The universal subleading spectrum of effective string theory

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Motivation

- ✔ We would like to understand the physics of confinement.
- ✔ A natural question to ask is how does colour flux behave as the sources (quarks) are stretched apart.
- ✔ For a theory with infinitely massive quarks the flux cannot break.
- ✔ We expect that as the quarks become widely separated the flux tube will behave like a string.
- ✔ In particular to a first approximation there will be a linear increase in the string energy.
- ✔ How small does the tube have to be before the string approximation breaks down?
- ✔ What are the subleading corrections to the string energy in an expansion in $1/R$?

Effective string theory

Original field theory has D-dimensional Poincaré invariance.

Long string-like configuration breaks $(D-2)$ transverse translations.

$\Longrightarrow$ $(D-2)$ massless modes on string worldsheet.

Outside $D = 26$ dimensions fundamental string quantisation leads to

- ✔ Light cone: Anomaly in Lorentz invariance.
- ✔ Covariant: $(D-1)$ massless modes.

Imagine rewriting the path integral in terms of effective X^μ variables.

$$\int \mathcal{D}X e^{iS_0(X)}, \qquad S_0(X) = \frac{1}{4\pi a^2} \int d\tau^+ d\tau^- \partial_+ X \cdot \partial_- X.$$

$\mathcal{D}X$ will contain some determinants.

Guess: Take Liouville field and replace intrinsic metric e^ϕ with induced metric $h_{+-} = \partial_+ X \cdot \partial_- X$.

[Polchinski,Strominger]

$$S_L = \frac{26-D}{48\pi} \int d\tau^+ d\tau^- \partial_+ \phi \partial_- \phi \longrightarrow \frac{26-D}{48\pi} \int d\tau^+ d\tau^- \frac{\partial_+^2 X \cdot \partial_- X \partial_+ X \cdot \partial_-^2 X}{(\partial_+ X \cdot \partial_- X)^2}$$

Effective string in conformal gauge [Polchinski,Strominger]

More systematically one can absorb the posssible determinants into the action.

Write down all possible terms of worldsheet dimension $(1,1)$ modulo

- ✔ terms proportional to leading order equations of motion $\partial_+\partial_- X^\mu$,
- ✔ terms proportional to leading order constraints $\partial_\pm X \cdot \partial_\pm X$.

We will be working in an effective theory. We need to classify terms according to some expansion.

Instead of a string with boundaries we will simplify the problem and wrap the flux around a compact dimension of radius R.

$$X^\mu(\tau, \sigma + 2\pi) = X^\mu(\tau, \sigma) + 2\pi R \delta_1^\mu .$$

$$\partial X \sim O(R), \qquad \partial^p X \sim O(1) \quad (p > 1).$$

Effective string action

Up to and including terms of order R^{-2} Polchinski and Strominger found,

$$S = \frac{1}{4\pi}\int d\tau^+ d\tau^- \left[\frac{1}{a^2}\partial_+ X\cdot\partial_- X + \beta\frac{\partial_+^2 X\cdot\partial_- X\partial_+ X\cdot\partial_-^2 X}{(\partial_+ X\cdot\partial_- X)^2}\right].$$

Classical conformal invariance:

$$\delta X = \epsilon^-(\tau^-)\partial_- X - \frac{\beta a^2}{2}\partial_-^2\epsilon^-(\tau^-)\frac{\partial_+ X}{\partial_+ X\cdot\partial_- X} + (+\longleftrightarrow -),$$

$$\delta S = O(R^{-2}).$$

Energy-momentum tensor:

$$T_{--} = -\frac{1}{2a^2}\partial_- X\cdot\partial_- X + \frac{\beta}{2}\frac{\partial_-^3 X\cdot\partial_+ X}{\partial_+ X\cdot\partial_- X} + O(R^{-2}).$$

Higher orders [JMD]

If one continues the classification to higher order one finds the next possible terms are $O(R^{-6})$:

$$L_1 = \frac{1}{Z^3}\partial_-^2 X\cdot\partial_-^2 X\partial_+^2 X\cdot\partial_+^2 X,$$
$$L_2 = \frac{1}{Z^3}\partial_-^2 X\cdot\partial_+^2 X\partial_-^2 X\cdot\partial_+^2 X,$$
$$L_3 = \frac{1}{Z^4}\partial_-^2 X\cdot\partial_+^2 X\partial_- X\cdot\partial_+^2 X\partial_-^2 X\cdot\partial_+ X,$$
$$L_4 = \frac{1}{Z^5}\partial_- X\cdot\partial_+^2 X\partial_- X\cdot\partial_+^2 X\partial_-^2 X\cdot\partial_+ X\partial_-^2 X\cdot\partial_+ X.$$

where

$$Z = \partial_+ X\cdot\partial_- X.$$

In particular they cannot effect the energy-momentum tensor at the next order in R^{-1}.

$$T_{--} = -\frac{1}{2a^2}\partial_- X\cdot\partial_- X + \frac{\beta}{2Z}\partial_-^3 X\cdot\partial_+ X$$
$$-\frac{\beta}{2Z^2}(-\partial_- X\cdot\partial_- X\partial_-^2 X\cdot\partial_+^2 X + \partial_-^2 X\cdot\partial_- X\partial_- X\cdot\partial_+^2 X + \partial_-^2 X\cdot\partial_+ X\partial_-^2 X\cdot\partial_+ X)$$
$$+O(R^{-3}).$$

Fluctuations around classical background

Lagrangian:

$$\mathcal{L} = -\frac{R^2}{8\pi a^2} + \frac{1}{4\pi a^2}\partial_+ Y \cdot \partial_- Y + \frac{\beta}{\pi R^2}\partial_-^2 Y \cdot e_+ e_- \cdot \partial_+^2 Y$$
$$+ \frac{\beta}{\pi R^3}[\partial_+^2 Y \cdot e_- \partial_+ Y \cdot \partial_-^2 Y + \partial_+^2 Y \cdot \partial_- Y e_- \cdot \partial_-^2 Y]$$
$$+ \frac{4\beta}{\pi R^3}\partial_+^2 Y \cdot e_- e_+ \cdot \partial_-^2 Y [e_+ \cdot \partial_- Y + e_- \cdot \partial_+ Y] + O(R^{-4}).$$

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$$= -\frac{R^2}{8\pi a^2} + \frac{1}{4\pi a^2}\partial_+ \hat{Y} \cdot \partial_- \hat{Y} + O(R^{-4}).$$

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Energy-momentum tensor:

$$T_{--} = -\frac{R}{a^2} e_- \cdot \partial_- \hat{Y} - \frac{1}{2a^2}\partial_- \hat{Y} \cdot \partial_- \hat{Y} - \frac{\beta}{R} e_+ \cdot \partial_-^3 \hat{Y}$$
$$- \frac{2\beta}{R^2}(e_+ \cdot \partial_-^3 \hat{Y} e_+ \cdot \partial_- \hat{Y} - e_+ \cdot \partial_-^2 \hat{Y} e_+ \cdot \partial_-^2 \hat{Y}) + O(R^{-3}).$$

Quantum conformal symmetry

OPE:

$$T_{--}(\tau^-)T_{--}(0) = \frac{12\beta + D}{2(\tau^-)^4} + \frac{2}{(\tau^-)^2}T_{--}(0) + \frac{1}{\tau^-}\partial_- T_{--}(0) + O(R^{-2}),$$

Equivalently - Virasoro algebra:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{12\beta + D}{12}(m^3 - m)\delta_{m,-n},$$

where

$$L_n = \frac{R}{a}e_- \cdot \hat{\alpha}_n + \frac{1}{2}\sum_{m=-\infty}^{\infty} :\hat{\alpha}_{n-m}\cdot\hat{\alpha}_m: + \frac{\beta}{2}\delta_{n,0}$$

$$- \frac{\beta a n^2}{R}e_+ \cdot \hat{\alpha}_n - \frac{\beta a^2 n^2}{R^2}e_{+\mu}e_{+\nu}\sum_{m=-\infty}^{\infty}\hat{\alpha}^\mu_{n-m}\hat{\alpha}^\nu_m + O(R^{-3}).$$

and

$$\partial_- \hat{Y}^\mu = a\sum_{m=-\infty}^{\infty}\hat{\alpha}^\mu_m e^{-im\tau^-} + O(R^{-4}).$$

Critical central charge [Polchinski,Strominger],

$$\beta = \frac{26 - D}{12}$$

Spectrum

Momentum:

$$p^\mu = \frac{R}{2a^2}(e_-^\mu + e_+^\mu) + \frac{1}{2a}(\alpha_0^\mu + \tilde{\alpha}_0^\mu) + O(R^{-4}).$$

Physical state conditions $L_0 = \tilde{L}_0 = 0$ imply spectrum is universal and equivalent to Nambu-Goto [Arvis] up to and including $O(R^{-3})$ terms.

e.g. ground state:

$$(-p^2)^{\frac{1}{2}} = \frac{R}{2a^2} + \frac{\beta - 2}{R} - \frac{a^2}{R^3}(\beta - 2)^2 + O(R^{-4}).$$

Discussion

- Polchinski-Strominger approach is conformal gauge approach to effective worldsheet theory.
- The theory has manifest D-dimensional Poincaré invariance.
- The action is singular unless expanded around the long string vacuum.
- Analysis can be pushed at least one order further than PS.
- Spectrum is universal up to and including $O(R^{-3})$ corrections to energy.
- The theory is essentially free up to this order.
- Higher order analysis needs to take into account genuine interactions.
- Quantum conformal invariance is crucial (gives a consistent algebra to impose physical state conditions).
- Puzzle: should be identical to static gauge analysis - (seems to be dimension dependent) ??.
- Parity odd terms...