k-strings: a review (lattice)

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Confining Flux Tubes and Strings, Trento, July 2010

Why studying k-strings on the lattice?

A first-principle computation can serve the following purposes:

- verify predictions from BSM computations (SUSY, MQCD, . . .)
- provide stringent tests for mechanisms of colour confinement
- shed more light on the dynamics of flux tubes

The lattice can in principle answer the following questions:

- Do k-string bound states form?
- If yes, what is the tension and how does it depend on k?
- Which mechanisms of confinement agree with numerical determinations of k-string tensions?
- What is the effective string description (tension, transverse size, Lüscher term, . . .) of a k-flux tube?
- Is the N-ality the only quantum number that characterises k-strings?

and others . . .

Outline

1. k-string properties from a variational procedure
2. Selected results from variational calculations
3. Recent developments
4. Conclusions

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k-string tensions (closed string channel)

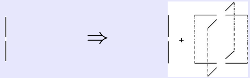
$$P^{\mathcal{R}}(t) = \frac{1}{d_{\mathcal{R}}} \sum_{\hat{k},\vec{n}\perp} \mathrm{Tr}_{\mathcal{R}} \prod_{j=0}^{l} U_k(\vec{n} + j\hat{k}, t)$$

$$C^{\mathcal{R}\mathcal{R}}(t) = \left\langle \left(P^{\mathcal{R}}(0)\right)^{\dagger} P^{\mathcal{R}}(t) \right\rangle = \sum_j |c_j^{\mathcal{R}\mathcal{R}}|^2 e^{-am_j^{\mathcal{R}} t} \underset{t\to\infty}{\longrightarrow} |c_l^{\mathcal{R}\mathcal{R}}|^2 e^{-am_l^{\mathcal{R}} t} \quad (1)$$

$$am_l^{\mathcal{R}} \simeq a^2 \sigma_{\mathcal{R}} l - c_{\mathcal{R}} \frac{\pi(D-2)}{6l}$$

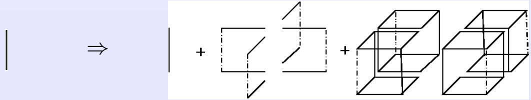
Link operators generate noisy correlators $\Rightarrow$ reduce ultraviolet fluctuations via

- Blocking



Fast increase of the size of the operators

- Smearing



Finer resolution

Correlation matrix

In a given representation, consider trial operators $\Phi_1(t), \dots, \Phi_n(t)$ with the quantum numbers of the state of interest

$$\begin{aligned}
C_{ij}(t) &= \langle 0|\left(\Phi_i(0)\right)^\dagger \Phi_j(t)|0\rangle \\
&= \langle 0|\left(\Phi_i(0)\right)^\dagger e^{-Ht}\Phi_j(0)e^{Ht}|0\rangle \\
&= \sum_n \langle 0|\left(\Phi_i(0)\right)^\dagger |n\rangle\langle n|e^{-Ht}\Phi_j(0)e^{Ht}|0\rangle \\
&= \sum_n e^{-\Delta E_{ln}t}\langle 0|\left(\Phi_i(0)\right)^\dagger |n\rangle\langle n|\Phi_j(0)|0\rangle \\
&= \sum_n c^*_{in}c_{jn}e^{-\Delta E_{ln}t} \underset{t\to\infty}{\longrightarrow} c^*_{i1}c_{j1}e^{-am_{l1}t}
\end{aligned}$$

Variational principle

1. Find the eigenvector v that minimises

$$am_{l1}(t_d) = -\frac{1}{t_d}\log\frac{v_i^* C_{ij}(t_d) v_j}{v_i^* C_{ij}(0) v_j}$$

   for some t_d
2. Fit $v(t)$ with the law $Ae^{-m_{l1}t}$ to extract m_{l1}
3. Find the complement to the space generated by $v(t)$
4. Repeat 1-3 to extract $E_{l2}, \ldots, E_{ln}$

A carefully constructed variational set allows to obtain a good overlap with the state of interest

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Expectations for σ_k

- No bound states $\sigma_k = k\sigma$
- SQRT Casimir scaling

$$\sigma_k = \sigma \sqrt{\frac{k(N-k)}{N-1}}$$

MIT bag model, wall vortex approximation

- (Asymptotic) Casimir Scaling

$$\sigma_k = \sigma \frac{k(N-k)}{N-1}$$

Dimensional reduction to D=2, one-gluon exchange, first order for Georgi-Glashow SU(N) model, U(1)^N Abelian Higgs model, ...

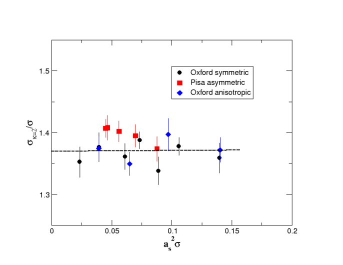
- Sine Law

$$\sigma_k = \sigma \frac{\sin(\pi k/N)}{\sin(\pi/N)}$$

SUSY, MQCD, spin systems in 2D, ...

Early results

1. A bound state forms [Wingate and Ohta, 2000]
2. In $D = 3$ the ratio of the string tensions is very close to Casimir scaling, while in $D = 4$ it is in-between Casimir Scaling and Sine Law [Lucini and Teper, 2000, 2001 and 2002; Lucini, Teper and Wenger, 2003]
3. The ratio is definitely Sine Law [Del Debbio, Panagopoulos, Rossi and Vicari, 2001 and 2002]



How definite is definite?

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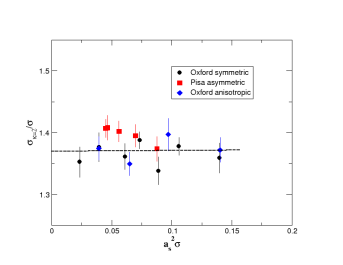
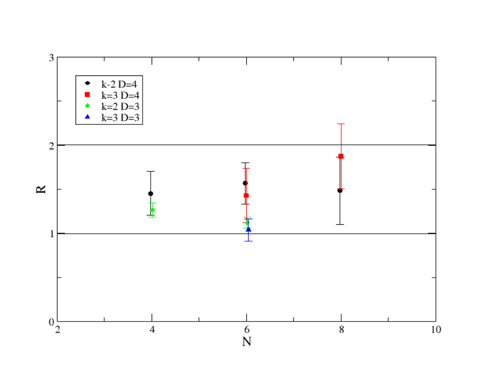


How definite is definite?

Casimir Scaling vs. Sine Law



There is no strong argument to expect exact CS or exact SL

$\hookrightarrow$ Better motivated questions are $1/N$ vs $1/N^2$ corrections for the ratio or centre vs. group for tensions

Leading behaviour as $N \to \infty$

1. k=2 (minimally saturated string)

   SL: $R/k \simeq 1 - \pi^2/2N^2$

   CS: $R/k \simeq 1 - 1/N$

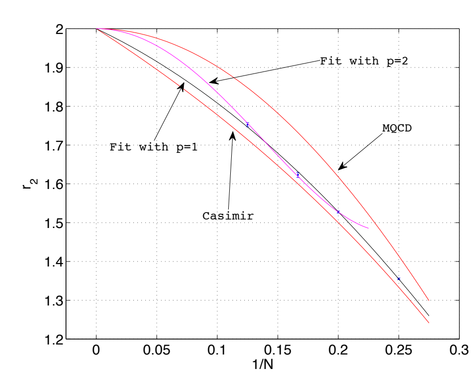
2. k=N/2 (maximally saturated string)

   SL: $R/N \simeq 2/\pi + \pi/(3N^2)$

   CS: $R/N \simeq 1/2 + 1/(2N)$

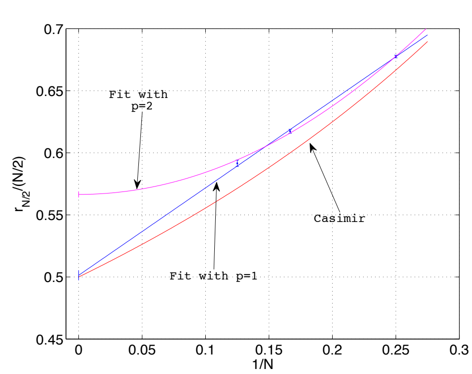
Better defined question for numerical simulations, but in principle we need calculations at fairly large N

σ_2/σ at Large N



Corrections $\mathcal{O}(1/N)$ seems to be preferred [Bringoltz and Teper, 2008]

$\sigma_{N/2}/\sigma$ at Large N



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Unstable strings

Numerical results [Lucini, Teper and Wenger, 2004; Bringoltz and Teper, 2006]:

- The lowest mass is absent in the spectrum of non-totally antisymmetric representations
- Casimir scaling is a good guidance to describe the ratios of tensions of unstable strings
- Unexpected pattern of degeneracy, e.g. $m_{3S} \simeq m^{*}_{3M} \simeq m^{**}_{3A}$

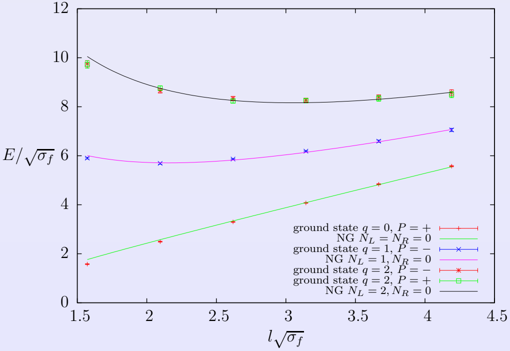
Interpretations

1. the asymptotic ground state does not develop at distances accessible to lattice simulations [Armoni and Lucini, 2006]
2. relevance of the whole gauge group [Bringoltz and Teper, 2008]

Decay amplitudes found to be smaller than 10^{-4} [Bringoltz and Teper, 2008]

$\hookrightarrow$ Enormously suppressed or zero?

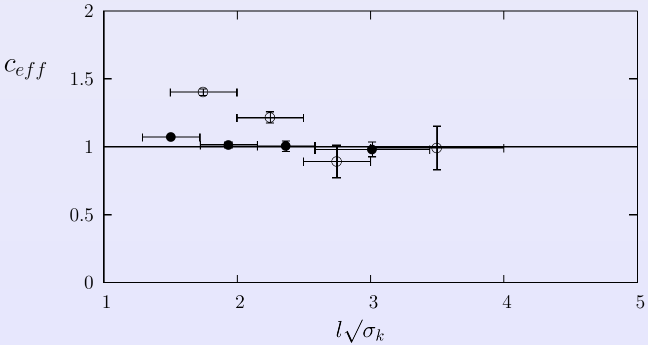
Higher excitations



Nambu-Goto good approximation at large l, corrections $\mathcal{O}(1)$ at low l

[Athenodorou, Bringoltz and Teper, 2008]

Lüscher term



Compatible with single-string oscillations [Teper and Meyer, 2004; Bringoltz and Teper, 2008]

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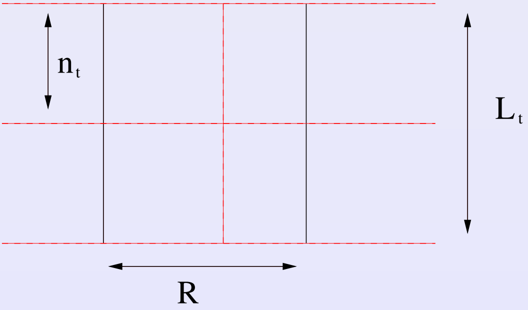
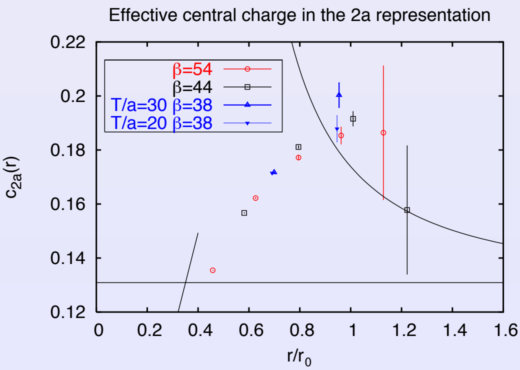
Lüscher–Weisz algorithm



- Temporal slicing: exponential increase in statistics
- Spacial slicing: quadratic increase in statistics

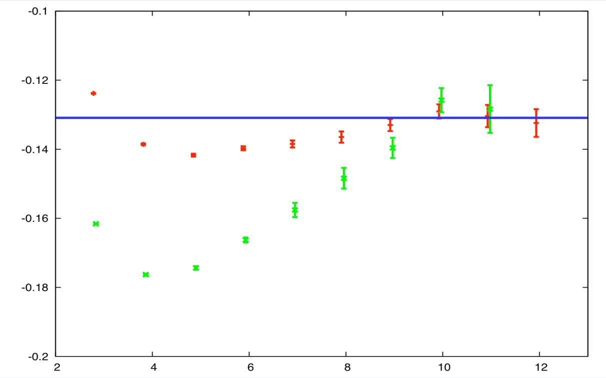
This algorithm allows to extract k-string tensions directly from link correlators on large lattices, but numerical simulations are challenging in D=3+1

Lüscher term – D=2+1 SU(5)



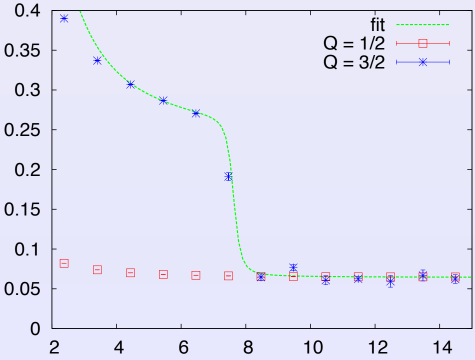
[Meyer, 2006]

Lüscher term – D=2+1 SU(4)



[Pepe, Plenary talk at Lattice 2010]

Decay of unstable strings – D=2+1 SU(2)



[Pepe and Wiese, 2009]

Decay amplitude – D=2+1 SU(3)

k-strings (lattice)
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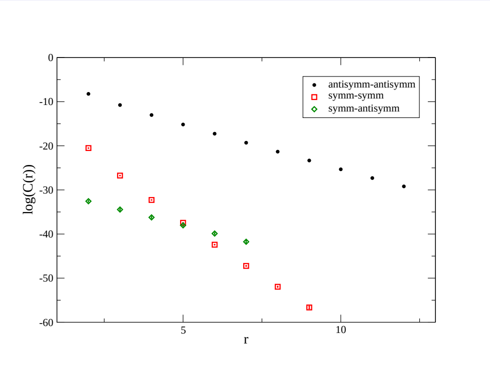
Variational procedure
Variational Calculations
Developments
Conclusions



[Lucini and Rago, to appear]

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Conclusions

- In SU(N) YM theory k-string bound states do form
- k-strings are well described by Nambu-Goto strings at large l
- The Lüscher term is the one expected for coherent oscillations
- Ratios of k-string tensions are numerically closer to but not compatible with CS in D=2+1, while nothing can be concluded in D=3+1
- The most accurate calculations to date find $1/N$ corrections for the ratio and classification according to the representation in D=2+1 $\Rightarrow$ more to be understood?
- More insights might come from studying excitations
- New studies exploiting more efficient algorithms are currently being performed, and new exciting results are starting to appear $\Rightarrow$ clearer picture expected to emerge soon