

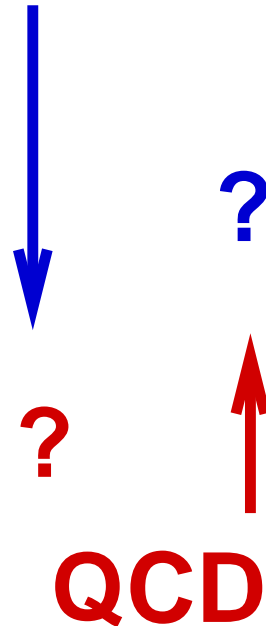
# **AdS/QCD and Charmonium at Finite Temperature — Bottom-up approach**

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with Grigoryan and Hohler, arXiv:1003.1138 [hep-ph]

# String dual



- Bottom-up approach:
  - Start with QCD and use holographic correspondence to deduce features to be expected from its string dual.
  - Even without known UV completion, this gives useful models for QCD.

# Motivation

- Matsui-Satz '86:  $J/\psi$  suppression — a signature for QGP.

Qualitative mechanism: Debye screening of chromoelectric attraction between  $c$  and  $\bar{c}$ .

Quantifying this effect proves difficult.

- Potential models of charmonium

Karsch-Mehr-Satz '88: color Coulomb + linear  $\longrightarrow$  Debye.

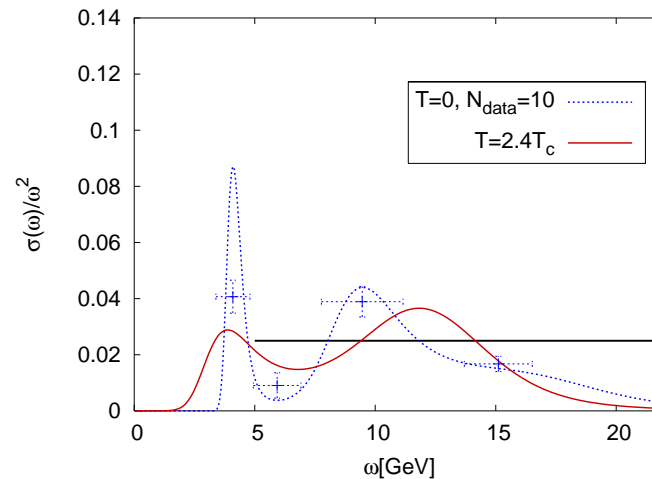
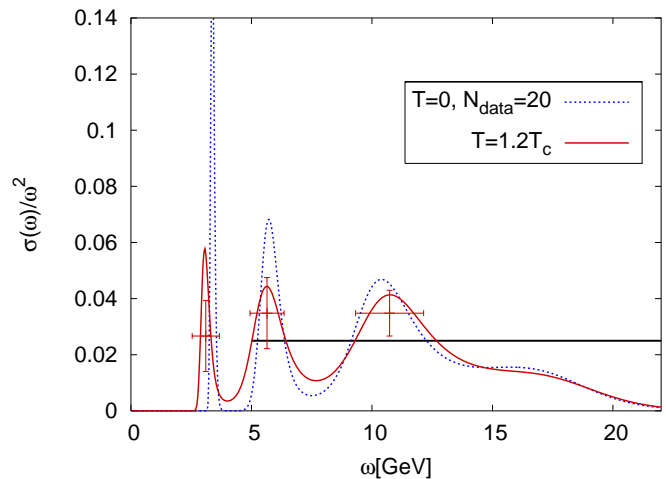
More recently: color potential measured on the lattice vs  $T$ .

Result:  $T_{\text{diss}} \sim 1.2 - 1.5T_c$ .

# Spectral function

- Charmonium spectral function:

Well defined QFT object. Measureable on the lattice.



from Bazavov *et al*

- Result: the peak persists at least until  $T \sim 2.4T_c$

- (Leaving aside the important/difficult question of how the spectral function is connected to experimentally measured  $J/\psi$  rates,)

What is the temperature dependence of the charmonium spectral function?

# Holographic approach

- A quark-antiquark pair in a strongly interacting medium — fertile ground for strong-coupling approaches, such as AdS/QCD.

Mateos, Myers, Thomson; Babington, Erdmenger, Evans, Guralnik, Kirsch, Threlfall; Hoyos-Badajoz, Landsteiner, Montero; Peeters, Sonnenschein, Zamaklar, Paredes; Liu, Rajagopal, Wiedemann, Ejaz, Faulkner; Greubel, Kaminski, Kerner, Pena-Benitez; ...

- “Top-down” AdS/CFT approach:  $\mathcal{N} = 4$  SSYM as the medium.

Quarks — D3-D7 strings (*a la* Karch-Katz).

Mesons — oscillations of the D7 (flavor) brane.

The CFT medium is not confining. No phase transition.

But at  $T \sim m_{\text{meson}}$  a transition occurs in the dynamics of the mesons — D7 brane connects to the black-hole — oscillations are “absorbed” — mesons acquire thermal widths.

In a stringy picture — meson string touches the horizon — string breaks.  
Spectral function?

# What do we want?

- We want meson spectrum to go asymptotically as  $m_n^2 \sim n$ , rather than  $m_n^2 \sim n^2$ .

And we want spectral function (e.g., to compare to Lattice).

- Fukushima *et al* take the “soft-wall” model of Karch *et al*, where  $m_n^2 = m_1^2 \times n$  and set  $m_1 = m_{J/\psi}$  (“rescaled  $\rho$ ” model).

Result:  $T_{\text{diss}} \sim 1.2T_c$ .

- But is the charmonium spectrum similar to  $m_n^2 = m_1^2 \times n$ ?

Rather, it is more like a 2-scale pattern:

$m_n^2 = m_1^2 + \mu^2(n - 1)$ , where  $\mu^2$  is the QCD string tension scale,

while  $m_1$  is determined by the heavy quark mass, unrelated to  $\Lambda_{\text{QCD}}$ .

- Let us, in the spirit of “bottom-up” AdS/QCD approach, try to match the spectrum, and then see what that means for the finite  $T$  spectral function.

# “Soft wall” model

- We want correlators of the operator  $J^\mu = \bar{c}\gamma^\mu c$ .

Holographic action for the field  $A_M$  dual to  $J^\mu$  — 5d Yang-Mills:

$$S = \int d^5x \sqrt{g} e^{-\Phi} \left\{ -\frac{1}{4g_5^2} F^{MN} F_{MN} \right\}$$

$g_{MN}$  and  $\Phi$  are metric and dilaton field background. We will choose them to match QCD/phenomenology. Assume that background is dynamically determined, but do not model the dynamics.

- For example,

the AdS metric  $ds^2 = z^{-2}(dx \cdot dx - dz^2)$  matches  $\Pi(Q^2) \sim \log(Q)$ ,

while  $\Phi \sim z^2$  gives linear spectrum:  $m_n^2 \sim n$  (Karch *et al*).

# Spectrum

- Poles in the correlator — normalizable modes of the (5d Maxwell) equation:

$$\partial_z \left( e^B \partial_z v_n \right) + m_n^2 e^B v_n = 0, \quad v_n(0) = 0,$$

with  $B \equiv -\log z - \Phi(z)$ . Substitute  $v_n = e^{B/2} \psi_n \longrightarrow$  Schrödinger form

$$-\psi_n'' + U(z)\psi_n = m_n^2 \psi_n, \quad U(z) = \frac{1}{4}(B')^2 + \frac{1}{2}B''.$$

- With  $\Phi = (az)^2$ :  $U = (a^2 z)^2 + \frac{3}{4z^2}$  — 2d harmonic oscillator ( $m = 1$ ).

$$m_n^2 = 4a^2 n, \quad n = 1, 2, \dots$$

- Generalization to  $S > 1$  gives  $m_n^2 = 4a^2(n + S - 1)$ .  
String-like spectrum without (explicit) string.

# Decay constants and the “dip”

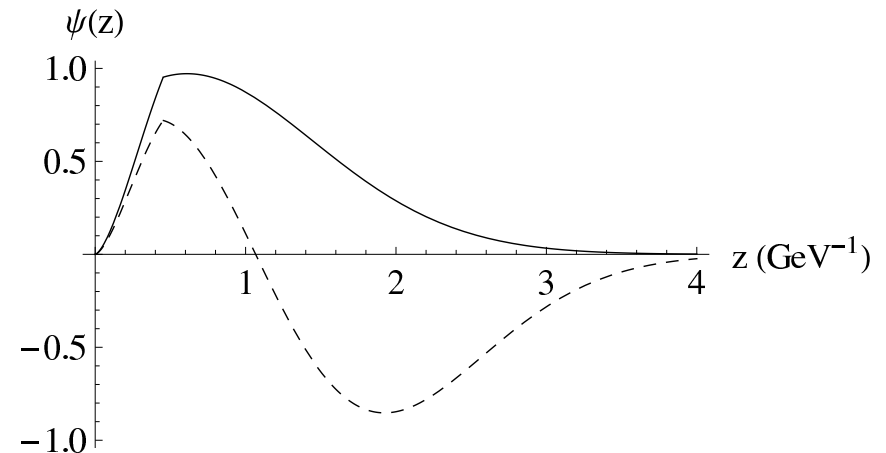
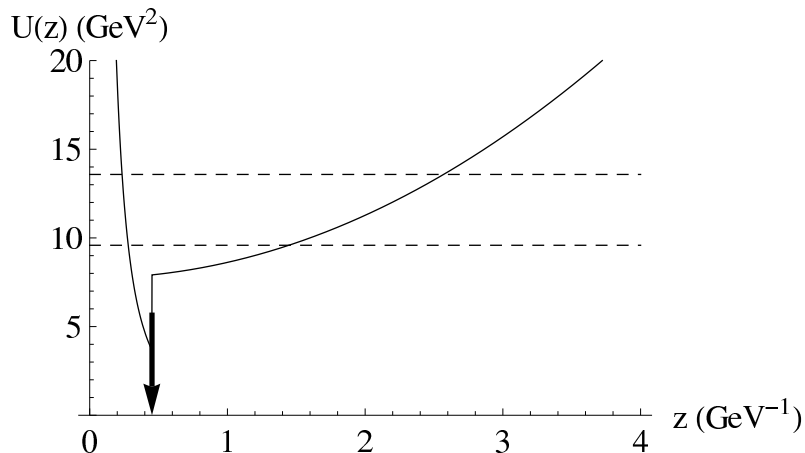
- Easy to modify  $U$  to get 2-scale pattern:  
 $U \rightarrow U + c^2$ .  
 (One can then solve for  $B$ .)

Observable	Experiment (MeV)	$U_{(a)}$ (MeV)	$U_{(a,c)}$ (MeV)
$m_{J/\psi}$	3096	3096*	3096*
$m_{\psi'}$	3685	<b>4378</b>	3685*
$f_{J/\psi}$	416	348	<b>145</b>
$f_{\psi'}$	296	348	173

- But  $f_{J/\psi}$  is too small!

To get finite- $T$  spectral function right — begin with the right  $T = 0$  spectral function.

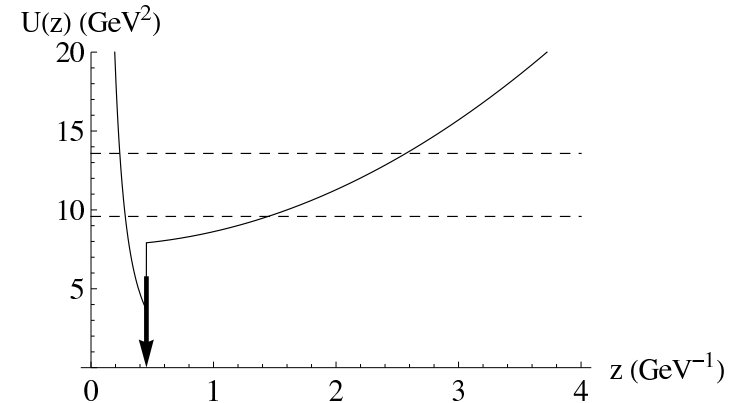
- Recall that  $f_n \sim \psi_n''(0)$ . Thus we need to make the wave-function “steeper” at small  $z$ , without increasing  $m_2^2 - m_1^2$ . A “dip” does it:



# Minimalistic potential

- Features:

- $3/(4z^2)$  “centrifugal core” at small  $z$ ;
- soft wall at large  $z$ ;
- vertical “shift”;
- narrow “dip”.



$$U(z) = \frac{3}{4z^2} \theta(z_d - z) + ((a^2 z)^2 + c^2) \theta(z - z_d) - \alpha \delta(z - z_d).$$

Minimalistically, we do not model the shape of the dip.

- Fit 4 parameters to 4 pieces of spectral data:  $m_{J/\psi}$ ,  $m_{\psi'}$ ,  $f_{J/\psi}$ ,  $f_{\psi'}$ :

$$a = 0.970 \text{ GeV}, \quad c = 2.781 \text{ GeV},$$

$$\alpha = 1.876 \text{ GeV}, \quad z_d^{-1} = 2.211 \text{ GeV}.$$

with  $g_5^2 = 12\pi^2/N_c$ , as usual, fit to reproduce  $\Pi(Q^2) \rightarrow N_c/(12\pi^2) \log Q$ .

# Finite $T$

- Corresponds to a black-hole background

$$ds^2 = e^{2A(z)} [h dt^2 - d\mathbf{x}^2 - h^{-1} dz^2] .$$

If the function  $h(z)$  has a simple zero,  $h(z_h) = 0$ , there is an event horizon at  $z = z_h$ . The temperature  $T$  corresponding to this background is related to  $z_h$  as

$$T = \frac{1}{4\pi} |h'(z_h)| .$$

- We shall minimalistically take the Ansatz:

$$h = 1 - (z/z_h)^4 ,$$

We combine  $A$  and  $\Phi$  into  $B = A - \Phi$  — the combination entering in the calculation of the spectral function —

and *assume*  $B$  does not change with  $T$  (for the lack of a more informed choice).

# Spectral function

- The spectral function is obtained using Son-Starinets prescription, by solving (5d Maxwell)

$$\partial_z (h e^B \partial_z V) + \omega^2 h^{-1} e^B V = 0 ,$$

with

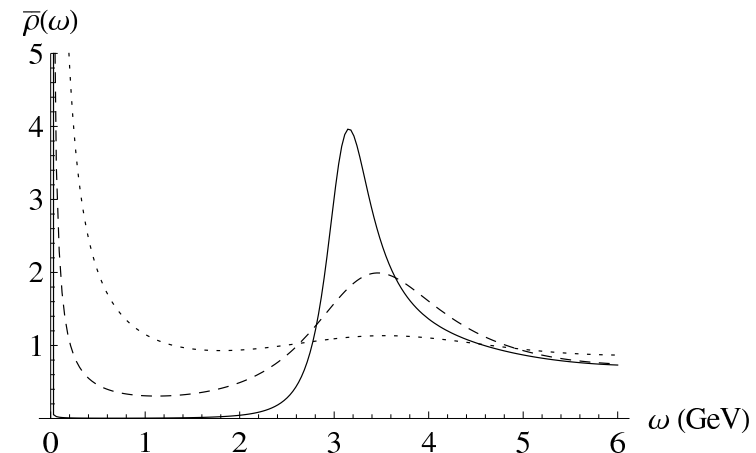
$$V(\omega, \epsilon) = 1 ; \quad V(\omega, z) \xrightarrow{z \rightarrow z_h} C(\omega) (1 - z/z_h)^{-i\omega/(4\pi T)} ,$$

$$G_R(\omega) = -\frac{1}{g_5^2} h e^B V'(z, \omega) \Big|_{z=\epsilon} = -\frac{1}{g_5^2} \frac{V'(\epsilon, \omega)}{\epsilon} ,$$

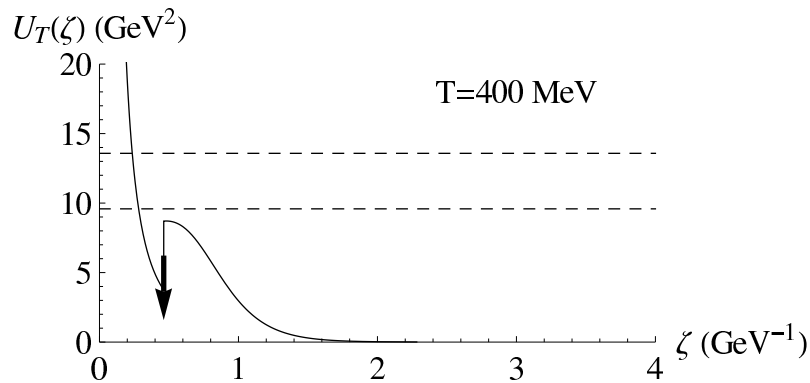
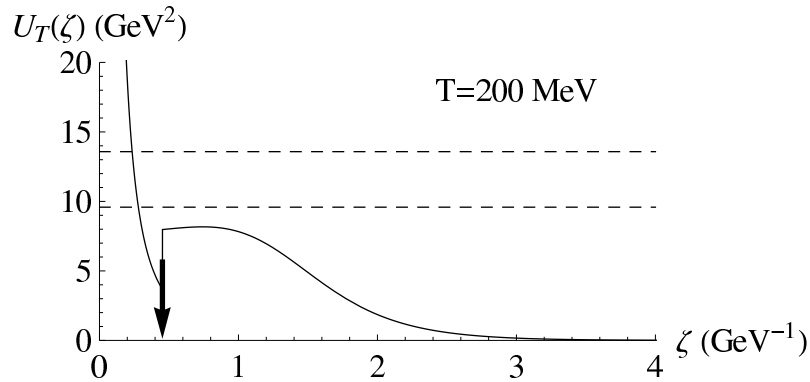
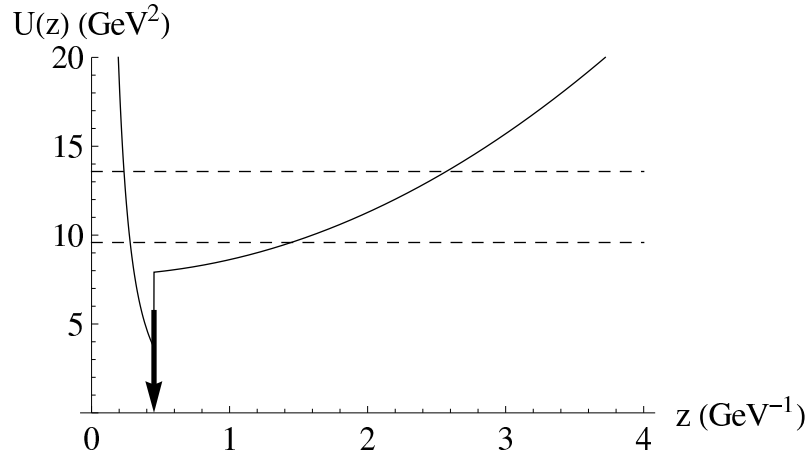
- For the spectral density:  $\rho = -\text{Im } G_R$ :

$$g_5^2 \rho(\omega) = \text{Im } \epsilon^{-1} V'(\epsilon, \omega) = |C(\omega)|^2 e^{B(z_h)} \omega ,$$

$$\bar{\rho}(\omega) \equiv \frac{2g_5^2}{\pi} \frac{\rho}{\omega^2} \quad T = 200, 400, 600 \text{ MeV}$$



# Holographic potential



$$\Psi(\zeta(z)) = e^{B(z)/2} V(z);$$

$$\zeta(z) = \int_0^z dz' / h(z');$$

$$-d^2 \Psi / d\zeta^2 + U_T(\zeta) \Psi = \omega^2 \Psi ,$$

where

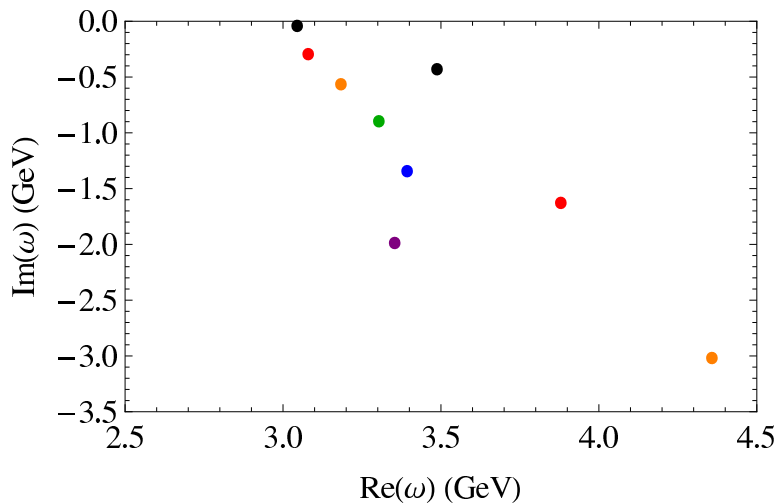
$$U_T(\zeta(z)) = \frac{d^2 B / d\zeta^2}{2} + \left( \frac{dB / d\zeta}{2} \right)^2$$

# Quasinormal modes

- Quasinormal modes are solutions of the same e.o.m. (5d Maxwell eq.) with the same *complex* b.c. at  $z_h$ :  $v_n \sim (z_h - z)^{-i\omega/(2\pi T)}$ , but with

$$v_n(0) = 0.$$

- The QNM frequencies are discrete and complex.  $\text{Im } \omega < 0$ . These frequencies are poles of  $G_R(\omega)$ .



$$\omega_1: T = 100 - 600 \text{ MeV}$$

$$\omega_2: T = 100 - 300 \text{ MeV}$$

# Normalizing QNM and residues

- Can the residue in the pole

$$r_n \equiv \lim_{\omega \rightarrow \omega_n} (\omega - \omega_n) G_R(\omega)$$

be expressed entirely in terms of  $v_n$ ?

- Yes, similar to decay constants:

$$r_n = \frac{1}{g_5^2} \frac{(v_n''(0))^2}{2\omega_n},$$

- But normalization must be

$$\lim_{\delta \rightarrow 0} \left[ \int_{\epsilon}^{(1-\delta)z_h} \frac{dz}{h} e^B v^2 + \frac{i}{2\omega} e^B v^2 \Big|_{z=(1-\delta)z_h} \right] = 1.$$

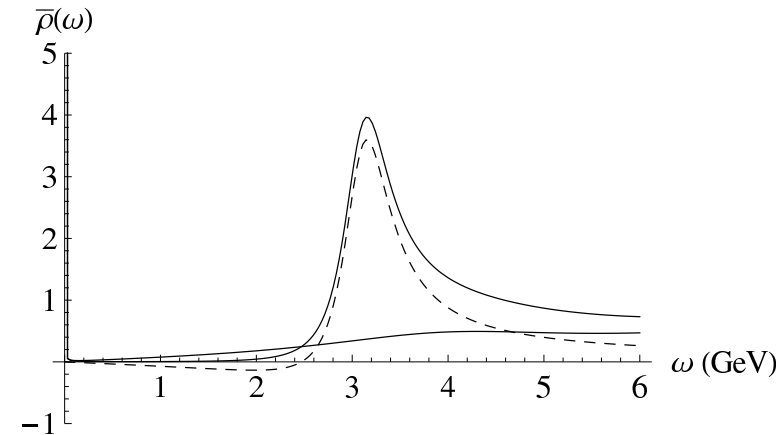
Note:  $v_n(z_h) = \infty$  since  $\text{Im } \omega_n < 0$ . Also, it is a *complex* condition.

In terms of  $\psi$  and  $\zeta$ ,  $\int d\zeta \psi^2 + \text{c.t.}$  — similar to Zeldovich's norm for quasinormal levels.

# QNM and peak “melting”

- The peak is due to the QNM:

$$\bar{\rho}_n(\omega) \equiv \text{Im} \left( \frac{\bar{r}_n}{\omega - \omega_n} \right),$$



- Comparison between models.  
Define the height:

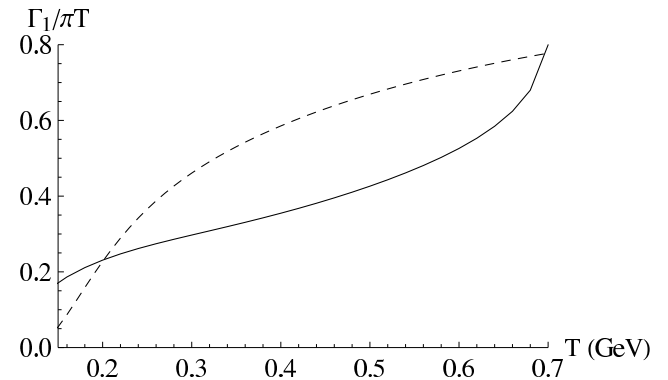
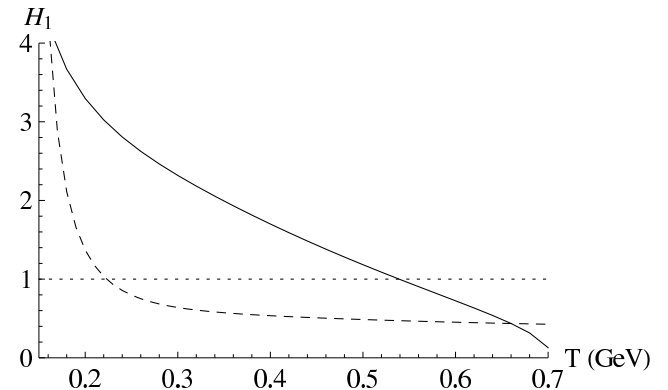
$$H_n \equiv \bar{\rho}_n(\omega) \Big|_{\omega = \text{Re } \omega_n} = \frac{\text{Re } \bar{r}_n}{\text{Im } \omega_n},$$

The width, as usual:

$$\Gamma_n \equiv -\text{Im } \omega_n.$$

dashed: “rescaled  $\rho$ ” —  $T_{\text{melt}} \approx 230 \text{ MeV}$

solid: “shift/dip” —  $T_{\text{melt}} \approx 540 \text{ MeV}$



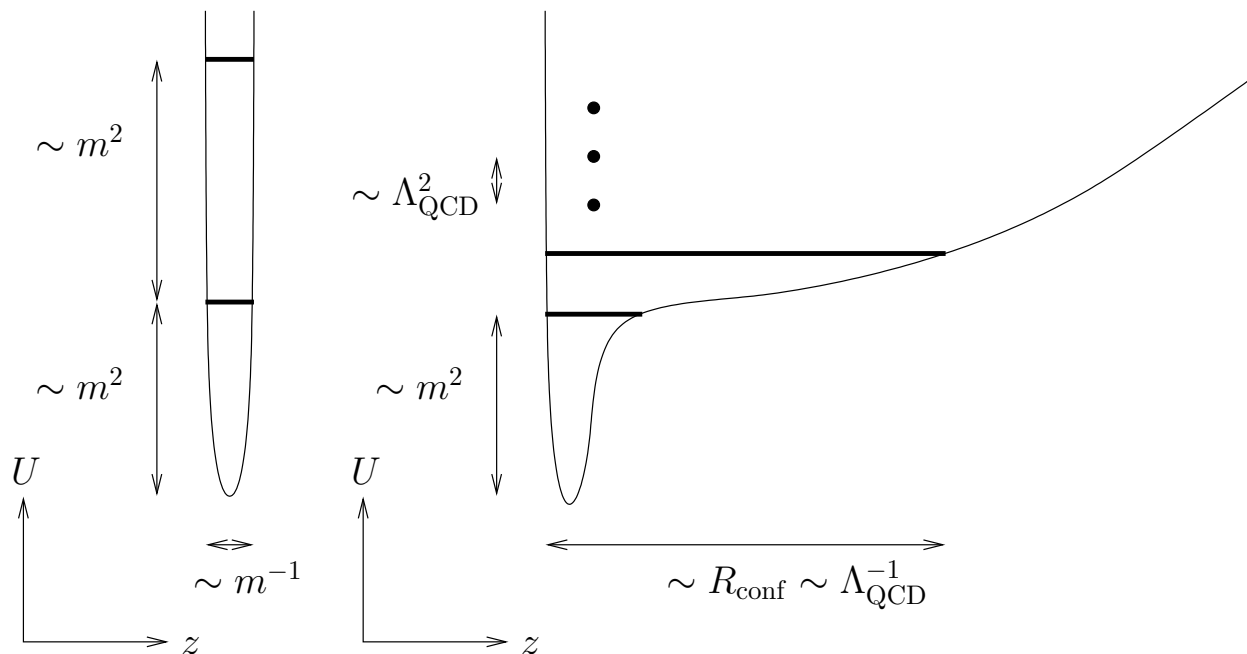
# Concluding remarks

- Matching the decay constant(s) is essential.

The “dip” increases  $f_{J/\psi}$  and  $T_{\text{diss}}$ .

Physically: larger  $f$  – more compact state, and more robust.

- The dip in “top-down” vs “bottom-up” potentials:



- Can such potentials arise dynamically? Or “top-down”?

- Can this be connected to the stringy description of mesons?