Interaction effects in bilayer graphene

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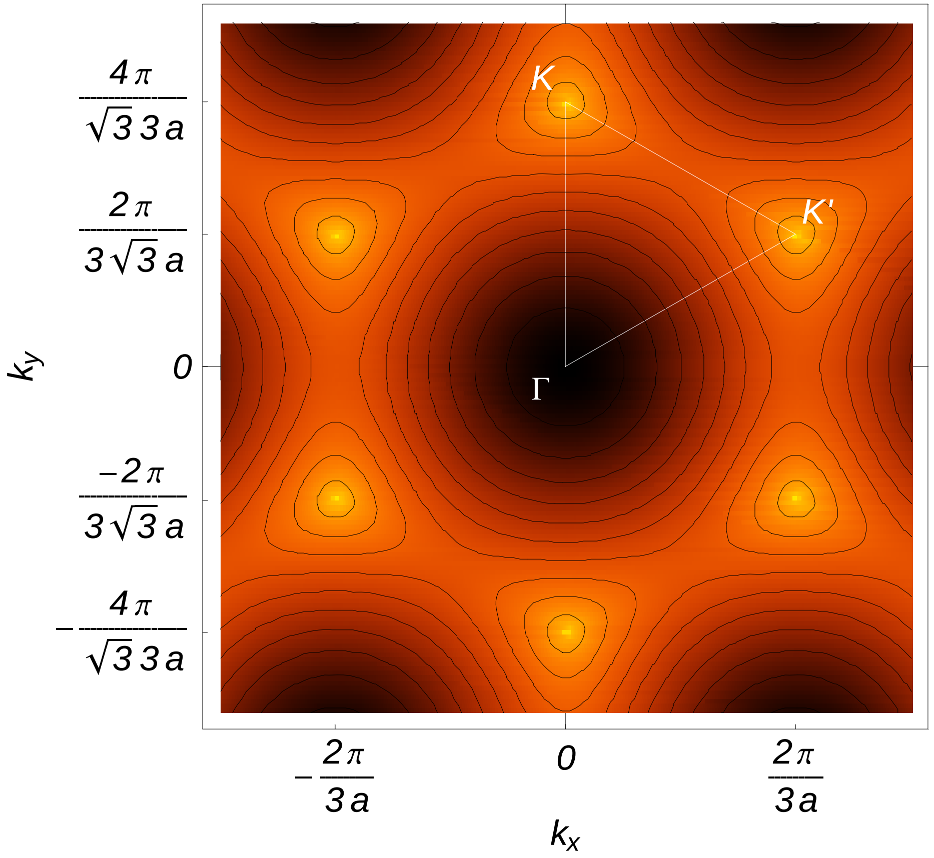
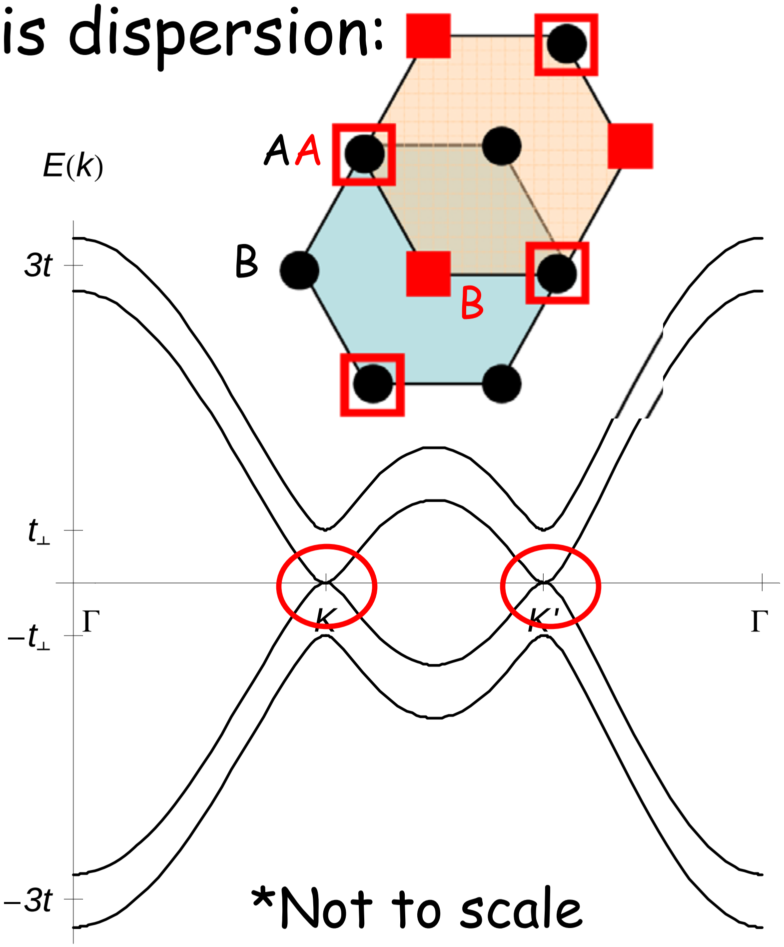
Motivation

- Motion of the electrons on the AB stacked honeycomb bilayer leads to this dispersion:





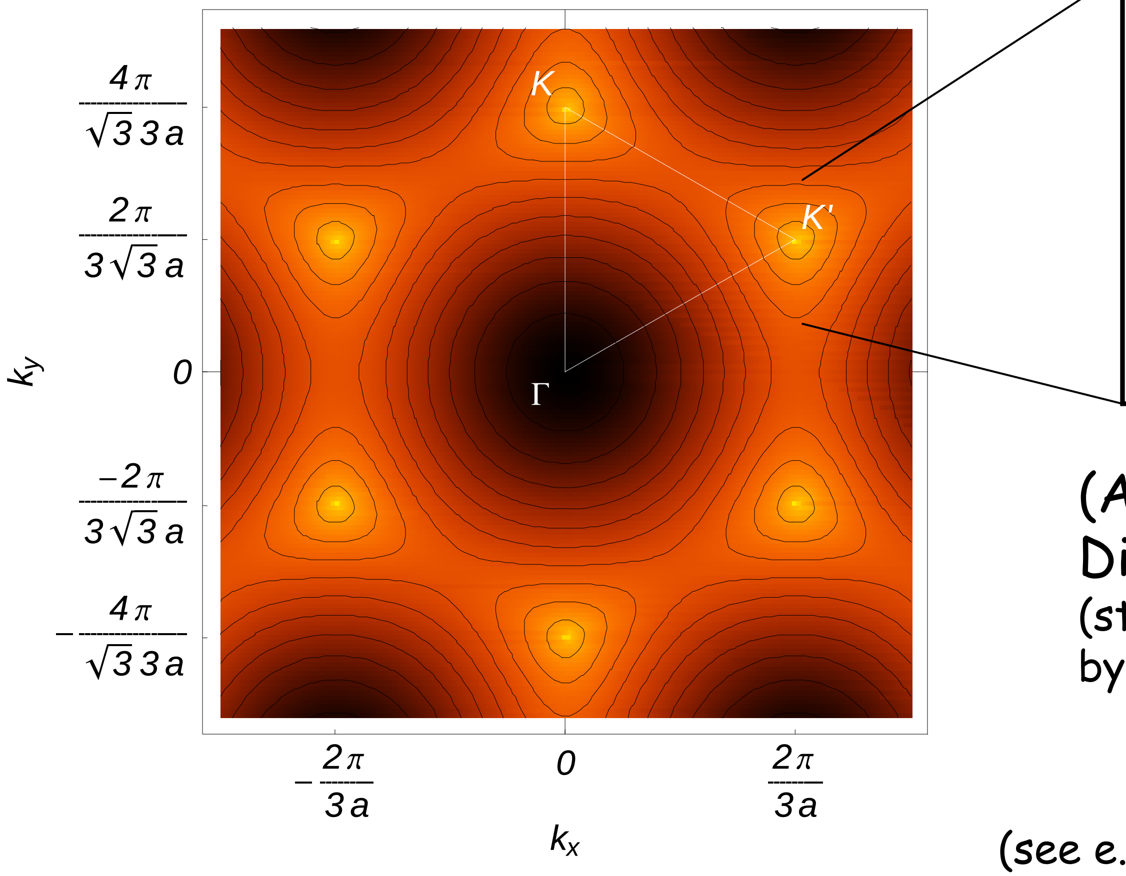
*Not to scale

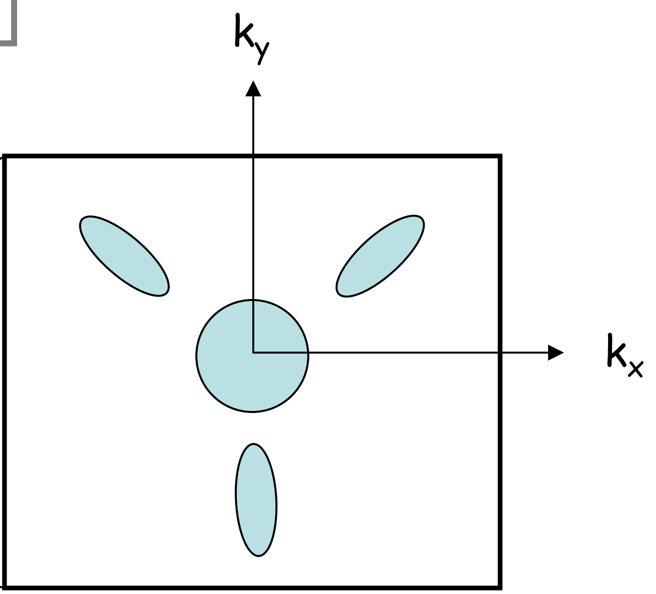
(and fine-tuned)

(see e.g. McCann and Falko PRL 2006 Nillson et.al. 2008, Castro Neto et.al. RMP 2009)

Trigonal warping effects

$$2\pi = (\pi+\pi+\pi)-\pi$$





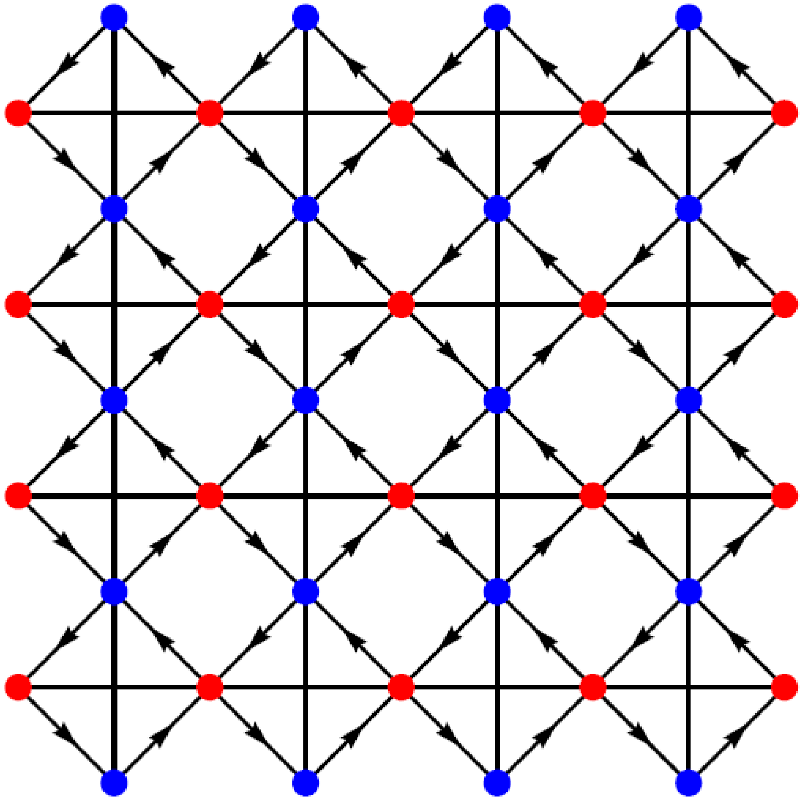
(Anisotropic) massless Dirac fermions

(stable at weak coupling as argued by previous speakers)

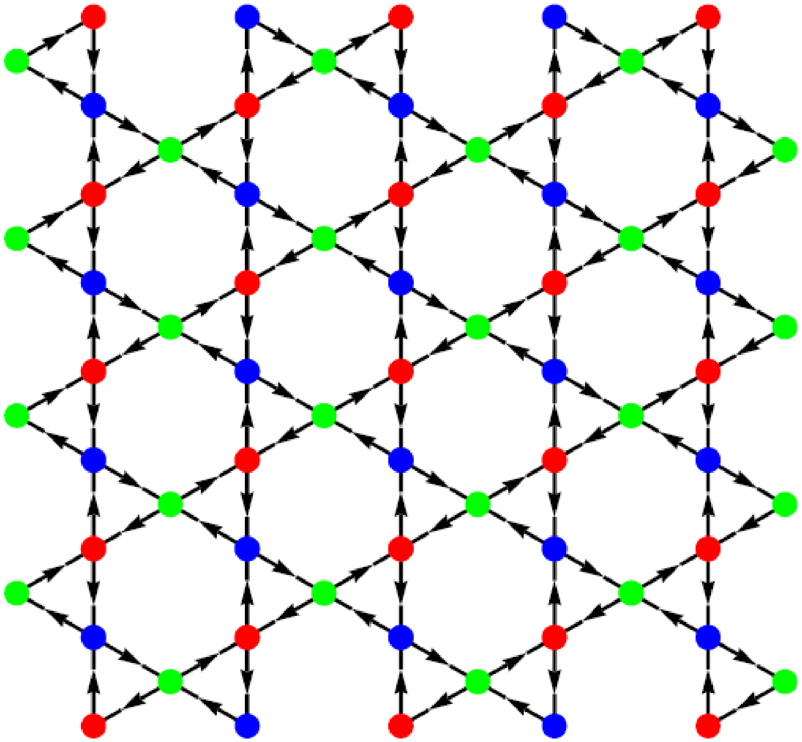
(see e.g. McCann and Falko PRL (2006)

Other models w/ parabolic touching

- Trigonal warping effects can be defeated for other lattices with 4 and 6 fold rotational symmetries



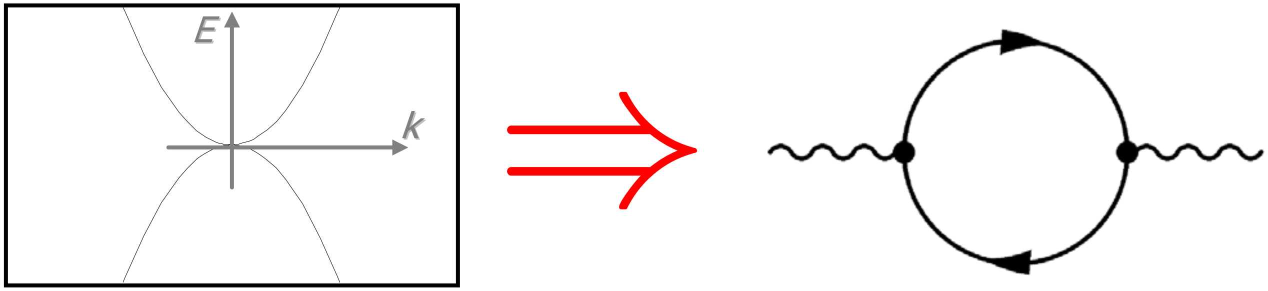
parabolic touching at (π,π)



parabolic touching at $(0,0)$

Kai Sun et.al. PRL 2009

Motivation



$\sim \ln\left(t_\perp / T\right)$

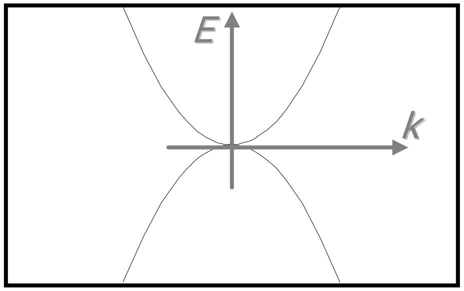
- The ordering susceptibility diverges as T->0 even in the non-interacting limit (argued by others, too (See e.g. H. Min et. al. PRB (2008), Castro Neto et.al. RMP 2009))

- Trigonal warping cuts-off the infra-red singularity so ordering is expected only at finite coupling

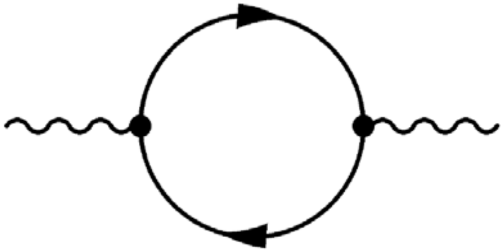
- Fine tuning is a small price to pay for controlled access to strong coupling phases from weak coupling

Vafek and Yang, PRB 81, 041401(R) (2010), (featured in Physics 3, 1 (2010))

Motivation



$\Rightarrow$



$\sim \ln\left(t_{\perp}/T\right)$

- Which ordering tendency dominates can be determined using weak-coupling Renormalization Group.

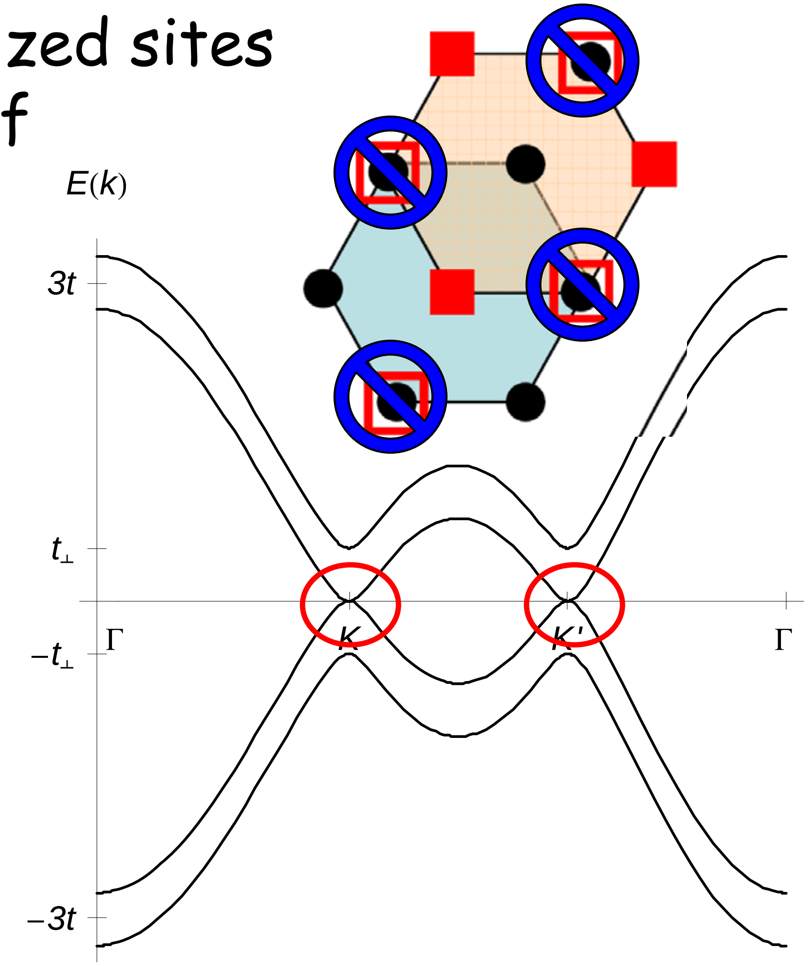
Vafek and Yang, PRB 81, 041401(R) (2010), (featured in Physics 3, 1 (2010))
For spinless case see H. Min et. al. PRB (2008).

Effective low energy theory

- We develop an effective field theory for the low energy modes

- Project out the dimerized sites using Schrieffer-Wolff transformation

(see e.g.
McCann and Falko PRL 2006
Nillson et.al. 2008,
Castro Neto et.al. RMP 2009)



Effective low energy theory

$$\mathcal{S}_0 = \int d\tau d^2\mathbf{r} \left[\psi^\dagger \left(\frac{\partial}{\partial \tau} + \sum_{a=x,y} \Sigma^a d^a_{\mathbf{k}} \right) \psi \right]$$

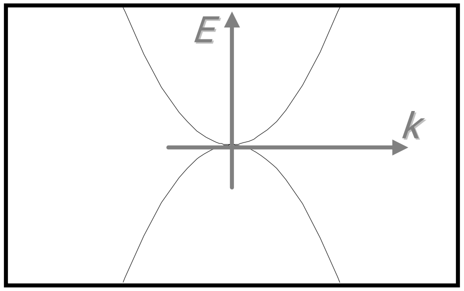
$$\Downarrow$$

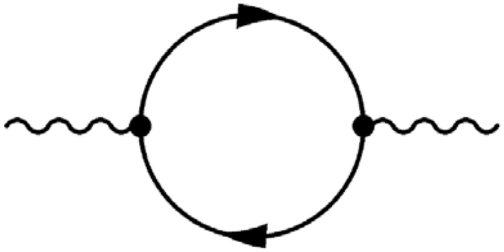
$$\boxed{G_{\mathbf{k}}(i\omega_n) = \frac{i\omega_n + \sum_{a=x,y} \Sigma^a d^a_{\mathbf{k}}}{\omega_n^2 + d^2_{\mathbf{k}}}}$$

$$\begin{aligned}
d^x_{\mathbf{k}} &= \frac{k_x^2 - k_y^2}{2m^*} \\
d^y_{\mathbf{k}} &= \frac{2k_x k_y}{2m^*} \\
\Sigma^x &= 1\sigma^x \otimes 1_N \\
\Sigma^y &= \tau^z \sigma^y \otimes 1_N \\
\Sigma^z &= \tau^z \sigma^z \otimes 1_N
\end{aligned}$$

valley layer

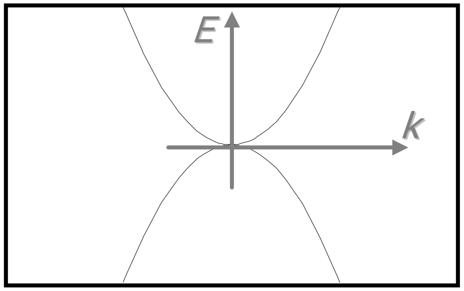
Motivation

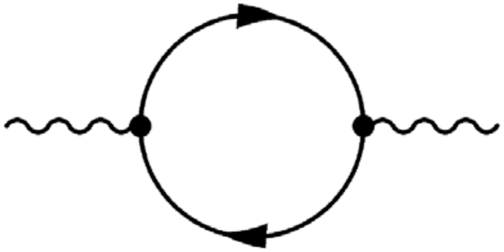




$$
\begin{aligned}
\chi_O &\approx -k_B T \sum_{n=-\infty}^{\infty} \int \frac{d^2\mathbf{k}}{A_{BZ}} Tr\left[O G_{\mathbf{k}}(i\omega_n) O G_{\mathbf{k}}(i\omega_n)\right] \\
&\approx -\int \frac{d^2\mathbf{k}}{(2\pi)^2} k_B T \sum_{n=-\infty}^{\infty} Tr\left[O \frac{i\omega_n + \Sigma\cdot d_{\mathbf{k}}}{\omega_n^2 + d_{\mathbf{k}}^2} O \frac{i\omega_n + \Sigma\cdot d_{\mathbf{k}}}{\omega_n^2 + d_{\mathbf{k}}^2}\right] \\
&\approx c_1 \left(1 + \frac{1}{2} \sum_{a=x,y} Tr\left[O\Sigma_a O\Sigma_a\right]\right) \\
&+ c_2 \left(1 - \frac{1}{2} \sum_{a=x,y} Tr\left[O\Sigma_a O\Sigma_a\right]\right) \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\tanh \frac{k^2}{T}}{k^2}
\end{aligned}
$$

Motivation





$$\begin{aligned}\chi_O &\approx c_1\left(1+\frac{1}{2}\sum_{a=x,y} Tr\left[O\Sigma_a O\Sigma_a\right]\right)\\ &+ c_2\left(1-\frac{1}{2}\sum_{a=x,y} Tr\left[O\Sigma_a O\Sigma_a\right]\right)\ln\frac{t_\perp}{T}\end{aligned}$$

Vanishes if $[O,\Sigma_a]$ vanishes
But finite if $[O,\Sigma_a]$ is finite

Effective low energy theory

$$\boxed{\mathcal{S}_0 = \int d\tau d^2\mathbf{r} \left[\psi^\dagger \left(\frac{\partial}{\partial \tau} + \sum_{a=x,y} \Sigma^a d^a_{\mathbf{k}} \right) \psi \right]}$$

$$\begin{aligned}
d^x_{\mathbf{k}} &= \frac{k_x^2 - k_y^2}{2m^*} \\
d^y_{\mathbf{k}} &= \frac{2k_x k_y}{2m^*} \\
\Sigma^x &= 1\sigma^x \otimes 1_N \\
\Sigma^y &= \tau^z \sigma^y \otimes 1_N \\
\Sigma^z &= \tau^z \sigma^z \otimes 1_N
\end{aligned}$$

valley

layer

- Scaling z=2 ($\omega \sim k^2$)
- By power counting, contact interactions are the only marginal coupling

Effective low energy theory

$$\begin{aligned}
\mathcal{S} &= \int d\tau d^2\mathbf{r} \left[\psi^\dagger \left(\frac{\partial}{\partial \tau} + \sum_{a=x,y} \Sigma^a d^a_{\mathbf{k}} \right) \psi \right] \\
&\ \text{screened Coulomb} \\
&+ \frac{1}{2} \boxed{g_1} \int d\tau d^2\mathbf{r} \psi^\dagger \psi(\mathbf{r},\tau) \psi^\dagger \psi(\mathbf{r},\tau) \\
&+ \frac{1}{2} g_2 \int d\tau d^2\mathbf{r} \psi^\dagger \Sigma^z \psi(\mathbf{r},\tau) \psi^\dagger \Sigma^z \psi(\mathbf{r},\tau) \\
&+ \frac{1}{2} g_3 \int d\tau d^2\mathbf{r} \sum_{a=x,y} \psi^\dagger \Sigma^a \psi(\mathbf{r},\tau) \psi^\dagger \Sigma^a \psi(\mathbf{r},\tau)
\end{aligned}$$

$$\begin{aligned}
d^x_{\mathbf{k}} &= \frac{k_x^2 - k_y^2}{2m^*} \\
d^y_{\mathbf{k}} &= \frac{2k_x k_y}{2m^*} \\
\Sigma^x &= 1\sigma^x \otimes 1_N \\
\Sigma^y &= \tau^z \sigma^y \otimes 1_N \\
\Sigma^z &= \underset{\text{valley}}{\underset{\uparrow}{\tau^z}} \underset{\text{layer}}{\underset{\uparrow}{\sigma^z}} \otimes 1_N
\end{aligned}$$

- Scaling z=2 ($\omega \sim k^2$)
- Contact interactions are the only marginal couplings
- We assume that screened Coulomb interaction dominates

Effective low energy theory

$$
\begin{aligned}
\mathcal{S} &= \int d\tau d^2\mathbf{r} \left[\psi^\dagger \left(\frac{\partial}{\partial\tau} + \sum_{a=x,y} \Sigma^a d^a_{\mathbf{k}}\right)\psi\right] \\
&+ \frac{1}{2} g_1 \int d\tau d^2\mathbf{r} \psi^\dagger\psi(\mathbf{r},\tau)\psi^\dagger\psi(\mathbf{r},\tau) \\
&+ \frac{1}{2} \boxed{g_2} \int d\tau d^2\mathbf{r} \psi^\dagger\Sigma^z\psi(\mathbf{r},\tau)\psi^\dagger\Sigma^z\psi(\mathbf{r},\tau) \\
&+ \frac{1}{2} g_3 \int d\tau d^2\mathbf{r} \sum_{a=x,y} \psi^\dagger\Sigma^a\psi(\mathbf{r},\tau)\psi^\dagger\Sigma^a\psi(\mathbf{r},\tau)
\end{aligned}
$$

Pseudo-spin Ising

$$
\begin{aligned}
d^x_{\mathbf{k}} &= \frac{k_x^2 - k_y^2}{2m^*} \\
d^y_{\mathbf{k}} &= \frac{2k_x k_y}{2m^*} \\
\Sigma^x &= 1\sigma^x \otimes 1_N \\
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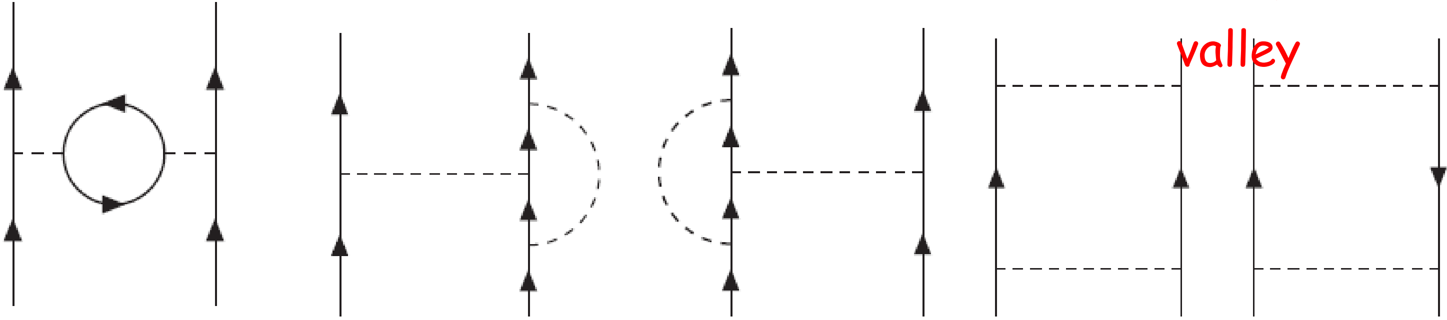
\Sigma^z &= \tau^z\sigma^z \otimes 1_N
\end{aligned}
$$

valley

layer



Effective low energy theory

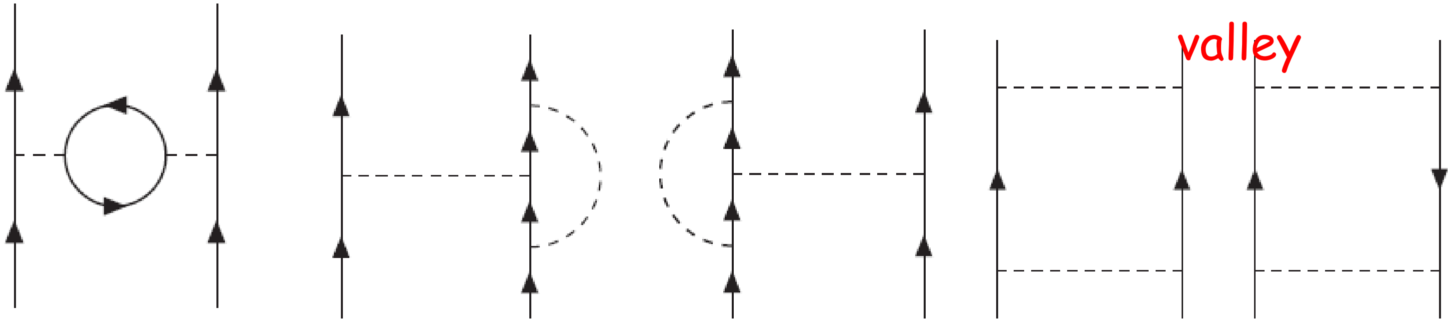
$$
\begin{aligned}
\mathcal{S} &= \int d\tau d^2\mathbf{r}\left[\psi^\dagger\left(\frac{\partial}{\partial\tau}+\sum_{a=x,y}\Sigma^a d^a_{\mathbf{k}}\right)\psi\right] \\
&+ \frac{1}{2}g_1\int d\tau d^2\mathbf{r}\psi^\dagger\psi(\mathbf{r},\tau)\psi^\dagger\psi(\mathbf{r},\tau) \\
&+ \frac{1}{2}g_2\int d\tau d^2\mathbf{r}\psi^\dagger\Sigma^z\psi(\mathbf{r},\tau)\psi^\dagger\Sigma^z\psi(\mathbf{r},\tau) \\
&+ \frac{1}{2}\boxed{g_3}\int d\tau d^2\mathbf{r}\sum_{a=x,y}\psi^\dagger\Sigma^a\psi(\mathbf{r},\tau)\psi^\dagger\Sigma^a\psi(\mathbf{r},\tau)
\end{aligned}
$$

Pseudo-spin XY

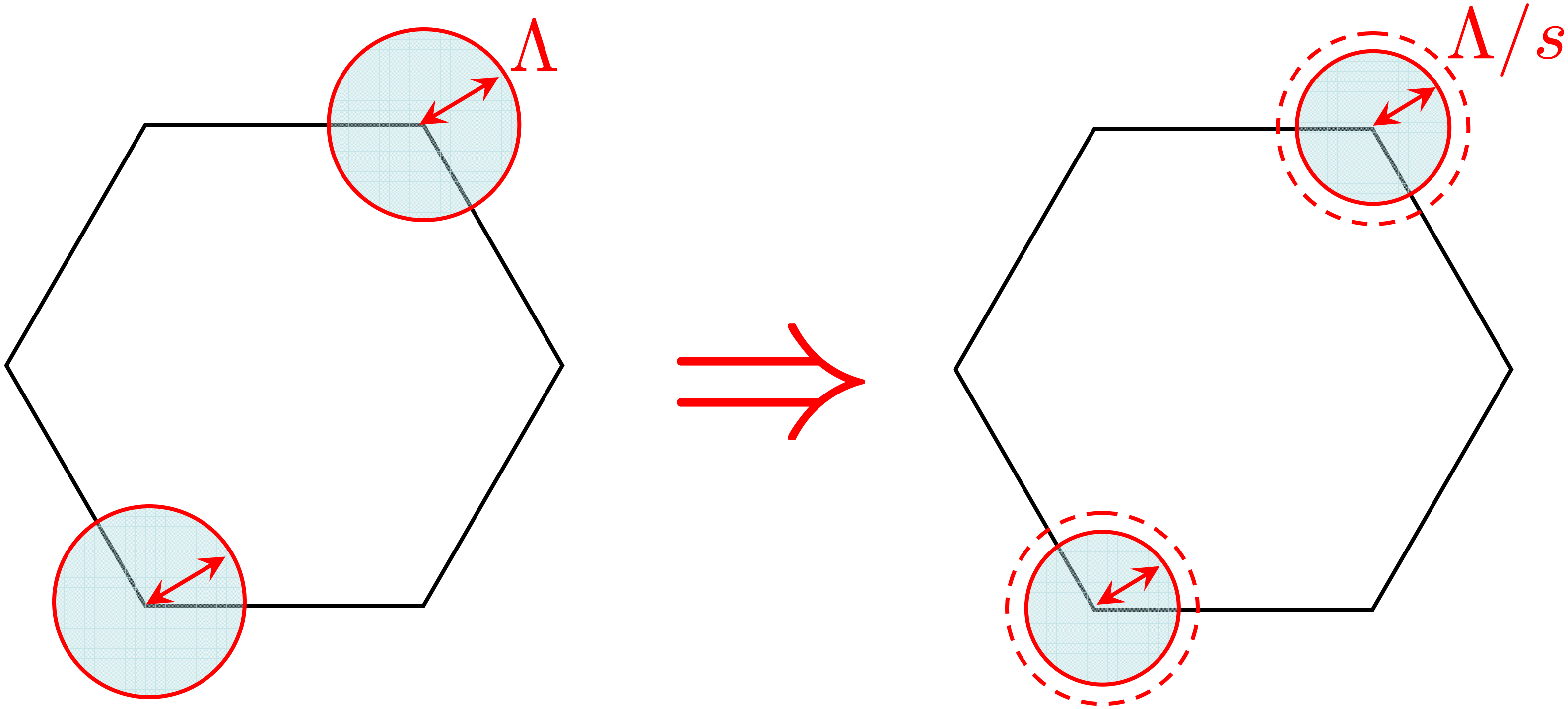
$$
\begin{aligned}
d^x_{\mathbf{k}} &= \frac{k_x^2-k_y^2}{2m^*} \\
d^y_{\mathbf{k}} &= \frac{2k_xk_y}{2m^*} \\
\Sigma^x &= 1\sigma^x\otimes 1_N \\
\Sigma^y &= \tau^z\sigma^y\otimes 1_N \\
\Sigma^z &= \tau^z\sigma^z\otimes 1_N
\end{aligned}
$$

valley

layer



Effective low energy theory: RG procedure



Effective low energy theory: RG equations

screened Coulomb

$$\frac{dg_1}{d\ln s} = \left[-4g_1g_3\right]\frac{m}{4\pi}$$

Pseudo-spin Ising

$$\frac{dg_2}{d\ln s} = \left[-4(N-1)g_2^2 + 4g_3^2 + 4g_1g_2 - 12g_2g_3\right]\frac{m}{4\pi}$$

Pseudo-spin XY

$$\begin{aligned}\frac{dg_3}{d\ln s} &= \left[-g_1^2 - g_2^2 - 2(N+2)g_3^2 + 2g_1g_3 + 2g_2g_3\right]\frac{m}{4\pi}\\ &= \left[-(g_1-g_3)^2 - (g_2-g_3)^2 - 2(N+1)g_3^2\right]\frac{m}{4\pi} < 0\end{aligned}$$

Effective low energy theory: RG equations

Since the system is autonomous, we can eliminate $\log s$ and arrive at a system

$$\frac{dg_1}{dg_3} = f\left(\frac{g_1}{g_3}, \frac{g_2}{g_3}\right)$$
$$\frac{dg_2}{dg_3} = g\left(\frac{g_1}{g_3}, \frac{g_2}{g_3}\right)$$

$$\begin{aligned} f\left(g_1, g_2\right) &= \frac{-4g_1}{-g_1^2 - g_2^2 - 2(N+2) + 2g_1 + 2g_2} \\ g\left(g_1, g_2\right) &= \frac{-4(N-1)g_2^2 + 4 + 4g_1 g_2 - 12 g_2}{-g_1^2 - g_2^2 - 2(N+2) + 2g_1 + 2g_2} \end{aligned}$$

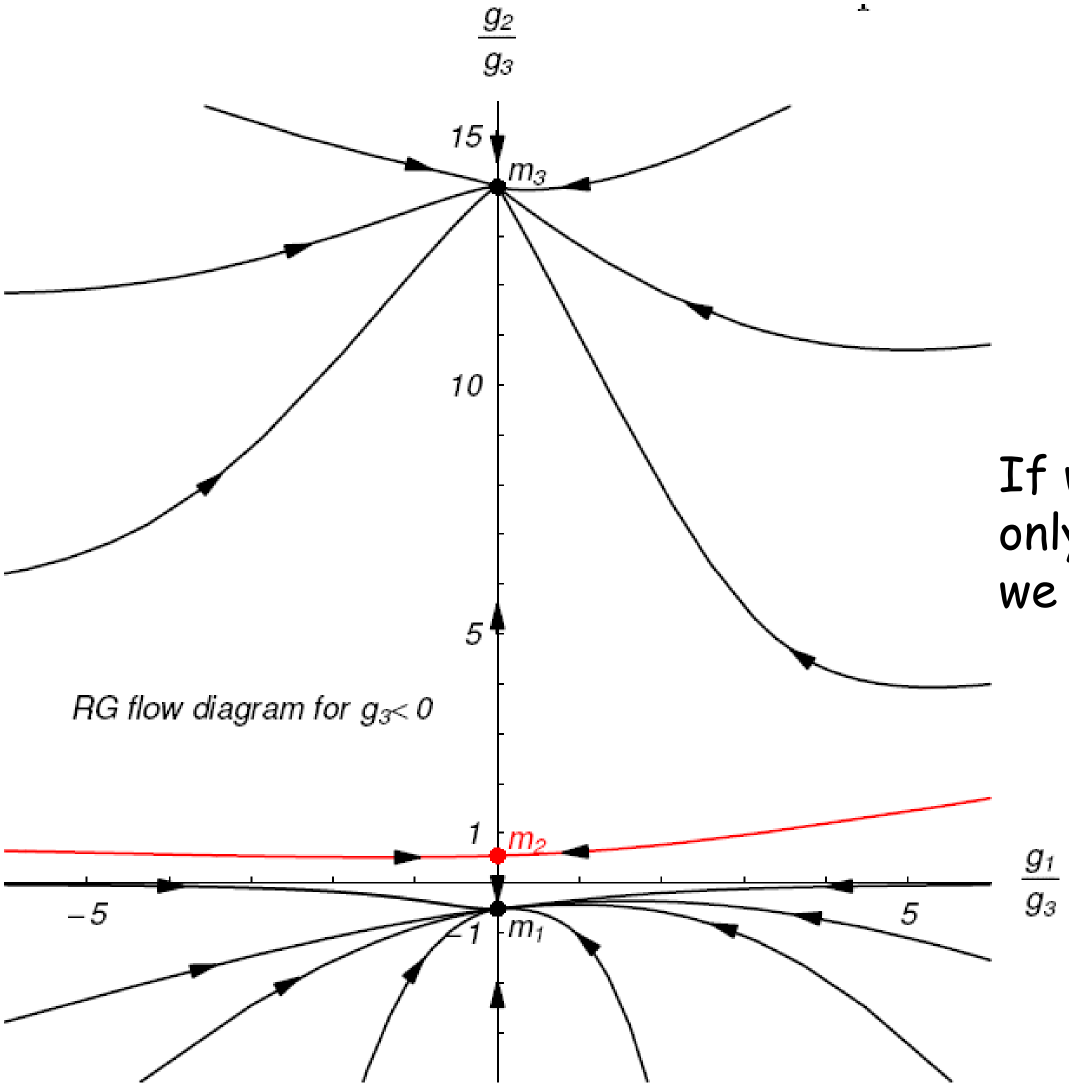
This system is in turn homogeneous, and it can be written as

$$\boxed{\begin{aligned} g_3 \frac{d\frac{g_1}{g_3}}{dg_3} &= -\frac{g_1}{g_3} + f\left(\frac{g_1}{g_3}, \frac{g_2}{g_3}\right) \\ g_3 \frac{d\frac{g_2}{g_3}}{dg_3} &= -\frac{g_2}{g_3} + g\left(\frac{g_1}{g_3}, \frac{g_2}{g_3}\right). \end{aligned}}$$

We can think of these as effective flow equations for the coupling constant ratios!

RG flow diagram

Pseudo-spin Ising/Pseudo-spin XY



screened Culomb/Pseudo-spin XY

If we start w/Coulomb interaction only (only g1 is finite), we start below the red separatrix

=> flow to m1

Analysis of the susceptibilities

Introduce additional source terms in the action, which correspond to possible broken symmetry states

$$\boxed{\Delta\mathcal{S} \quad = \quad -\Delta_{ph}^{\mathcal{O}_i}\int d\tau d^2\mathbf{r}\,\left[\psi^\dagger\mathcal{O}_i\psi\right] - \Delta_{pp}^{\mathcal{O}_i}\int d\tau d^2\mathbf{r}\,\left[\psi_{\alpha\sigma}\mathcal{O}^i_{\alpha\beta,\sigma\sigma'}\psi_\beta\right]}$$

The question of instability is answered by finding Δ with the strongest RG divergence.

P-H

$$\mathcal{O}_i = \tau^\mu\sigma^\nu \text{ where } \mu,\nu = 0,1,2,3 \text{ and } \tau_0 = \sigma_0 = 1,$$

$$\Delta_{ph,ren}^{\tau^\mu\sigma^\nu} \quad = \quad \Delta_{ph}^{\tau^\mu\sigma^\nu}\left(1 + \left[Ag_1 + Bg_2 + Cg_3\right]\frac{m}{4\pi}\ln s\right)$$

P-P

$$\psi_{\alpha\sigma}\mathcal{O}^{(i)}_{\alpha\beta}\psi_{\beta\sigma'}$$

$$\Delta_{pp,ren}^{\tau^\mu\sigma^\nu} \quad = \quad \Delta_{pp}^{\tau^\mu\sigma^\nu}\left(1 + \left[A'g_1 + B'g_2 + C'g_3\right]\frac{m}{4\pi}\ln s\right)$$

Analysis of the susceptibilities

P-H $$\Delta^{\tau^\mu\sigma^\nu}_{ph,ren} = \Delta^{\tau^\mu\sigma^\nu}_{ph}\left(1+[Ag_1+Bg_2+Cg_3]\,\frac{m}{4\pi}\ln s\right)$$

P-P $$\Delta^{\tau^\mu\sigma^\nu}_{pp,ren} = \Delta^{\tau^\mu\sigma^\nu}_{pp}\left(1+[A'g_1+B'g_2+C'g_3]\,\frac{m}{4\pi}\ln s\right)$$

$\psi^\dagger\tau^\mu\sigma^\nu\psi$	$\nu=0$	$\nu=x$	$\nu=y$	$\nu=z$
$\mu=0$	$0,0,0$	$\boxed{1,-1,-2N}$	$1,-1,0$	$2,2,-4$
$\mu=x$	$1,-1,0$	$0,0,0$	$2,2,-4$	$1,-1,0$
$\mu=y$	$1,-1,0$	$0,0,0$	$2,2,-4$	$1,-1,0$
$\mu=z$	$0,0,0$	$1,-1,0$	$\boxed{1,-1,-2N}$	$2,2-4N,-4$
$\psi_{\alpha s}(\tau^\mu\sigma^\nu)_{\alpha\beta}\psi_{\beta s'}$				
$\mu=0$	$-1,-1,0$	$-2,2,-4$	$0,0,0$	$-1,-1,0$
$\mu=x$	$-2,2,-4$	$-1,-1,0$	$-1,-1,0$	$0,0,0$
$\mu=y$	$-2,2,-4$	$-1,-1,0$	$-1,-1,0$	$0,0,0$
$\mu=z$	$-1,-1,0$	$-2,2,-4$	$0,0,0$	$-1,-1,0$

Table 1: (Upper half) The susceptibility coefficients A,B,C for different particle-hole order parameters $\psi^\dagger\mathcal{O}_i\psi$. In the physical case $N=4$. (Lower half) The susceptibility coefficients A',B',C' for different particle-particle order parameters $\psi_{\alpha\sigma}\mathcal{O}^{(i)}_{\alpha\beta}\psi_{\beta\sigma'}$.

Conclusion

Starting with screened Coulomb interaction only, the strongest divergence is found for the operators

$$\boxed{\psi^\dagger 1\sigma_x\psi(\mathbf{r}) \text{ and } \psi^\dagger \tau_z\sigma_y\psi(\mathbf{r})}$$

This corresponds to
an electronic nematic state:

- Breaks rotational symmetry
- Even under π rotation
- Does not break translational symmetry (unlike stripes)

Conclusion

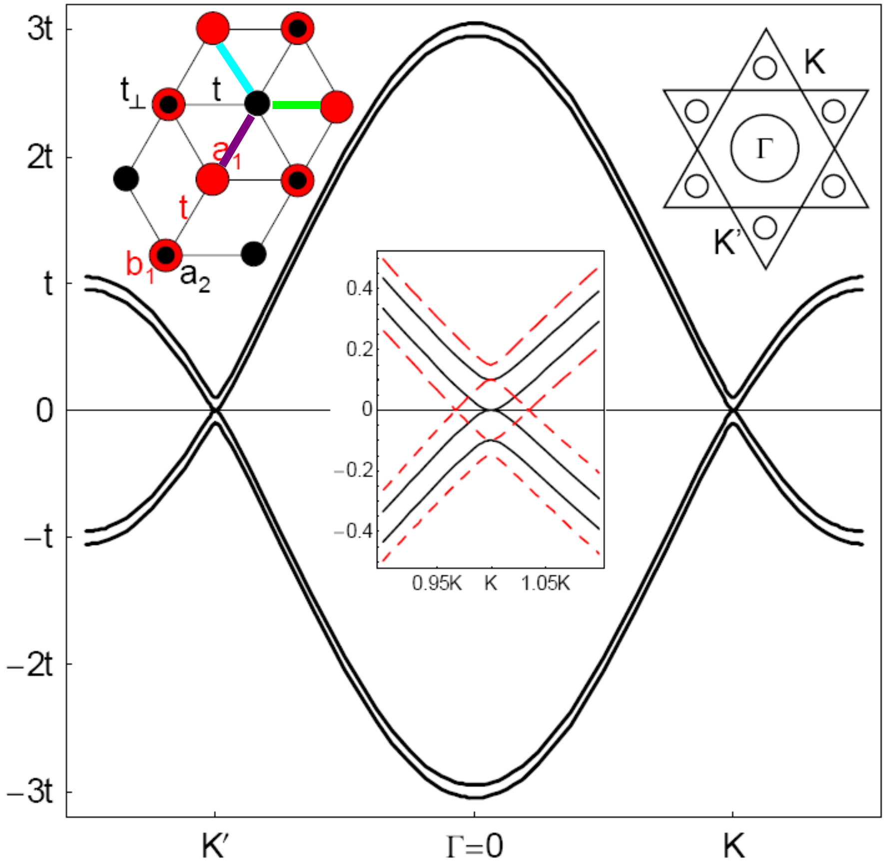
Starting with screened Coulomb interaction only, the strongest divergence is found for the operators

$$\boxed{\psi^\dagger 1\sigma_x\psi(\mathbf{r}) \text{ and } \psi^\dagger\tau_z\sigma_y\psi(\mathbf{r})}$$

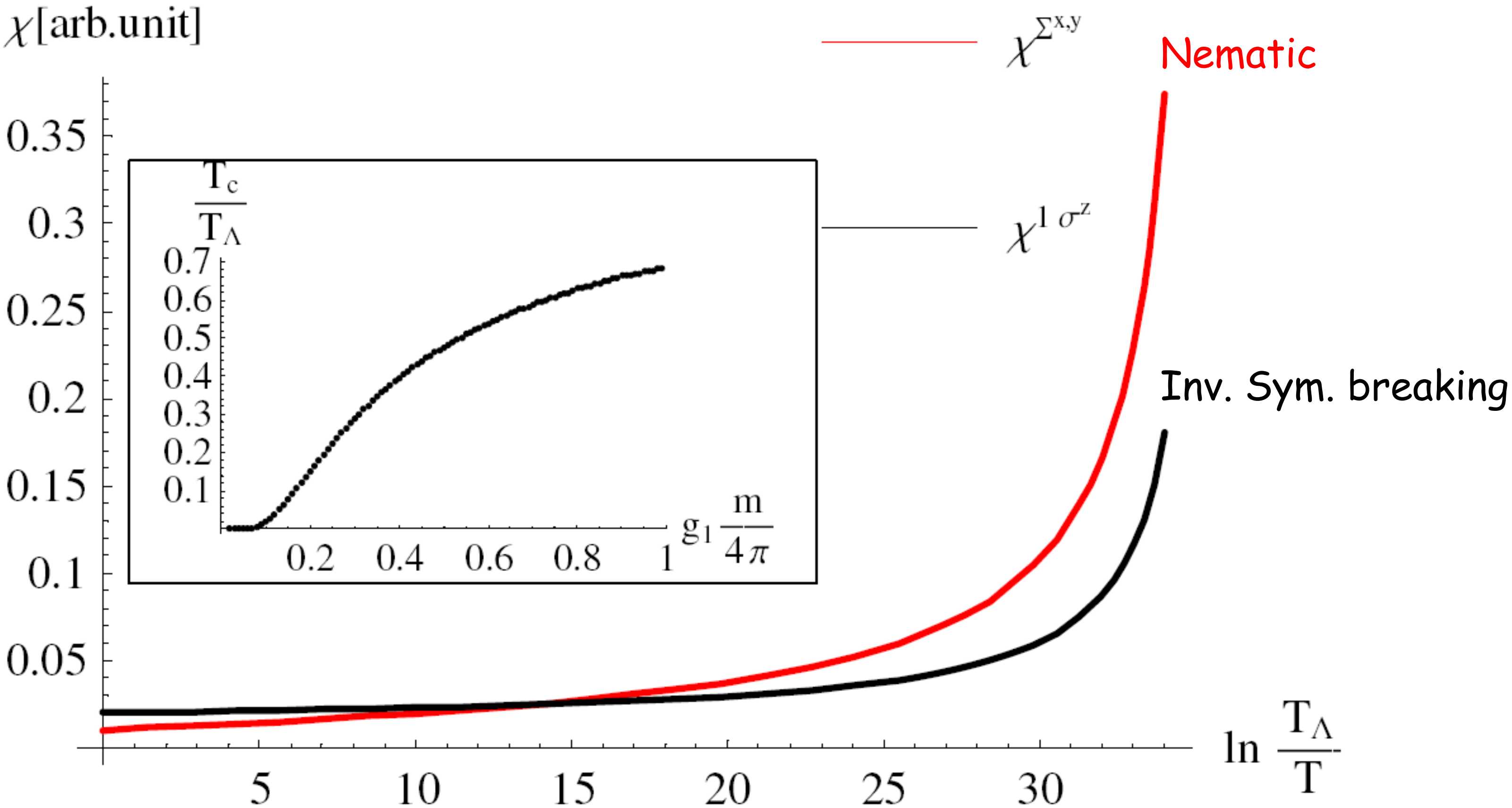
This corresponds to
an electronic nematic state

The transition is 2nd order
(3 state Potts model universality class)

Each parabolic touching is split into two conical (Dirac) touchings.



Conclusion


χ[arb.unit]
0.35
0.3
0.25
0.2
0.15
0.1
0.05
5
10
15
20
25
30
ln T_Λ / T
χ^{Σ^{x,y}}
χ^{1 σ^z}
Nematic
Inv. Sym. breaking
T_c / T_Λ
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.2
0.4
0.6
0.8
1
g_1 m / 4π