Intrinsic Width of Flux Tubes from Gauge/Gravity Duality

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Confining flux tubes and strings, ECT, Trento 5-9 July 2010,
Based on: V. Vyas, arXiv:1004.2679

Outline

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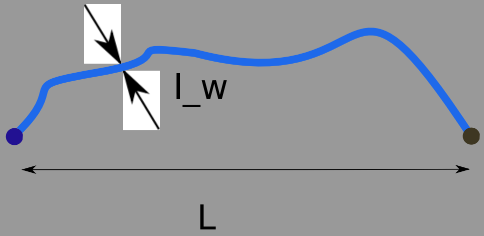
Fluctuating Flux-Tubes

- A fluctuating flux-tube of finite intrinsic thickness is formed between a static quark and an anti-quark.



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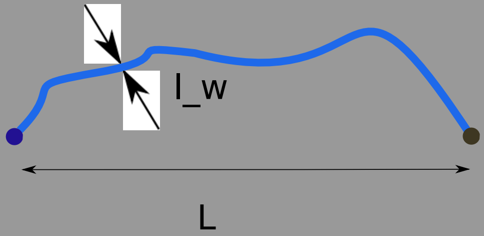

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Salient Features of Numerical Simulations

- Ground state and most of the excited states are very well described by the Nambu-Goto string in four-dimensions.
- There are few excited states of closed flux-tubes which show clear deviations from the Nambu-Goto predictions.
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Deviations from the Nambu-Goto String

Evidence of fluctuating intrinsic thickness?

- Can the deviation from the predictions of the Nambu-Goto string be excited modes of intrinsic thickness?

Intrinsic Thickness

A Model for a Thick String?

To evaluate the effects of intrinsic thickness we need a model that has intrinsic thickness as an explicit degree of freedom.

Use Holography via Gauge/Gravity Duality

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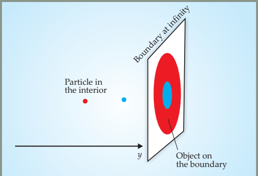
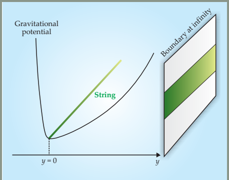
- 



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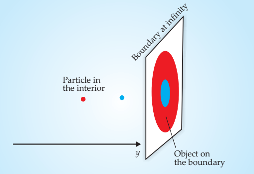
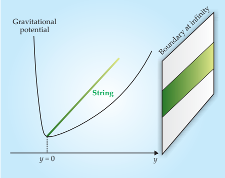
- 



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<Wilson Loops> from 5-Dimensional Fundamental String

Assumed Duality

$$< \mathrm{Tr}\hat{P}\exp\left\{i\oint_\Gamma A\right\} >_{YM} = \int [DX]\exp\left\{-T_o\int d^2\sigma\sqrt{\gamma[X]}\right\}$$

Strings Live in a Five Dimensional Curved Space

- $$ds^2 = g_{mn}dx^m dx^n = F(y)\left(dx_4^2 + dx_1^2 + dx_2^2 + dx_3^2 + dy^2\right)$$
- $$\gamma_{ab} = g_{mn}\frac{\partial X^m}{\partial \sigma^a}\frac{\partial X^n}{\partial \sigma^b},$$

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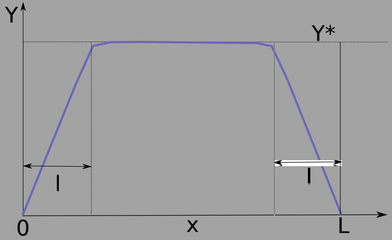
Confining geometry

Assumed Properties of the Warp Factor

- $F(y)$ has minimum at $y = y^*$
- $\lim_{y \to 0} F(y) \approx \frac{R^2}{y^2}$

Minimal String in Five-Dimensions $Y_c(x_1)$



Position Dependent String Tension

Action of the Minimal Surface in Static Gauge

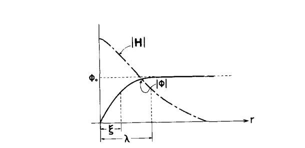
$$S_c = \int_{-\infty}^{\infty} dt \int_0^L dx T[Y_c(x)]$$

Position Dependent String Tension

$$T[Y_c(x)] = \begin{cases} T_0 F[Y_c](1 + Y'^2)^{1/2} & 0 \leq x \leq l \\ T_0 F[Y^*] & l < x < L - l \,, \\ T_0 F[Y_c](1 + Y'^2)^{1/2} & L - l \leq x \leq L \end{cases}$$

Nielsen-Olesen Vortex Line

Thickness of a Vortex Line H. Nielsen and P. Olesen. *Nucl. Phys., B61:45–61, 1973.*



The string tension of this vortex-line goes as

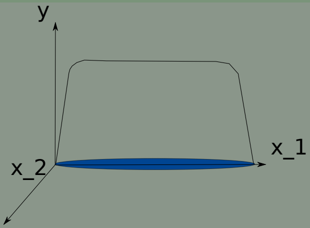
$$T \sim \frac{1}{\lambda^2}.$$

Intrinsic Thickness of a Flux-Tube

A suggestive interpretation of the position-dependent string tension

$$l_w^2(x_1) \sim \frac{1}{T_s[Y_c(x_1)]}$$

Intrinsic thickness of the flux-tube changes with the position of the string in the fifth-dimension



- Consistent with the holographic interpretation.

Static Potential: Quadratic Approximation

Massless and Massive Fluctuations

- Fluctuations along the Y directions, fluctuation in the intrinsic thickness, are massive.
- Fluctuations along x_T are massless.

Including small fluctuations around the minimal surface gives rise to

$$V(L) = T_0 F[Y^*] L - \frac{\pi}{12} \cdot \frac{1}{L} - \frac{1}{4\sqrt{\pi} L} \sqrt{ML} \exp(-2ML).$$

- $M \sim m_{glueball} \sim 1\mathrm{GeV}$

Need for a Non-Perturbative Approximation

The quadratic approximation is inadequate

- Lattice results indicates that the data fits very well to the Arvis's formula

$$V[L] = \frac{L}{l_s^2}\left(1 - \frac{L_c^2}{L^2}\right)^{\frac{1}{2}}; \quad L_c^2 = \frac{\pi d\, l_s^2}{12}.$$

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What is Large D_T Approximation?

Fluctuations in Static Gauge

$$X(\sigma) = \left\{ t, x, \vec{\phi}_T, Y_c(x) + \phi_y(t,x) \right\}, \quad \vec{\phi} = \left\{ \vec{\phi}_T, \phi_y \right\}.$$

The Large D_T Expansion

- Assume large number of flat transverse dimensions, D_T.
- Very similar in spirit to the large N analysis of $O(N)$ model, but with a "magnetic field" along one of the directions.
- For the Nambu-Goto string in flat space gives the exact result (Alvarez, 1981)

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Outline of the Calculation

The Large D Expansion, an Outline

1. Approximate the metric near $Y_c = Y^*$ as $F[Y] = F^*(1 + \frac{1}{2}M^2\phi_Y^2)$
2. Introduce an auxiliary field: $g_{ab} = \partial_a\vec{\phi}.\partial_b\vec{\phi}$ and a delta functional $\delta(g_{ab}(\sigma) - \partial_a\vec{\phi}.\partial_b\vec{\phi})$ in the functional integral.
3. Write the delta functional using Lagrangian multiplier field N_{ab}.
4. Integrate the fluctuations $\vec{\phi}$ which now appear only quadratically.
5. Minimize the effective action $S_{eff}[g, N]$, which is proportional to D_T, to get the leading contribution in $1/D_T$ expansion.

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Result

The Ground State Energy: Fixed λ, $D_T \to \infty$

$$V[L] = \frac{L}{l_s^2}\left(1 - \frac{L_c^2}{L^2}\right)^{\frac{1}{2}} - \frac{1}{4\sqrt{\pi L}} C[L]\sqrt{ML}\exp\{-2B(L)ML\}$$

- $\lambda = \frac{L_{c^2}}{2L^2}$ $\quad L_c^2 = \frac{\pi D_T l_s^2}{12}$ $\quad B(L) = (1-\lambda)^{\frac{1}{2}}$ $\quad C[L] = \frac{(1-\lambda)^{\frac{1}{4}}}{(1-2\lambda)^{\frac{1}{2}}}$
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- The first term: the Arvis's formula.
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Comparison with Effective Stings in Four Dimensions

Rigid String: Relevant and Marginal Terms

$$S_{rigid} = M_0^2 \int d^2\sigma \sqrt{\gamma} + \frac{1}{2e^2} \int d^2\sigma \sqrt{\gamma} (\triangle(\gamma) X^\mu)^2$$

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Conclusion: Flux-Tubes from Gauge/Gravity

Gauge/Gravity provides a model for the flux-tubes with intrinsic thickness

- For the **assumed** confining geometry, and in the large d_T limit, it implies that the effective strings that describe long flux-tubes are just the Nambu-Goto strings. Marginal and irrelevent terms, must be $O(1/d_T)$.

Conclusion: Deviations from Nambu Goto

Breathing Modes

- According to the present model, the leading source of the deviations in the spectrum of flux-tube from Nambu-Goto spectrum must be due to excited modes of intrinsic thickness - or excited modes polarised along the curved fifth-dimension (or their mixing.)

Conclusion:Lattice Simulations

Confining Geometries and Lattice Simulations

- This conclusion may not hold for a different confining geometry (e.g. Greensite and Olesen - 1999)
- Lattice simulations that can distinguish ground state energy at the level of $O(1/L^4)$ can delineate confining geometries.

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