

THE HAMILTONIAN APPROACH TO YANG-MILLS (2+1)

SYSTEMATICS & UPDATE

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The key points of our analysis are:

1. Parametrization of potentials: $A_0 = 0$ and we use complex coordinates $z = x_1 - ix_2$ with

$$\frac{1}{2}(A_1 + iA_2) = -\partial M M^{-1}, \quad \frac{1}{2}(A_1 - iA_2) = M^{\dagger-1} \bar{\partial} M^{\dagger}$$

$M \in SL(N, \mathbb{C})$, for gauge group $SU(N)$. (More generally, $G \Rightarrow G^{\mathbb{C}}$.)

2. Wave functions are gauge-invariant (Gauss law) and depend on $H = M^{\dagger} M$ with the inner product

$$\langle 1|2 \rangle = \int d\mu(H) \exp[2 c_A S_{wzw}(H)] \Psi_1^* \Psi_2$$

$$S_{wzw}(H) = \frac{1}{2\pi} \int \text{Tr}(\partial H \bar{\partial} H^{-1}) + \frac{i}{12\pi} \int \epsilon^{\mu\nu\alpha} \text{Tr}(H^{-1} \partial_{\mu} H H^{-1} \partial_{\nu} H H^{-1} \partial_{\alpha} H)$$

$d\mu(H)$ = Haar measure for H ; $c_A = N$ = adjoint Casimir value.

3. $H \in SL(N, \mathbb{C})/SU(N)$ is the basic gauge-invariant variable for the theory.

- The Hamiltonian and the wave functions can be expressed as functions of the (scaled version of the) current $J = (2/e)\partial H H^{-1}$.
- The Wilson loop operator is given by

$$W(C) = \text{Tr } \mathcal{P} e^{-\oint A} = \text{Tr } \mathcal{P} \exp\left(\frac{e}{2} \oint J\right)$$

All gauge-invariant quantities can be made from J .

- The Hamiltonian is $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$ with

$$\begin{aligned} \mathcal{H}_0 &= m \int_z J_a(\vec{z}) \frac{\delta}{\delta J_a(\vec{z})} + \frac{2}{\pi} \int_{z,w} \frac{1}{(z-w)^2} \frac{\delta}{\delta J_a(\vec{w})} \frac{\delta}{\delta J_a(\vec{z})} \\ &\quad + \frac{1}{2} \int_x : \bar{\partial} J^a(x) \partial J^a(x) : \\ \mathcal{H}_1 &= i e f_{abc} \int_{z,w} \frac{J^c(\vec{w})}{\pi(z-w)} \frac{\delta}{\delta J_a(\vec{w})} \frac{\delta}{\delta J_b(\vec{z})} \end{aligned}$$

where $m = e^2 c_A / 2\pi$.

All calculations have to be done with proper regularization.

- We start with a regularization of the δ -function

$$\delta^{(2)}(u, w) \implies \sigma(\vec{u}, \vec{w}, \epsilon) = \frac{1}{\pi\epsilon} \exp\left(-\frac{|u-w|^2}{\epsilon}\right)$$

- This is equivalent to

$$\begin{aligned} \bar{G}(\vec{x}, \vec{y}) &= \frac{1}{\pi(x-y)} \\ \implies \bar{G}(\vec{x}, \vec{y}) &= \int_u \bar{G}(\vec{x}, \vec{u}) \sigma(\vec{u}, \vec{y}; \epsilon) H(u, \vec{y}) H^{-1}(y, \vec{y}) \end{aligned}$$

- This simplifies as

$$\bar{G}_{ma}(x, y) = \frac{1}{\pi(x-y)} \left[\delta_{ma} - e^{-\frac{(x-y)^2}{\epsilon}} [H(x, \vec{y}) H^{-1}(y, \vec{y})]_{ma} \right]$$

All results checked using regularized expressions, with a single regulator from beginning to end.

7. By comparison to a resummed perturbation theory, m can be identified as the magnetic mass, relevant for the QCD plasma.
8. For $SU(2)$, this value is $m \approx 0.32 e^2$ which can be compared with other estimates:

$m/e^2 =$	0.35	Common factor for glueball masses (lattice, PHILIPSEN)
	0.51	Max. Abelian gauge (lattice, KARSCH ET AL)
	0.52	Landau gauge (")
	0.44	$\lambda_3 = 2$ gauge (")
	0.38	Resummation of P.T. (ALEXANIAN & VPN)
	0.28	Resummation of P.T. (BUCHMULLER & PHILIPSEN, JACKIW & PI)
	0.37	Gauge-invariant lattice definition (PHILIPSEN)

9. The vacuum wave function was found as $\Psi_0 = \exp\left[\frac{1}{2}F(H)\right]$,

$$F(H) = - \int \bar{\partial} J_a \left[\frac{1}{(m + \sqrt{m^2 - \nabla^2})} \right] \bar{\partial} J_a \\ + 2f_{abc} \int f^{(3)}(\vec{x}, \vec{y}, \vec{z}) J_a(\vec{x}) J_b(\vec{y}) J_c(\vec{z}) + \mathcal{O}(J^4)$$

10. This has the approximate forms

$$F \approx - \int B \frac{1}{\sqrt{-\nabla^2}} B \quad (\text{high momenta}) \\ \approx \int \frac{B^2}{2me^2} \quad (\text{low momenta})$$

11. This gave values of string tension

$$\sqrt{\sigma_R} = e^2 \sqrt{\frac{c_A c_R}{4\pi}},$$

in agreement with lattice values to within 1 – 3%.

Group	Representations					
	k=1 Fund.	k=2 antisym	k=3 antisym	k=2 sym	k=3 sym	k=3 mixed
$SU(2)$	0.345 0.335					
$SU(3)$	0.564 0.553					
$SU(4)$	0.772 0.759	0.891 0.883		1.196 1.110		
$SU(5)$	0.977 0.966					
$SU(6)$	1.180 1.167	1.493 1.484	1.583 1.569	1.784 1.727	2.318 2.251	1.985 1.921
$SU(N)$ $N \rightarrow \infty$	0.1995 N 0.1976 N					

Comparison of $\sqrt{\sigma}/e^2$ with lattice estimates (lower entry, in red) from [LUCINI & TEPER](#), [BRINGOLTZ & TEPER](#). k is the rank of the representation.

- String tension for $SU(N)$ at $N \rightarrow \infty$ was also calculated by KISKIS & NARAYANAN, who obtained $0.1964 N$, a deviation of $\sim 1.55\%$
- Analysis of theories without center such as G_2, F_4, E_8 : Motivated by the fact that in these theories one expects string breaking for all representations.
- Calculation of string tension (at distances before string breaking) done for G_2 , for representations $\underline{7}, \underline{14}, \underline{27}, \underline{64}, \underline{77}, \underline{77'}, \underline{182}, \underline{189}$, giving

$$\sigma_R = \sigma_7 \times \frac{c_R}{c_7}, \quad \text{to 1\% accuracy}$$

(GREENSITE *et al*, LIPTAK & OLEJNIK, WELLEGHAUSEN, WIPF & WOZAR)

- Calculate $\beta^2 \sigma_7$ for different values of $\beta = 30, 35, 40$ and find $\beta^2 \sigma_7 = 41.4, 41.7, 38.4$.
- String breaking calculated for $\underline{7}$ and $\underline{14}$ and breaking energy agrees with $2 \times$ glue-lump energy.

- Absorb $\exp(2c_A S_{wzw})$ from the inner product into the wave function by $\Psi = e^{-c_A S_{wzw}(H)} \Phi$.

The Hamiltonian acting on Φ is

$$\mathcal{H} \rightarrow e^{-c_A S_{wzw}(H)} \mathcal{H} e^{-c_A S_{wzw}(H)}$$

- Consider $H = e^{t^a \varphi^a} \approx 1 + t^a \varphi^a + \dots$, a small φ limit appropriate for a (resummed) perturbation theory. The new Hamiltonian is

$$\mathcal{H} = \frac{1}{2} \int \left[-\frac{\delta^2}{\delta \phi^2} + \phi(-\nabla^2 + m^2)\phi + \dots \right]$$

where $\phi_a(\vec{k}) = \sqrt{c_A k \bar{k} / (2\pi m)} \varphi_a(\vec{k})$.

- The vacuum wave function is

$$\Phi_0 \approx \exp \left[-\frac{1}{2} \int \phi^a \sqrt{m^2 - \nabla^2} \phi^a \right]$$

- Transforming back to Ψ ,

$$\Psi_0 \approx \exp \left[-\frac{c_A}{\pi m} \int (\bar{\partial} \partial \varphi^a) \left[\frac{1}{m + \sqrt{m^2 - \nabla^2}} \right] (\bar{\partial} \partial \varphi^a) + \dots \right]$$

- The full wave function must be a functional of J . The only form consistent with the above is

$$\Psi_0 = \exp \left[-\frac{2\pi^2}{e^2 c_A^2} \int \bar{\partial} J^a(x) \left[\frac{1}{m + \sqrt{m^2 - \nabla^2}} \right]_{x,y} \bar{\partial} J^a(y) + \dots \right]$$

since $J \approx (c_A/\pi) \partial \varphi + \mathcal{O}(\varphi^2)$.

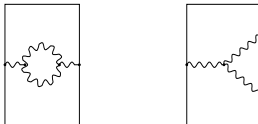
- This indicates the robustness of the Gaussian term in Ψ_0 , since this argument only presumes
 - Existence of a regulator, so that the transformation $\Psi \iff \Phi$ can be carried out
 - The two-dimensional anomaly calculation

- The large N limit of string tension differs from lattice value by only $\sim 1\%$ (BRINGOLTZ & TEPER) to $\sim 1.55\%$ (KISKIS & NARAYANAN).
- The difference is small, but considered statistically significant.
- How can such corrections arise?
 - Higher terms in recursive solution of Schrödinger equation: The expression for Ψ_0 can get corrections.
 - For string tension, corrections can also arise from $\mathcal{O}(J^3)$ and higher terms in Ψ_0 , when the integration in $\int W(C)\Psi_0^*\Psi_0$ is carried out.
- To get parametric control on the calculation with higher terms, define an extension of the theory with e^2 and m as independent parameters.
- We set $m = e^2 c_A / 2\pi$ at the end. The smallness of the corrections have to come from numerical factors.

- Two types of corrections possible
 - Corrections to coupling, numerical and R -independent \Rightarrow ratios σ_R/σ_F unaffected.

For this, we will calculate the correction to the coefficient of $\bar{\partial}J\bar{\partial}J$ in Ψ_0 in a systematic expansion scheme (KARABALI, NAIR, YELNIKOV)

- Corrections via new diagrams to Wilson line expectation value (under investigation, related to screening and string breaking)



We do not have a calculation of the matrix element, but the energy E_* at which the string breaks can be estimated (AGARWAL, KARABALI & NAIR)

- Solve Schrödinger equation as a power series in e , treating m and e as independent parameters
- This gives $\Psi_0^* \Psi_0 = e^F$ with

$$\begin{aligned}
 F = & \int f_{a_1 a_2}^{(2)}(x_1, x_2) J^{a_1}(x_1) J^{a_2}(x_2) + \frac{e}{2} f_{a_1 a_2 a_3}^{(3)}(x_1, x_2, x_3) J^{a_1}(x_1) J^{a_2}(x_2) J^{a_3}(x_3) \\
 & + \frac{e^2}{4} f_{a_1 a_2 a_3 a_4}^{(4)}(x_1, x_2, x_3, x_4) J^{a_1}(x_1) J^{a_2}(x_2) J^{a_3}(x_3) J^{a_4}(x_4) + \dots
 \end{aligned}$$

$f^{(2)}, f^{(3)}$, etc., are determined recursively, by solving the Schrödinger equation, $\mathcal{H}\Psi_0 = 0$.

- Recall that the Hamiltonian is given as $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$,

$$\begin{aligned}
 \mathcal{H}_0 = & m \int_z J_a(\vec{z}) \frac{\delta}{\delta J_a(\vec{z})} + \frac{2}{\pi} \int_{z,w} \frac{1}{(z-w)^2} \frac{\delta}{\delta J_a(\vec{w})} \frac{\delta}{\delta J_a(\vec{z})} \\
 & + \frac{1}{2} \int_x : \bar{\partial} J^a(x) \bar{\partial} J^a(x) : \\
 \mathcal{H}_1 = & i e f_{abc} \int_{z,w} \frac{J^c(\vec{w})}{\pi(z-w)} \frac{\delta}{\delta J_a(\vec{w})} \frac{\delta}{\delta J_b(\vec{z})}
 \end{aligned}$$

- This leads to recursion rules for f 's of the form

$$\begin{aligned}
 & mpf_{a_1 \dots a_p}^{(p)} + \sum_{n=2}^p n(p+2-n) f_{a_1 \dots a_{n-1} a}^{(n)} (\bar{\Omega}^0)_{ab} f_{ba_n \dots a_p}^{(p-n+2)} \\
 & + \sum_{n=2}^{p-1} n(p+1-n) f_{a_1 \dots a_{n-1} a}^{(n)} (\bar{\Omega}^1)_{aba_p} f_{ba_n \dots a_{p-1}}^{(p-n+1)} \\
 & + e^2 \left[\frac{(p+1)(p+2)}{2} f_{a_1 \dots a_p ab}^{(p+2)} (\bar{\Omega}^0)_{ab} + \frac{p(p+1)}{2} f_{a_1 \dots a_{p-1} ab}^{(p+1)} (\bar{\Omega}^1)_{aba_p} \right] = 0
 \end{aligned}$$

$$(\bar{\Omega}^0)_{ab}(x, y) = \delta_{ab} \partial_y \bar{G}(x, y)$$

$$(\bar{\Omega}^1)_{abc}(x, y, z) = -\frac{i}{2} f^{abc} [\delta(z-y) + \delta(z-x)] \bar{G}(x, y)$$

$$V_{ab}(x, y) = \delta_{ab} \int_z \bar{\partial}_z \delta(z-x) \bar{\partial}_z \delta(z-y)$$

- Use these to calculate f 's to the required order.

- Each function $f^{(n)}$ has a series expansion in powers of e^2 ,

$$f_{a_1 a_2}^{(2)}(x_1, x_2) = f_{0 a_1 a_2}^{(2)}(x_1, x_2) + e^2 f_{2 a_1 a_2}^{(2)}(x_1, x_2) + \dots$$

$$f_{0 a_1 a_2 a_3}^{(3)}(x_1, x_2, x_3) = f_{0 a_1 a_2 a_3}^{(3)}(x_1, x_2, x_3) + e^2 f_{2 a_1 a_2 a_3}^{(3)}(x_1, x_2, x_3) + \dots$$

$$f_{0 a_1 \dots a_4}^{(4)}(x_1, \dots, x_4) = f_{0 a_1 \dots a_4}^{(4)}(x_1, \dots, x_4) + e^2 f_{2 a_1 \dots a_4}^{(4)}(x_1, \dots, x_4) + \dots$$

- To the lowest order,

$$f_{0 a_1 a_2}^{(2)}(x_1, x_2) = \delta_{a_1 a_2} \left[-\frac{\bar{q}^2}{m + E_q} \right]_{x_1, x_2}$$

- We calculate corrections to order e^2 . For this, we need $f^{(3)}$ and $f^{(4)}$ to the lowest nontrivial order in e . For example,

$$f_{0 a_1 a_2 a_3}^{(3)}(k_1, k_2, k_3) = -\frac{f_{a_1 a_2 a_3}}{24} (2\pi)^2 \delta(k_1 + k_2 + k_3) g^{(3)}(k_1, k_2, k_3)$$

$$g^{(3)}(k_1, k_2, k_3) = \frac{16}{E_{k_1} + E_{k_2} + E_{k_3}} \left\{ \frac{\bar{k}_1 \bar{k}_2 (\bar{k}_1 - \bar{k}_2)}{(m + E_{k_1})(m + E_{k_2})} + \text{cycl. perm.} \right\}$$

There is a similar, but more complicated, formula for $f^{(4)}$.

- The correction to $f^{(2)}$ from the recursive solution of the Schrödinger equation is

$$\begin{aligned}
 e^2 f_2^{(2)}(q) &= \frac{m}{E_q} \left(\int \frac{d^2k}{32\pi} \frac{1}{k} g^{(3)}(q, k, -k - q) + \int \frac{d^2k}{64\pi} \frac{k}{k} g^{(4)}(q, k; -q, -k) \right) \\
 &\approx \frac{\bar{q}^2}{2m} (1.1308) + \dots
 \end{aligned}$$

(We have set $e^2 = 2\pi m/c_A$.)

- Seemingly, this is about 113% correction, but other “loop-contributions” are important.
- Integration over J 's in $\int W(C) \Psi_0^* \Psi_0$ can be expressed as the functional integral of a two-dimensional field theory for a complex **unconstrained** bosonic field.

- The expectation value of an observable \mathcal{O} is

$$\begin{aligned}
 \langle \mathcal{O} \rangle &= \int d\mu(H) e^{2c_A S_{wzw}(H)} \Psi_0(J) \Psi_0(J) \mathcal{O}(J) \\
 &= \int \underbrace{d\mu(H) e^{2c_A S_{wzw}(H)}} e^{F(J)} \mathcal{O}(J) \\
 &= \left[\mathcal{O}(\hat{J}) e^{F(\hat{J})} \right] \int d\mu(H) e^{2c_A S_{wzw}(H)} e^{-\frac{c_A}{\pi} \int \bar{C}^a (\partial H H^{-1})^a} \Big]_{\bar{C}=0}
 \end{aligned}$$

where $\hat{J}^a = -\sqrt{2\pi/mc_A} \frac{\delta}{\delta \bar{C}^a}$.

- The remaining integral can be evaluated as

$$\begin{aligned}
 \int d\mu(H) e^{2c_A S_{wzw}(H)} e^{-\frac{c_A}{\pi} \int \bar{C}^a (\partial H H^{-1})^a} &= \int d\mu(H) e^{2c_A S_{wzw}(UH) - 2c_A S_{wzw}(U)} \\
 &= e^{-2c_A S_{wzw}(U)} \\
 &= \int [d\varphi d\bar{\varphi}] e^{-\int \bar{\varphi}(\bar{\partial} + \bar{C})\varphi}
 \end{aligned}$$

where $\bar{C} = U^{-1} \bar{\partial} U$ and we used the Polyakov-Wiegmann identity

$$S_{wzw}(H) - \frac{1}{\pi} \int \text{Tr}(\bar{C} \partial H H^{-1}) = S_{wzw}(UH) - S_{wzw}(U)$$

- The complex boson field φ transforms as the adjoint representation of $SU(N)$. Use in equation for $\langle \mathcal{O} \rangle$ to get


$$\langle \mathcal{O} \rangle = \int [d\varphi d\bar{\varphi}] e^{-S(\varphi)} \mathcal{O}(\sqrt{2\pi/mc_A} \bar{\varphi} t^a \varphi)$$

where

$$S(\varphi) = \int (Z_2 \bar{\varphi} \partial \varphi + Z_1 \bar{\varphi} \bar{C} \varphi) - F(Z_1 \sqrt{2\pi/mc_A} \bar{\varphi} t^a \varphi)$$

(\bar{C} can be set to 0 at this point.)

- Notice that $J^a \rightarrow Z_1 \sqrt{2\pi/mc_A} \bar{\varphi} t^a \varphi$.
- $F(Z_1 \sqrt{2\pi/mc_A} \bar{\varphi} t^a \varphi)$ contains vertices, $F^{(2)}$ with two currents (quartic in $\varphi, \bar{\varphi}$), $F^{(3)}$ with three currents, etc. For example,

$$F^{(2)} = \frac{2\pi}{mc_A} \int (\bar{\varphi} t^a \varphi)_x f^{(2)}(x, y) (\bar{\varphi} t^a \varphi)_y =$$


- Evaluate loop corrections to the action in this two-dimensional field theory
- For string tension, in particular, we need corrections to $f^{(2)}$. We should calculate the corrected quartic vertex, i.e., **correction to $F^{(2)}$** (like a Wilsonian RG reduction).
- So our strategy is:
 - Obtain all loop corrections to the given order from the interaction terms
 - Numerical values of loop corrections decrease with increasing powers of m/E_k , so group loop corrections by powers of $m/E_k \equiv m/\sqrt{k^2 + m^2}$, calculate all contributions to a given order in m/E_k .
 - Set $m = e^2 c_A / 2\pi$ at the end

$$\Psi_0^* \Psi_0 = \exp \left(-\frac{2\pi^2}{me^2 c_A^2} \int \bar{\partial} J \bar{\partial} J + \dots \right) \implies \exp \left(-\frac{2\pi^2}{g_*^2 c_A^2} \int \bar{\partial} J \bar{\partial} J + \dots \right)$$

We need g_* .

- Vertices $F^{(3)}, F^{(4)}$, etc., can be treated in a perturbation series (loop expansion)
- But we need to sum all contributions due to $F_0^{(2)}$, because it does not have e^2 .

$$\langle \bar{\varphi} t^a \varphi(x) \bar{\varphi} t^b \varphi(y) \rangle = x \text{---} \text{loop} \text{---} y + x \text{---} \text{loop} \text{---} \text{blob} \text{---} \text{loop} \text{---} y +$$

$$+ x \text{---} \text{loop} \text{---} \text{blob} \text{---} \text{loop} \text{---} \text{blob} \text{---} \text{loop} \text{---} y + \dots$$

- The current correlator becomes

$$\langle \bar{\varphi} t^a \varphi(x) \bar{\varphi} t^b \varphi(y) \rangle = \delta^{ab} \frac{C_A}{\pi} \int \frac{d^2 k}{(2\pi)^2} e^{ik(x-y)} \frac{k}{\bar{k}} \left(\frac{m}{E_k} \right)$$

- The effective current vertex is

$$\left[\bar{\varphi} t^a \varphi(x) \right]_{eff} = \int \frac{d^2 k}{(2\pi)^2} e^{ik(x-z)} \frac{m}{E_k} (\bar{\varphi} t^a \varphi)(z)$$

as given by the diagram

$$\left[\bar{\varphi} t^a \varphi(x) \right]_{eff} = x \text{---} \text{blob} + x \text{---} \text{loop} \text{---} \text{blob} +$$

$$+ x \text{---} \text{loop} \text{---} \text{blob} \text{---} \text{loop} \text{---} \text{blob} + \dots$$

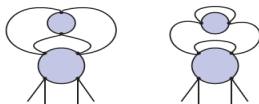
STRATEGY:

- Construct loop diagrams generated by $F^{(3)}$ (3 factors of $\bar{\varphi}t^a\varphi$) and $F^{(4)}$ (4 factors of $\bar{\varphi}t^a\varphi$).
- They can have arbitrary insertions of $F^{(2)}$'s; e.g.,







This leads to a factor of m/E_k .

- There are also renormalizations (due to $F^{(2)}$) we have to take into account; e.g.,

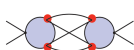


- Sum up $F^{(2)}$ insertions in all diagrams (of order e^2) generated by $F^{(3)}$ and $F^{(4)}$.
- Classify and group these by the number of factors of m/E_k .
- We will compute corrections of order e^2 and up to 4 powers of m/E_k .

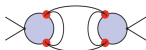
<u>Diagram</u>	<u>Powers of m/E_k</u>	<u>Low momentum value</u>
	m/E_k	$\frac{\bar{q}^2}{2m} (-0.58118) + \dots$
	$(m/E_k)^2$	$\frac{\bar{q}^2}{2m} (-0.47835) + \dots$
	$(m/E_k)^2$	$\frac{\bar{q}^2}{2m} (0.20169) + \dots$
	$(m/E_k)^3$	$\frac{\bar{q}^2}{2m} (-0.23569) + \dots$

Red dots indicate factors of m/E_k due to resummation of $F_0^{(2)}$ insertions.

There are 6 diagrams at the $(m/E_k)^4$ -order:



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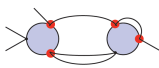
$\frac{\bar{q}^2}{2m} (0.02083) + \dots$



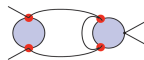
$\frac{\bar{q}^2}{2m} (-0.06893) + \dots$



$\frac{\bar{q}^2}{2m} (-0.01216) + \dots$



$\frac{\bar{q}^2}{2m} (0.06824) + \dots$



$\frac{\bar{q}^2}{2m} (-0.1037) \text{ to } (-0.166)$

- We can now sum up the contributions, including the correction from the recursion rules,

$$\Delta f_{Rec}^{(2)} \approx \frac{\bar{q}^2}{2m} (1.1308) + \dots$$

- Let C_n be the sum of corrections up to terms with $(m/E_k)^n$
- These partial sums are

$$C_0 = 1.13082$$

$$C_1 = 0.54964$$

$$C_2 = 0.27298$$

$$C_3 = 0.03729$$

$$C_4 = \begin{cases} -0.05843 \\ -0.00583 \end{cases}$$

- Remarks
 - The partial sums are systematically decreasing in value
 - The corrections to this order are small, much smaller than might be expected on parametric grounds or by computing one or two diagrams.

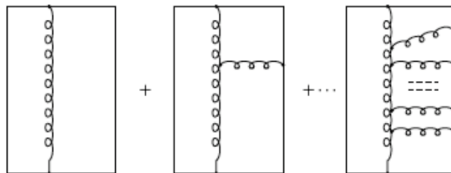
- The correction to string tension is then

$$\sqrt{\sigma_R} = e^2 \sqrt{\frac{c_A c_R}{4\pi}} \begin{cases} (1 - 0.02799 + \dots) \\ (1 - 0.0029 + \dots) \end{cases}$$

- This correction, of the order of -2.8% to -0.03% is entirely consistent with lattice calculations.
- Terms of order $(m/E_k)^5$ are expected to contribute only at the level of a fraction of 1% .
- Diagrams with two current-loops will also have powers of (m/E_k) , so their contributions might also be small.

A systematic expansion scheme is possible for $YM(2+1)$, despite lack of obvious parameters, with calculable small corrections to string tension.

- This is due to the breaking of the string to glue-lump states. The glue-lump is made of the external charge and a “gluon”.
- There are two issues here:
 1. The matrix element for *String* \implies *Glue-lumps*
 2. The energy at which the string breaks (We estimate this.)
- The bound state can arise from diagrams like this which are in the integral over $\Psi_0^* \Psi_0$



- A glue-lump state is given by

$$\Psi_G = \int d^2x d^2y f(\vec{x}, \vec{y}) \bar{\partial} J^a(\vec{x}) \left[\mathcal{P} e^{\int_{\vec{y}}^{\vec{x}} \partial_{HH}^{-1}} \right]^{ab} \chi^b(\vec{y}) \Psi_0$$

$\chi = M^\dagger \phi M^{\dagger-1} =$ A heavy scalar field of mass μ

- We obtain an eigenstate of the Hamiltonian if $f(x, y)$ obeys the equation

$$\left[\mu + m - \frac{\nabla_x^2}{2m} + \sigma_A |\vec{x} - \vec{y}| \right] f(\vec{x}, \vec{y}) \approx E f(\vec{x}, \vec{y})$$

(This equation is valid when e^2 is large, $\mu \rightarrow \infty$ and $N \rightarrow \infty$.)

- A variational estimate gives $E_* \approx 7.92 m$.
- Differs by $\sim 8.8\%$ from the lattice estimate of $\approx 8.68 m$ for $SU(2)$ (DE FORCRAND & KRATOCHVILA).
- Will be interesting to compare the case of G_2 , for which WELLEGHAUSEN, WIPF & WOZAR have E_* for representations 7 and 14.

- Where are we?

- Vacuum wave function is of the form

$$\Psi_0 = \exp \left[-\frac{2\pi^2}{e^2 c_A^2} \int \bar{\partial} J^a(x) \left[\frac{1}{m + \sqrt{m^2 - \nabla^2}} \right]_{x,y} \bar{\partial} J^a(y) + \dots \right]$$

- A systematic expansion scheme for which this is the lowest order result
- String tensions: lowest order and corrections of order a few percent
- Possibility of screening of $W_R(C)$ via string-breaking in some representations
- Extension to YM-CS theory (KARABALI, KIM, NAIR) and to $\mathbb{R} \times S^2$ (AGARWAL, NAIR)

- More questions and comments

- Use of $\mathbb{R} \times S^2$ to reorganize the expansion scheme
- Properties of the measure and the configuration space $\mathcal{A}/\mathcal{G}_*$
- Glueballs
- Calculations on the torus, deconfinement
- More accurate calculation of higher order corrections to string tension
- Fermions, supersymmetric cases

- The measure has the property

$$\int d\mu(C) = \int d\mu(H) e^{2c_A S_{wz\bar{w}}(H)} < \infty$$

This is related to the “mass gap”.

- The Hamiltonian has the form

$$\mathcal{H} = \frac{1}{2} \int \left[e^2 E^2 + B^2/e^2 \right]$$

$[E, B] \sim p$ (in momentum space) $\implies \Delta E \Delta B \sim p$, or $\Delta E \sim p/\Delta B$

$$\mathcal{E} = \langle \mathcal{H} \rangle \approx \frac{1}{2} \left[e^2 \frac{p^2}{(\Delta B)^2} + \frac{(\Delta B)^2}{e^2} \right]$$

Minimize with respect to $\Delta B \implies (\Delta B)^2 \sim p \implies \mathcal{E} \sim p$. This is the **photon**.

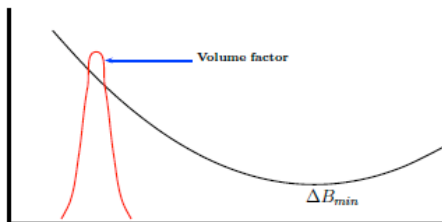
- For us

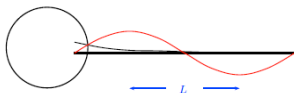
$$\langle \mathcal{H} \rangle = \int d\mu(H) \exp[2 c_A S_{wz\bar{w}}(H)] \int \frac{1}{2} [e^2 E^2 + B^2/e^2]$$

- Expanding the WZW action

$$\langle \mathcal{H} \rangle \approx \int d\mu(H) \exp \left[-\frac{c_A}{2\pi} \int B \frac{1}{p^2} B + \dots \right] \int \frac{1}{2} \left[e^2 E^2 + \frac{B^2}{e^2} \right]$$

Gaussian distribution for B Width $(\Delta B)^2 \sim \pi p^2 / c_A$, very narrow for small momenta, \implies mass gap $\sim e^2 c_A / 2\pi$.





- Even though the total volume is finite, the configuration space has “spikes”.
- Define length between two configurations as

$$L^2(A, A') = \text{Inf}_g \int d^2x \text{Tr}(A' - A^g)^2$$

- In $SU(2)$ gauge theory,

$$A = (-it^3) \ln(z\bar{z})^{n-1} \frac{(z d\bar{z} - \bar{z} dz)}{[1 + (z\bar{z})^n]}$$

has $s_C^2(A, 0) = 8\pi n$, so that for $n \geq (\lambda^2/8\pi)$, its distance from $A = 0$ is arbitrarily large.

- The spikes have zero measure, which is why a gap is possible.

- There may be a useful generalization to 3+1 dimensions. Define

$$d\mu(\mathcal{C})_{3d} = \frac{[dA]}{\text{vol}(\mathcal{G}_*)} \exp\left(-\frac{1}{4\mu} \int F^2\right) \Big]_{\mu \rightarrow \infty}$$

- This leads to

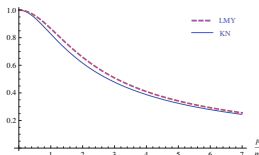
$$\begin{aligned} \int d\mu(\mathcal{C})_{3d} &= \int \frac{[dA]}{\text{vol}(\mathcal{G}_*)} e\left(-\frac{1}{4\mu} \int F^2\right) \Big]_{\mu \rightarrow \infty} \\ &= \langle 0 | e^{-\beta\mathcal{H}} | 0 \rangle \Big]_{\beta, \mu \rightarrow \infty} = \int d\mu(\mathcal{C})_{2d} \Psi_0^* \Psi_0 \Big]_{\mu \rightarrow \infty} \\ &= \int d\mu(\mathcal{C})_{2d} \exp\left(-\frac{\pi}{2\mu^2 c_A} \int F^2\right) < \infty \end{aligned}$$

- There has been an analysis of glueball states (LEIGH, MINIC, YELNIKOV)
 - The (ansatz for the) vacuum wave function is taken to be

$$\Psi_0 = \exp\left[-\frac{1}{4m} \int \bar{\partial} J K[L] \bar{\partial} J\right] \quad K[L] = \frac{J_2(4\sqrt{L})}{\sqrt{L} J_1(4\sqrt{L})}$$

Here $L = \mathcal{D}\bar{\partial}/m^2$, and J_1, J_2 are Bessel functions of orders 1 and 2 respectively.

- The kernel, despite appearances, is very close to the KKN kernel.



- Correlators of some gauge-invariant operators can be estimated and leads to values for glueball masses which are to within a few percent of lattice values.

State	LMY Calculation	Lattice
0^{++}	4.098	4.065 ± 0.055
0^{++*}	5.407	6.18 ± 0.13
0^{++**}	6.716	7.99 ± 0.22
0^{++***}	7.994	9.44 ± 0.38
0^{--}	6.15	5.91 ± 0.25
0^{--*}	7.46	7.63 ± 0.37
0^{--**}	8.77	8.96 ± 0.65
2^{++}	6.72	6.88 ± 0.16
2^{++*}	7.99	8.62 ± 0.38
2^{++**}	9.26	9.22 ± 0.32
2^{+-}	8.76	8.04 ± 0.50
2^{--}	8.76	7.89 ± 0.35
2^{+-*}	10.04	9.97 ± 0.91
2^{--*}	10.04	9.46 ± 0.46

Lattice values are from MEYER & TEPER and TEPER

- A better strategy might be to extract an effective string action from our formalism

- On the torus, the Hamiltonian has a part which is Laplacian for the zero modes,

$$A_z = M \left[\frac{i\pi a}{\text{Im } \tau} \right] M^{-1} - \partial_z M M^{-1}$$

- As one of the torus directions becomes small, there is an accumulation of the eigenvalues of the Laplacian for the zero modes.
- This could be the signal for deconfinement.