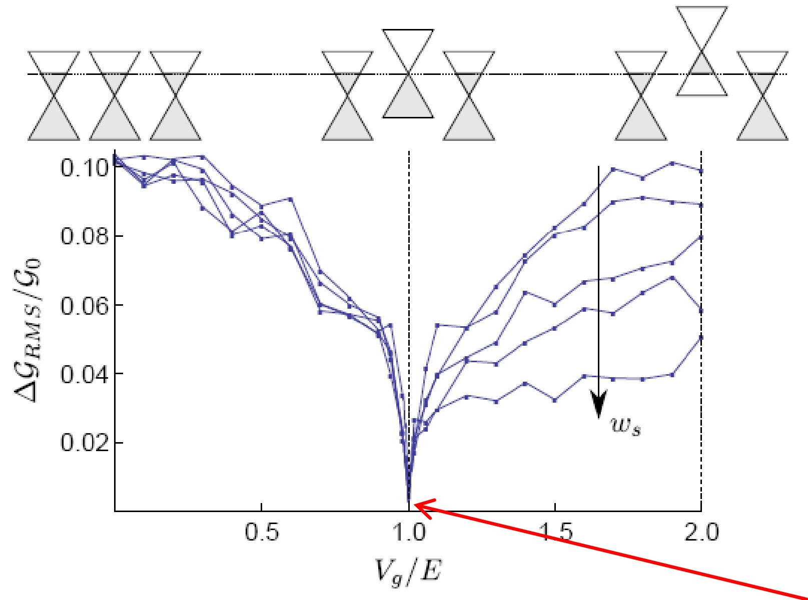
frequency limit of high
pass frequency filter
used for background subtraction

Interplay of AB effect and Klein tunneling

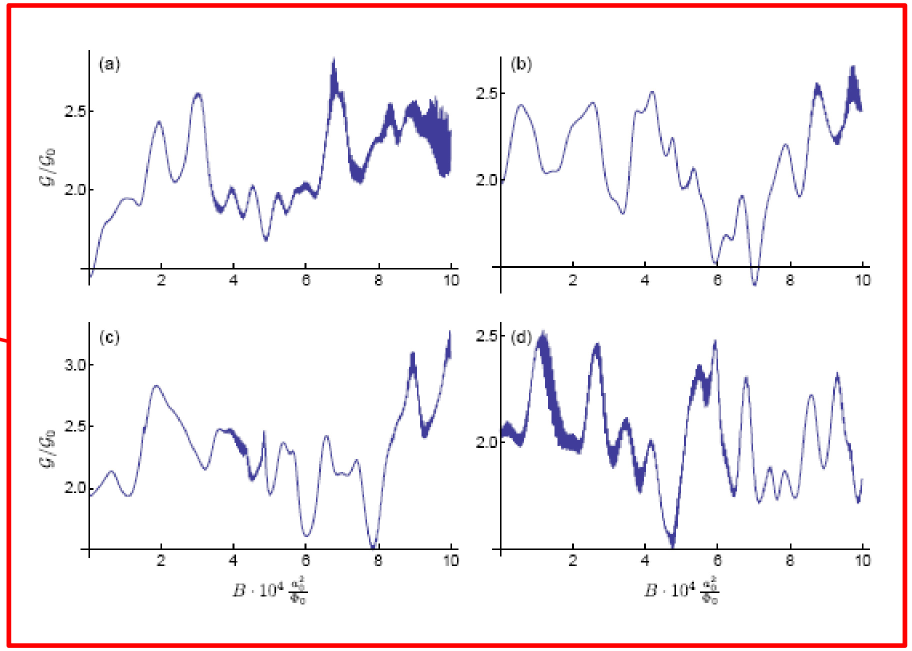


Schelter, Bohr & BT arXiv 2010

Interesting Dirac point physics?

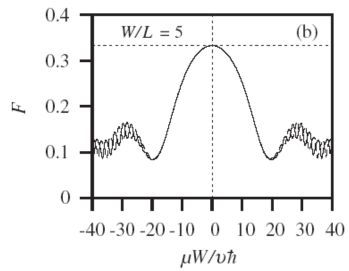


Schelter, Bohr & BT arXiv 2010



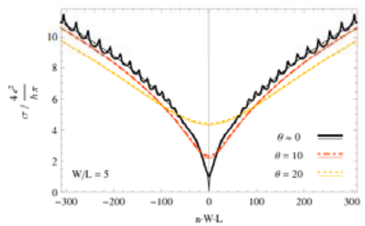
Summary

- ballistic transport in graphene contains unexpected physics: conductance ∝ L^{-1}

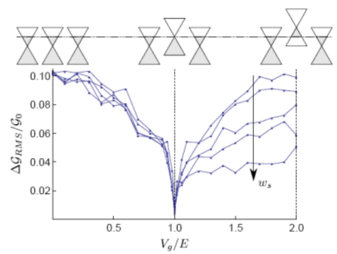


- conductivity has minimum at Dirac point & shot noise has maximum at Dirac point

- temperature dependence of ballistic transport is important in suspended graphene devices

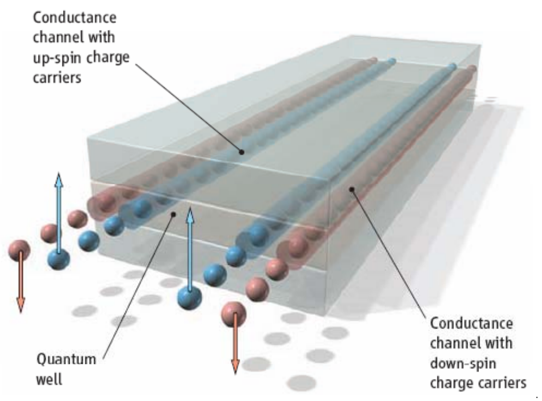


- prediction of scaling function of conductivity at finite n and T



- Interplay of AB effect and Klein tunneling can be nicely tested in graphene rings

Outlook

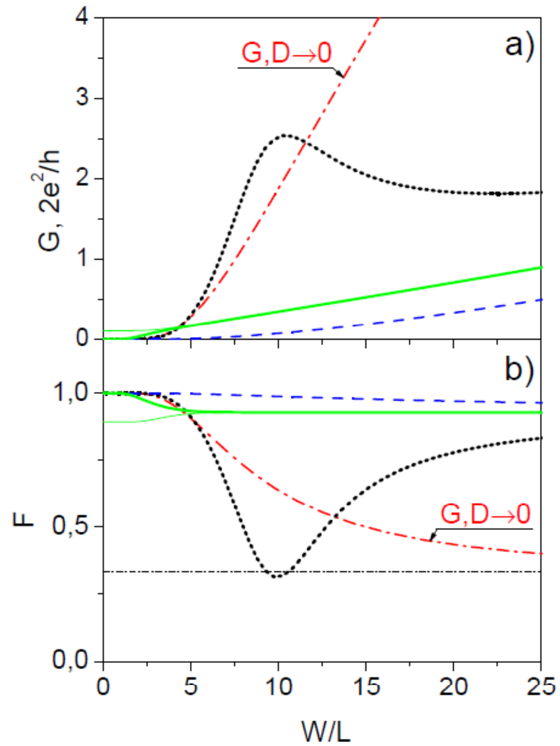


König et al. Science 2007

$$H = \begin{pmatrix} h(\vec{k}) & 0 \\ 0 & h^*(-\vec{k}) \end{pmatrix} \quad \text{with} \quad h(\vec{k}) = \varepsilon(\vec{k}) + d_a(\vec{k})\sigma^a$$

$$\varepsilon(\vec{k}) = C - Dk^2, \;\; \vec{d}(\vec{k}) = (Ak_x, -Ak_y, M - Gk^2)$$

Bernevig, Hughes & Zhang Science 2006



Novik, Recher, Hankiewicz & BT arXiv 2009