Linear increase of the flux tube width at finite T.

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Plan of the Talk

- Effective string predictions for the flux tube thickness at finite T
- dimensional reduction and the Svetistky-Yaffe conjecture

Main goal: show that the effective string approach predicts a log to linear transition in the R dependence of the flux tube width and test this result with MC simulations and dimensional reduction .

References

M.C, F.Gliozzi, U.Magnea, S.Vinti
, "Width of long colour flux tubes in lattice gauge systems"
Nucl.Phys. B460 (1996) 397

M.C., P. Grinza and N. Magnoli
" Study of the flux tube thickness in 3d LGT's by means of 2d spin models."
J. Stat. Mech. (2006) P11003

A. Allais and M.C
"Linear increase of the flux tube width at high temperature."
JHEP 0901:073,2009

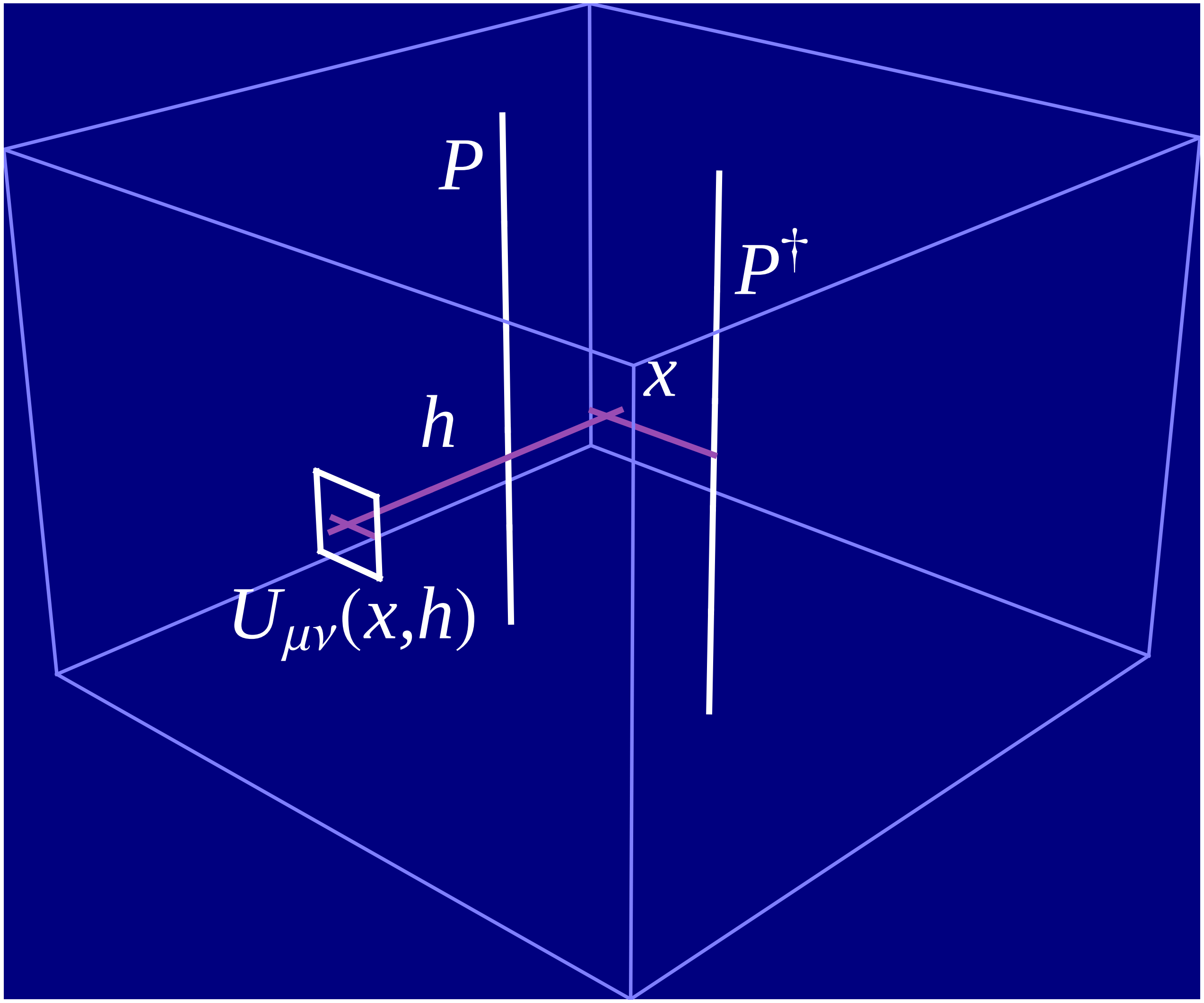
M.C
"Flux tube delocalization at the deconfinement point."
arXiv:1004.3875

The flux tube thickness.

The flux density in presence of a pair of Polyakov loops is:

$$< \mathcal{F}_{\mu,\nu}(x_0, x_1, h, R) >= \frac{< P(0,0)P^{+}(0,R)U_{\mu,\nu}(x_0, x_1, h) >}{< P(0,0)P^{+}(0,R) >} - < U_{\mu,\nu} >$$

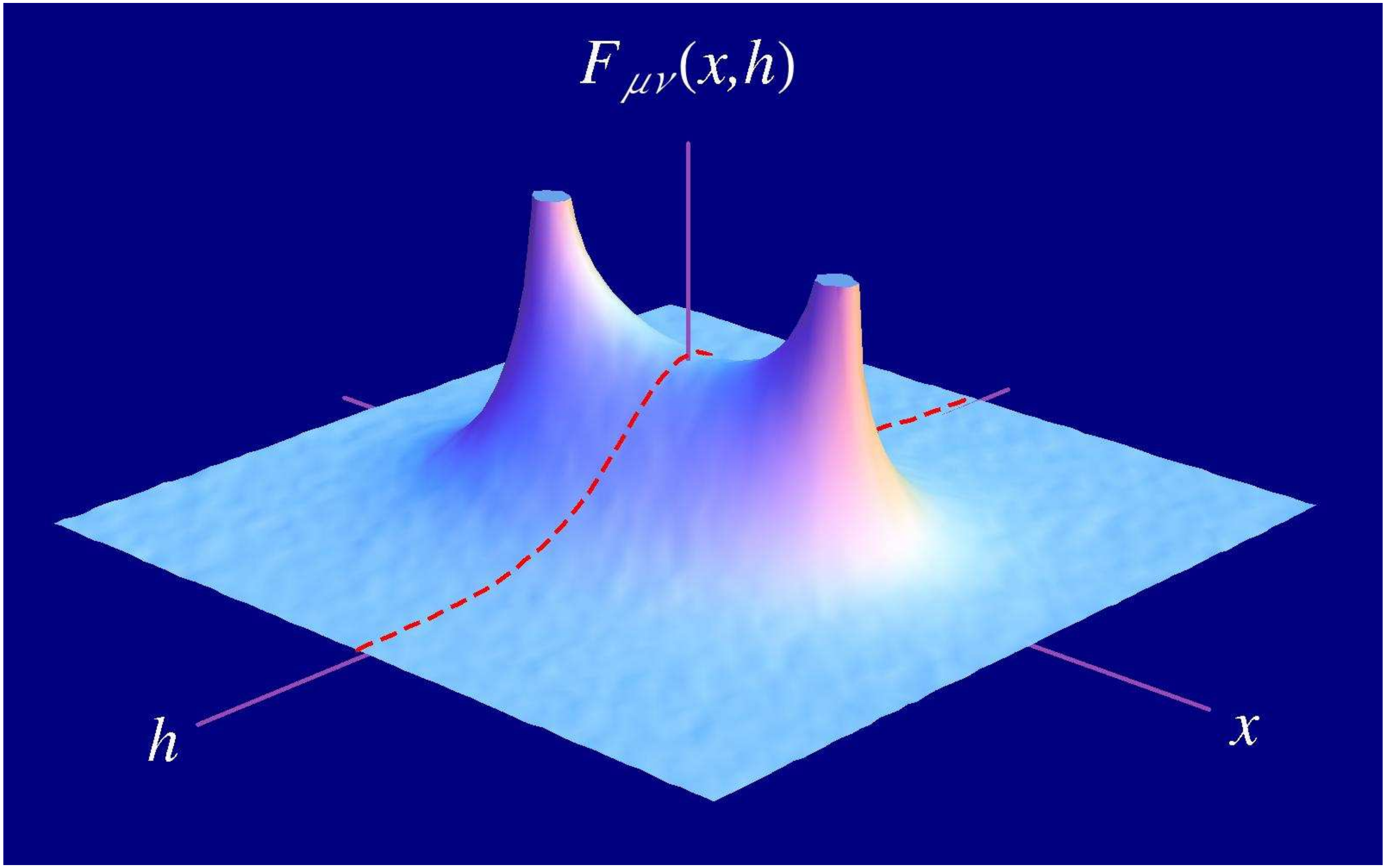
where x_0 denotes the timelike direction, x_1 is the direction of the axis joining the two Polyakov loops and h denotes the transverse direction.

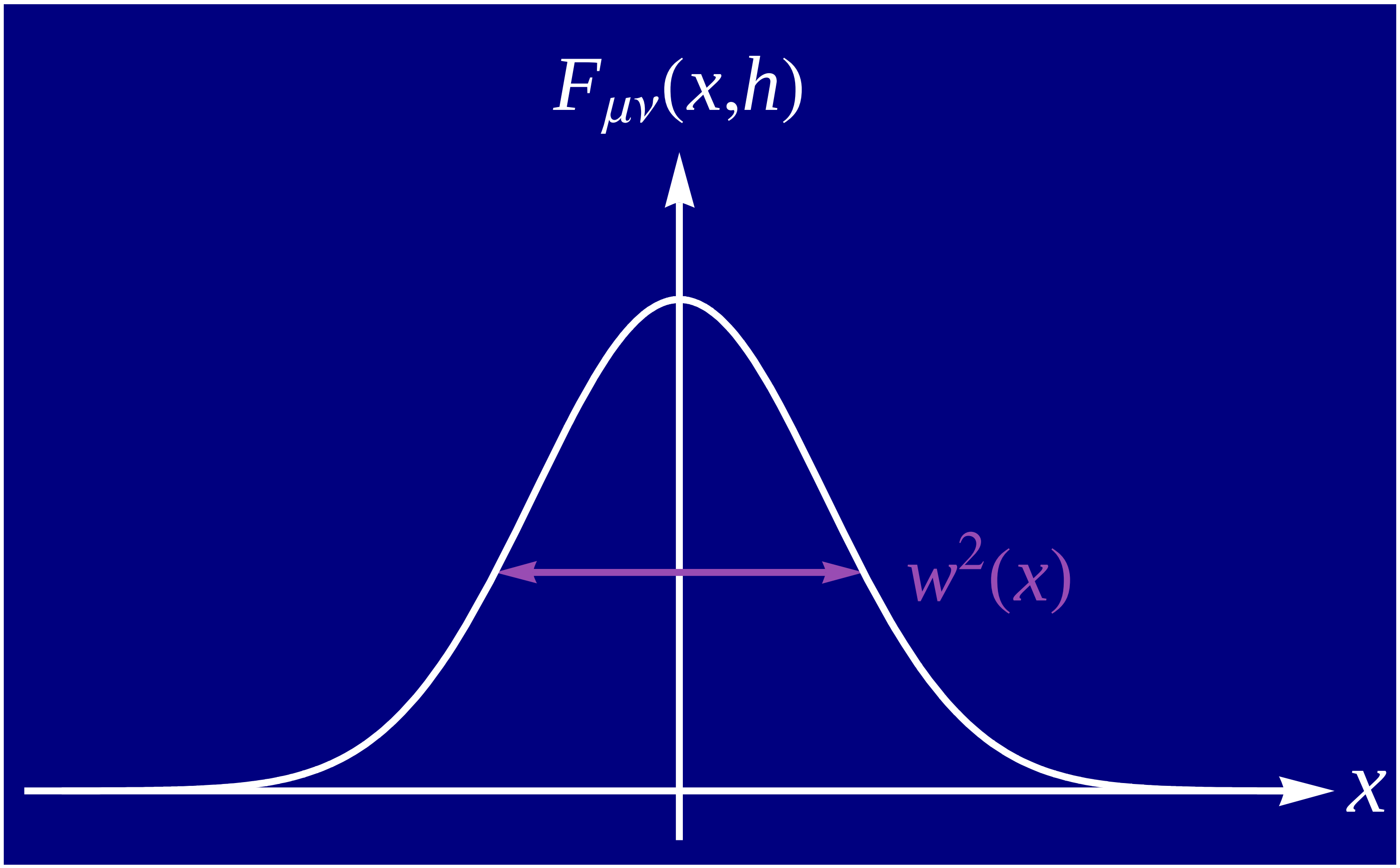

P
$P^\dagger$
x
h
$U_{\mu\nu}(x,h)$

To evaluate the flux tube thickness we fix $x_2 = R/2$ to minimize boundary effects. (thanks to the periodic b.c. in the "temperature" direction we can instead choose any value of x_0)

In the x_1 direction the flux density shows a gaussian like shape, the width of this gaussian is the "flux tube thickness": $w(R, L)$. $w(R, L)$ only depends on the interquark distance R and on the lattice size in the compactified timelike direction L, i.e. on the inverse temperature of the model

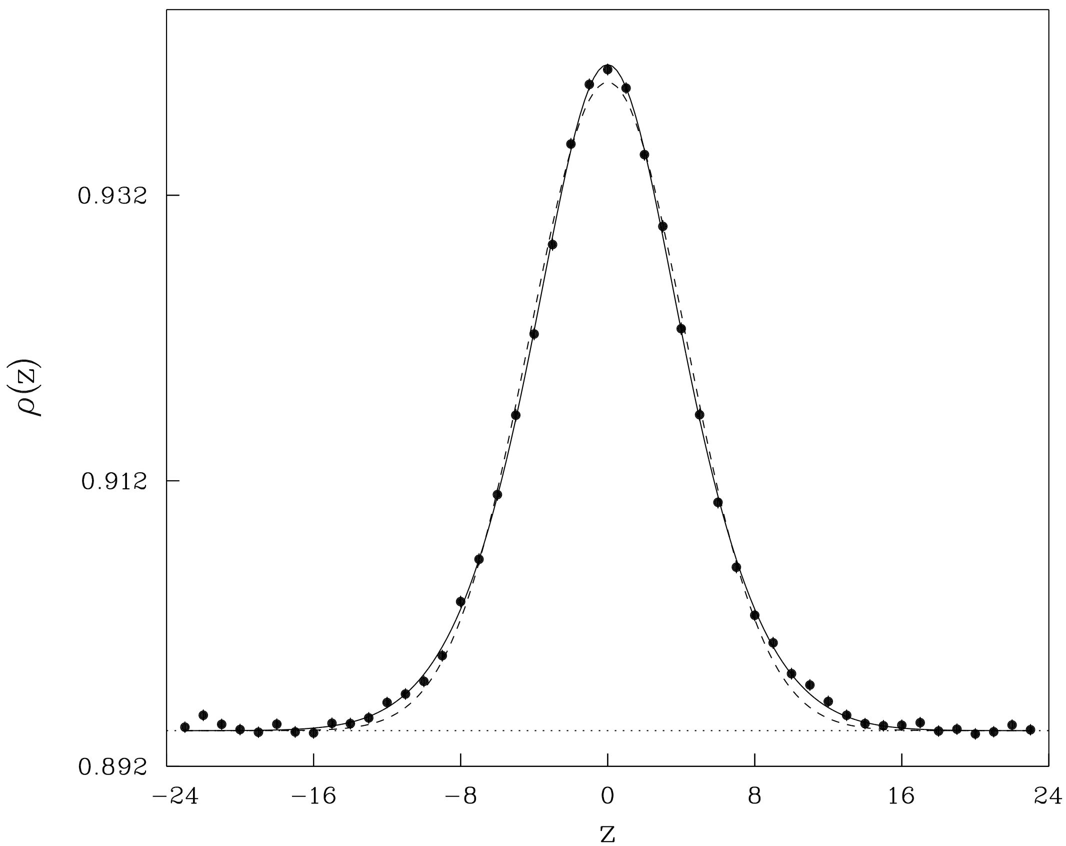
By tuning L we can thus study the flux tube thickness in the vicinity of the deconfinement transition


$F_{\mu\nu}(x,h)$
h
x


$F_{\mu\nu}(x,h)$
$w^2(x)$
x

Shape of the flux density generated by a 30×30 Wilson loop in the Ising gauge model (at $\beta = 0.7460$). The dashed line is the gaussian fit.

fig. 2

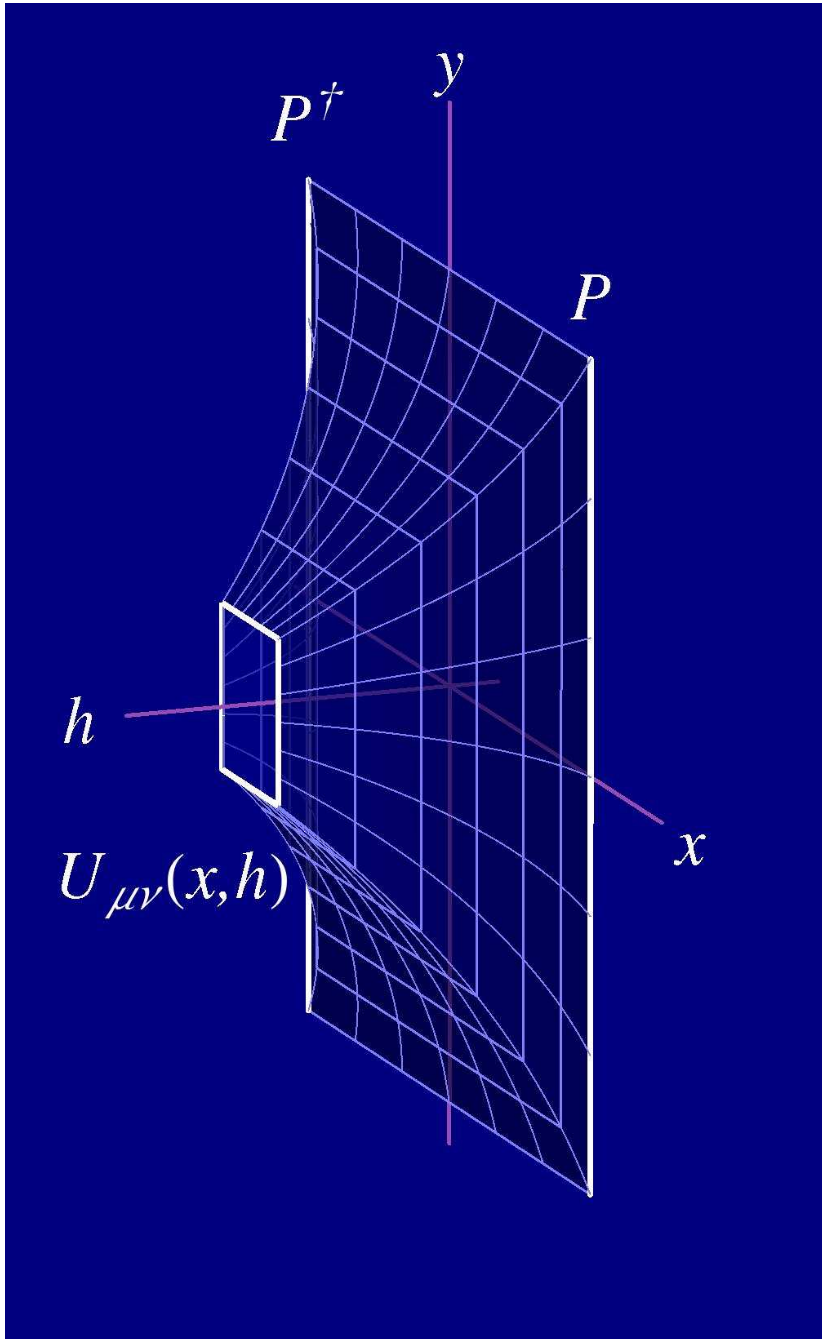


Effective string predictions for the flux tube thickness.

In the Nambu-Goto framework one should sum over all the surfaces bordered by the Polyakov loops and the plaquette with a weight proportional to the surface area.

$$\mathcal{F}_{\mu\nu}(x,h) = \sum_{\text{surf}} e^{-\sigma \text{Area[surf]}}$$

$$w^2(x) = \frac{\int \mathrm{d}h\ h^2\ \mathcal{F}_{\mu\nu}(x,h)}{\int \mathrm{d}h\ \mathcal{F}_{\mu\nu}(x,h)} = \frac{\int \mathrm{d}h\ h^2\ \sum_{\text{surf}} e^{-\sigma \text{Area[surf]}}}{\int \mathrm{d}h\ \sum_{\text{surf}} e^{-\sigma \text{Area[surf]}}}$$


y
$P^\dagger$
P
h
x
$U_{\mu\nu}(x,h)$

If we assume the size of the plaquette to be negligible with respect to the other scales, perform a "physical" gauge fixing and denote as h_0 the transverse coordinate of the plaquette then:

$$w^2(\vec{x}_0) = \frac{\int \mathrm{d}h_0 \; {h_0}^2 \int_{h(\vec{x}_0)=h_0} [\mathcal{D}h(\vec{x})] \;\; e^{-\sigma S[h]}}{\int \mathrm{d}h_0 \int_{h(\vec{x}_0)=h_0} [\mathcal{D}h(\vec{x})] \;\; e^{-\sigma S[h]}}$$

which can be rewritten as

$$w^2(\vec{x}_0) = \frac{\int [\mathcal{D}h(\vec{x})] \;\; h(\vec{x}_0)^2 e^{-\sigma S[h]}}{\int [\mathcal{D}h(\vec{x})] \;\; e^{-\sigma S[h]}} \equiv \langle h^2(\vec{x}) \rangle$$

with $S[h] = \sigma \int \mathrm{d}^2x \; \sqrt{1 + (\nabla h)^2}$

This expectation value is singular and must be regularized using for instance a point splitting prescription.

$$\sigma w^2(\vec{x}) = \langle h(\vec{x}+\vec{\epsilon})h(\vec{x})\rangle \equiv \mathcal{G}(\vec{x}+\vec{\epsilon};\vec{x})$$

The U.V. cutoff ϵ has a natural physical meaning: $\epsilon \sim$ plaquette size.

Dealing with the whole NG action turns out to be too difficult. We resort again to the free boson approximation ("SOS picture")

$$S[h] \simeq \sigma \int \mathrm{d}^2x\ [1 + 1/2(\nabla h)^2]$$

Then $\mathcal{G}(\vec{x}+\vec{\epsilon};\vec{x})$ is the Green function of the Laplacian on the cylinder.

which can be written (choosing the reference frame so as to have the two loops located in $\pm R/2$) as:

$$G_2(z;z_0) = -\frac{1}{2\pi}\log\left|\frac{\theta_1\left[\pi(z-z_0)/2R\right]}{\theta_2\left[\pi(z+\bar{z}_0)/2R\right]}\right|$$

with $q = e^{-\pi L/2R}$

Setting $z_0 = z+\epsilon$ and performing an expansion in ϵ one finds:

$$\sigma w^2(z) = -\frac{1}{2\pi}\log\frac{\pi|\epsilon|}{R} + \frac{1}{2\pi}\log\left|2\theta_2\left(\pi \text{ Re } z/R\right)/\theta_1'\right|$$

A similar calculation in the Wilson loop case gives

$$\sigma w^2(z) = -\frac{1}{2\pi}\log\frac{\pi|\epsilon|}{R} + \frac{1}{2\pi}\log\left|\frac{2\theta_2(\pi\ \mathrm{Re}\ z/R)\theta_4(i\pi\ \mathrm{Im}\ z/R)}{\theta_1'\ \theta_3(\pi z/R)}\right|$$

with $q = e^{-\pi L/R}$.

In both cases the dominant term diverges as $\frac{1}{2\pi}\log R$

Both results assume $L >> R$

Performing a dual tranformation we can obtain the behaviour for $R >> L$ i.e. in the high T regime. In this case the Green function can be written as:

$$G(z;z_0) = -\frac{1}{2\pi}\log\left|\frac{\theta_1\left[\pi(z-z_0)/L\right]}{\theta_4\left[\pi(z-\bar{z}_0)/L\right]}\right| - \frac{\mathrm{Im}z\ \mathrm{Im}z_0}{LR} \qquad q = e^{-2\pi R/L}$$

and we have:

$$\sigma w^2(z) = -\frac{1}{2\pi}\log\frac{\pi|\epsilon|}{L} + \frac{1}{2\pi}\log|\theta_4(2\pi i\ \mathrm{Im}\ z/L)/\theta_1'| - \frac{(\ \mathrm{Im}\ z)^2}{LR}$$

Expanding in powers of R/L we find

$$\sigma w^2(z) = -\frac{1}{2\pi}\log\frac{2\pi|\epsilon|}{L} + \frac{R}{4L}$$

This time the R dependence is linear !!

Summary of the result for the Polyakov loop correlator

- Low temperature

$$w^2 \sim \frac{1}{2\pi\sigma} \log(\frac{R}{R_c}) + \ldots \qquad\qquad (L >> R >> 0)$$

- High temperature (but below the deconfinement transition)

$$w^2 \sim \frac{1}{2\pi\sigma}(\frac{\pi R}{2L} + \log(\frac{L}{2\pi|\epsilon|}) + \ldots) \qquad\qquad (R >> L)$$

Log increase of the flux tube width at zero temperature but Linear increase near the deconfinement transition!

Comparison with MC simulations.

The 3d gauge Ising model is perfectly suited for studying the flux tube width.

Thanks to duality one can create a "vacuum" containing the Wilson loop or the Polyakov loop correlators by simply frustrating the links in the dual lattice orthogonal to a surface bordered by the loops.

Any choice of the surface is equivalent.

Measuring the energy operator (which is the dual of the plaquette operator) in this vacuum one can thus evaluate the ratio:

$$\frac{< P(0,0)P^{+}(0,R)U_{\mu,\nu}(x_0,x_1,h) >}{< P(0,0)P^{+}(0,R) >}$$

for any value of R and L with the same statistical uncertainty of the expectation value of the plaquette in the usual vacuum $< U_{\mu,\nu} >$.

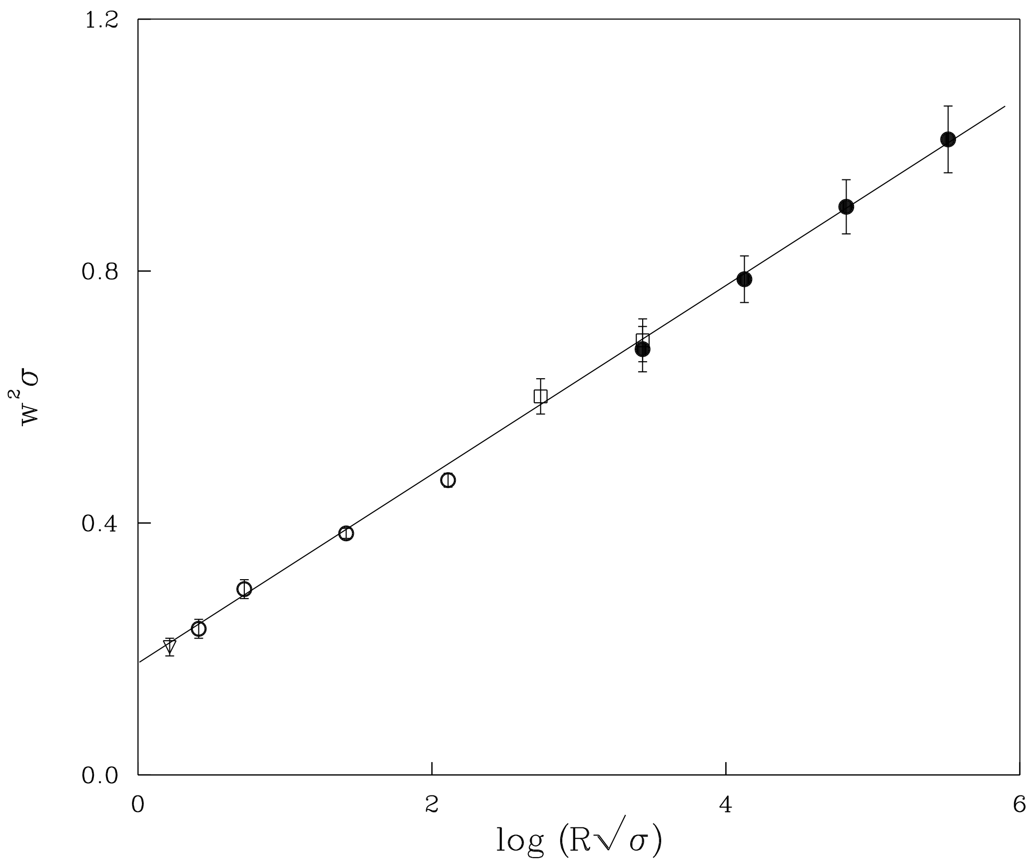
We used Wilson loops to test the low T predictions and Polyakov loop correlators for the high T ones.

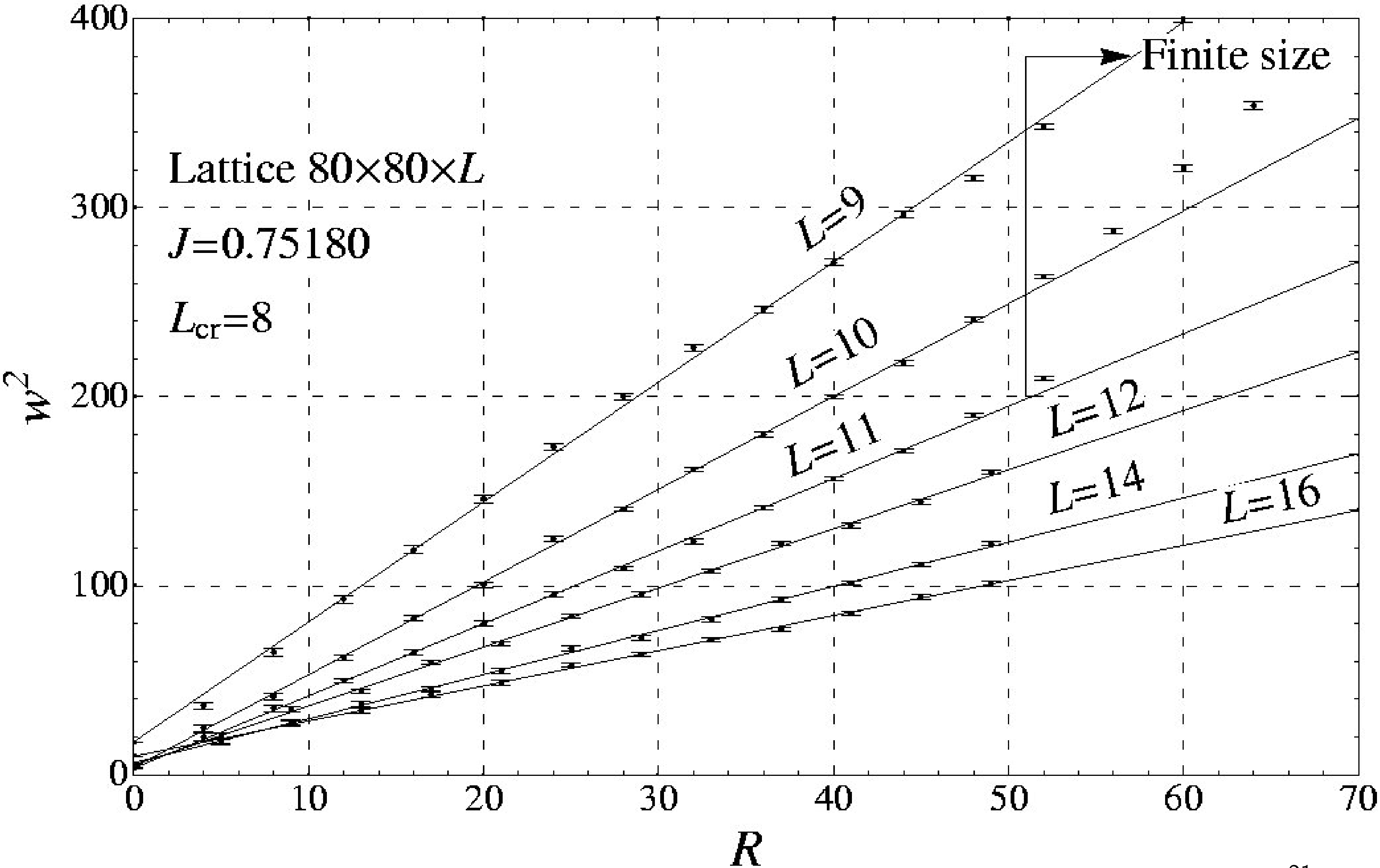
- **Low** T we tested several values of β in the range $0.6543 \leq \beta \leq 0.7516$ and several sizes of the Wilson loops (ranging from 12^2 to 64^2) so as to test a wide range of $R\sqrt{\sigma}$ values. The log fit of σw^2 as a function of $R\sqrt{\sigma}$ shows a very good χ^2 with an angular coef. 0.150(5) to be compared with $1/2\pi \sim 0.15915...$

- **High** T we studied the model at $\beta = 0.75180$ which corresponds to $aT_c = 1/8$ and $a^2\sigma = 0.0105(2)$ and tested lattice sizes $9 \leq L \leq 16$ i.e. temperatures ranging from $T = \frac{T_c}{2}$ to $T = \frac{8}{9}T_c$ For all the values $L \geq 10$ the linear fit has very good χ^2, but the ang.coef. shows deviations with respect to the expected value $1/(4\sigma L)$.

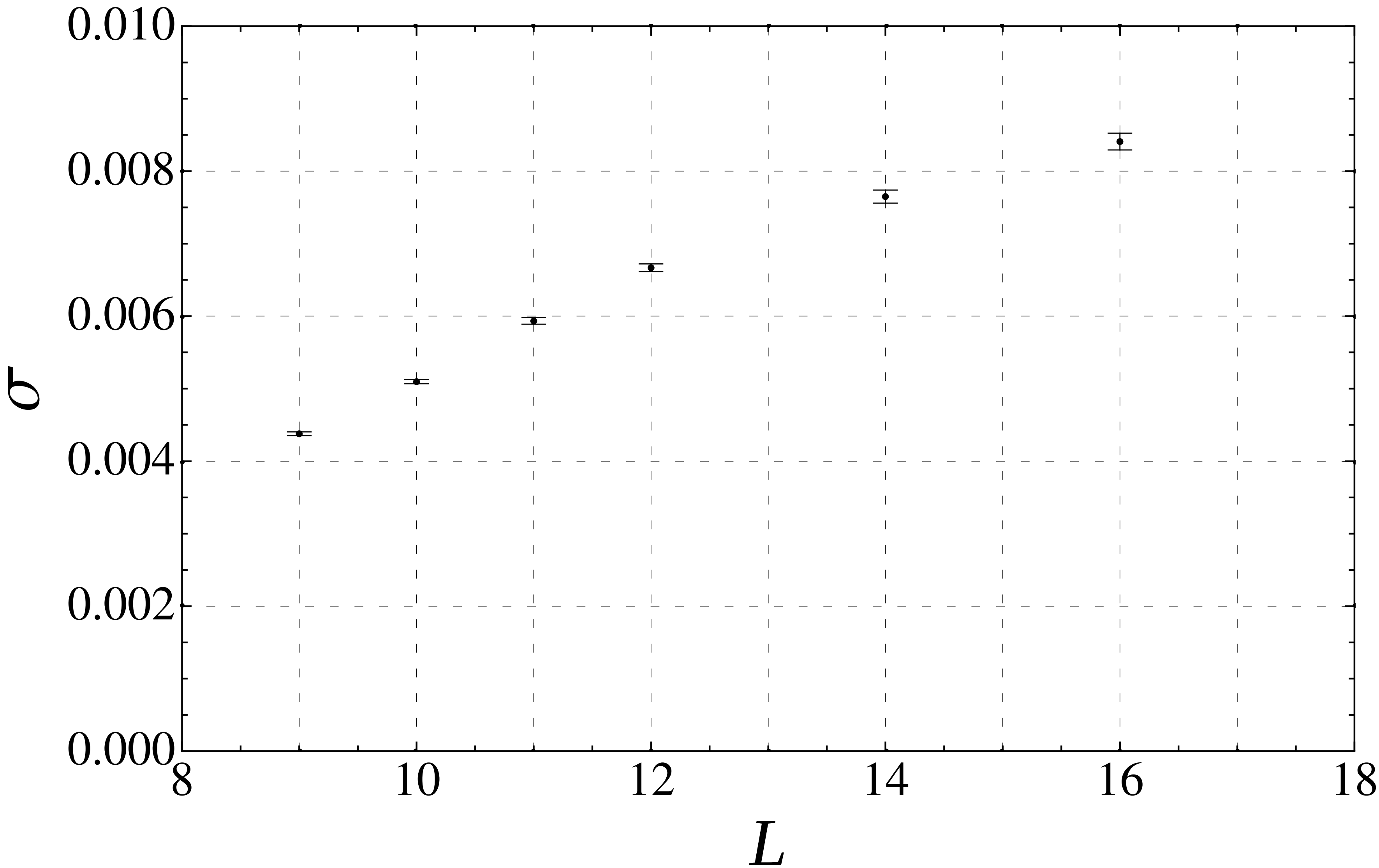
Squared width of the flux tube in units of sigma for the $\mathbb{Z}_2$ gauge theory. The open symbols are Wilson loop data while the black circles refer to the (dual) Ising interface.

fig. 3





Finite size
Lattice 80×80×L
J=0.75180
Lcr=8
L=9
L=10
L=11
L=12
L=14
L=16
400
300
200
100
0
0
10
20
30
40
50
60
70
w2
R



0.010
0.008
0.006
0.004
0.002
0.000
σ
8
10
12
14
16
18
L

Dimensional reduction and the Svetitsky-Yaffe conjecture

- "weak form of the S-Y conjecture:"

  The high temperature behaviour of a (d+1) LGT with gauge group G can be effectively described by a spin model in d dimensions with (global) symmetry group C (the Center of G).

- "strong form of the S-Y conjecture:"

  If both the spin model and the LGT have continuous phase transitions then they share the same universality class.

The mapping between the LGT and the effective spin model is based on the following identifications

LGT	spin model
Low T confining phase	High T symmetric phase
Polyakov loop ("C-odd")	spin operator
Plaquette operator ("C-even")	energy operator
Thermal perturbation	energy perturbation
string tension (σ/T)	mass of the theory
Polyakov loop correlator	spin-spin correlator

These mappings should be intended in a "renormalization group sense" i.e. the Polyakov loop operator is mapped into a linear combination of all the (C-odd) operators in the spin model. For instance, in the 2d Ising case the whole conformal family of the spin operator. This combination will be dominated by the relavant(s) operator(s). In the Ising case only one: the spin operator.

In the case of the plaquette operator the mapping will be in general a linear combination of the energy and the identity families.

Agreement between 2d spin model estimates and effective string predictions.

With the Nambu-Goto effective string we obtain for the Polyakov loop correlator:

$$\langle P(0,0)P(0,R)^{+}\rangle = \sum_{n=0}^{\infty} |v_n|^2 \left(\frac{\tilde{E}_n}{\pi}\right) K_0(\tilde{E}_n R).$$

This expression is expected to be reliable in the large distance limit. In this limit only the lowest state $(n = 0)$ survives and we end up with a single K_0 function:

$$\lim_{R\to\infty} \langle P(0,0)P(0,R)^+\rangle \sim K_0(\tilde{E}_0 R).$$

The spin-spin correlator of any 2d spin model in the symmetric phase is given by

$$\lim_{R\to\infty} \langle \sigma(0,0)\sigma(0,R)^+\rangle \sim K_0(mR).$$

The two expression coincide, they are universal (no dependence on the symmetry group) and allow to identify m with $\tilde{E}_0 = \sigma L \left\{1 - \frac{\pi}{3\sigma L^2}\right\}^{1/2}$. which, at the first order in $1/L$ becomes

$$m \leftrightarrow \sigma L = \sigma/T$$

Effective spin model description of the the flux tube thickness.

Following the S-Y mapping the flux density in the Ising LGT in (2+1) dimensions becomes the ratio of *connected* correlators:

$$\frac{\langle\sigma(x_1)\epsilon(x_2)\sigma(x_3)\rangle}{\langle\sigma(x_1)\sigma(x_3)\rangle}$$

in the high temperature phase of the 2d Ising model in zero magnetic field.

The large distance behaviour of this correlator can be evaluated using the Form Factor approach.

Width of the flux tube

The width of the flux tube evaluated at the midpoint between the two spins is given by

$$w^2(r) \;=\; \frac{r^2}{2K_0(2mr)} \int_{-\infty}^{\infty} dx \, \frac{x^2}{1+x^2} e^{-2mr\sqrt{1+x^2}}.$$

where $R \equiv 2r$ is the distance between the two spins.

In the large R limit we thus obtain

$$w^2 \simeq \frac{R}{4m} + \dots$$

to be compared with the effective string result:

$$w^2 \sim \left(\frac{R}{4\sigma L} + \log(\frac{L}{2\pi}) + \dots\right) \qquad (R >> L)$$

linear increase of the width in both cases but with a different T dependence of the coefficent $m = \tilde{E}_0 = \frac{\sigma}{T}\sqrt{1 - \frac{\pi T^2}{3\sigma}} = \sigma L \sqrt{1 - \frac{\pi}{3\sigma L^2}}$

This discrepancy is due to the free bosonic approx in the string calculation.

This can be tested using the 2 loop calculation recently obtained by Gliozzi, Pepe and Wiese.

$$w^2_{nlo} = \left(1 + \frac{4\pi f(\tau)}{\sigma r^2}\right) w^2_{lo}(r/2) - \frac{f(\tau) + g(\tau)}{\sigma^2 r^2},$$

$$f(\tau) = \frac{E_2(\tau) - 4E_2(2\tau)}{48},$$

$$g(\tau) = i\pi\tau \left(\frac{E_2(\tau)}{12} - \frac{qd}{dq}\right) (f(\tau) + \frac{E_2(\tau)}{16}) + \frac{E_2(\tau)}{96},$$

The dominant term in the $R >> L$ limit (i.e. $\tau \rightarrow 0$) turns out to be again linear in R:

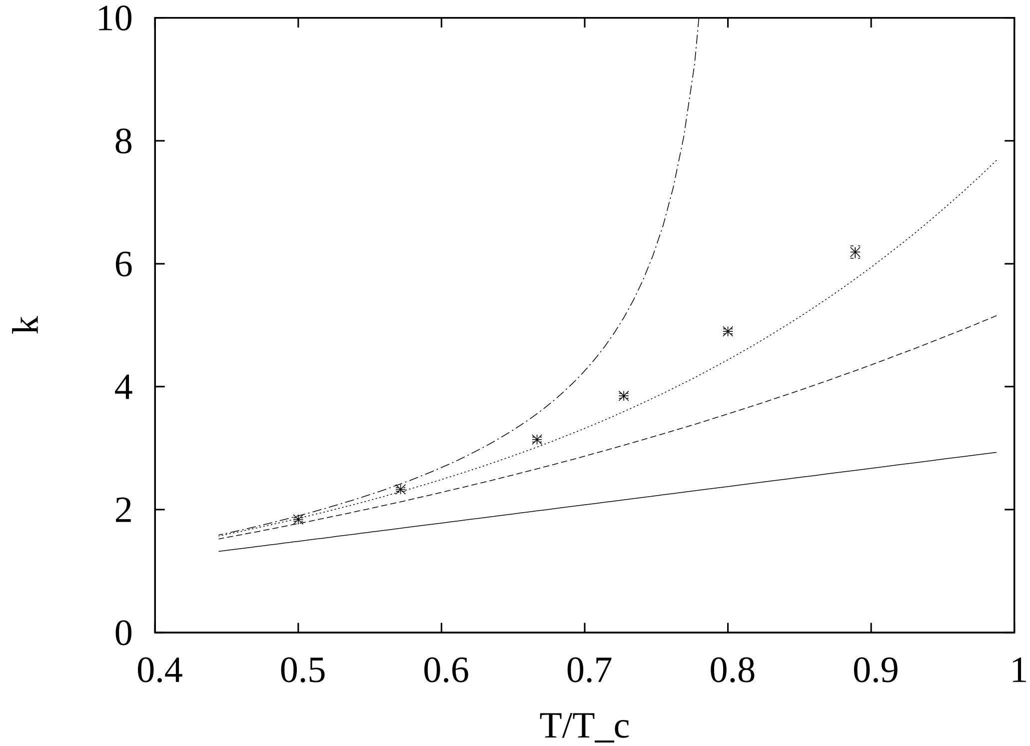
$$\sigma w^2_{nlo} \sim \frac{\pi R}{24\sigma^2 L^3} + \cdots$$

Combining together the leading and subleading corrections we end up with the following expression:

$$\sigma w^2 = \frac{R}{4\sigma L}\left(1 + \frac{\pi}{6\sigma L^2}\right) + \cdots$$

which is exactly the first term in the $1/L^2$ expansion of

$$\sigma w^2 = \frac{R}{4m} = \frac{R}{4\sigma L\sqrt{1 - \frac{\pi}{3\sigma L^2}}}$$



10
8
6
4
2
0
k
0.4
0.5
0.6
0.7
0.8
0.9
1
T/T_c

Conclusions

- The effective string approach predicts a logarithmic increase of the flux tube width at low temperature and a linear increase at high T (but still in the confining regime)

- This scenario is confirmed by MC simulations in the 3d gauge Ising model both at low and at high T, but an increasing discrepancy in the coefficient of the linear term appears as T_c is approached if one truncates the Nambu-Goto action at the first order.

- Dimensional reduction also predicts a linear increase of the flux tube thickness at high temperature, with a T dependence of the coefficient which suggests how to resum to all orders the Nambu-Goto action.

- This conjecture agrees with the two loop calculation of Gliozzi Pepe and Wiese.

- Comparing this prediction with the Ising model we find as usual a truncation to the quartic correction.

- We guess instead that in the $SU(2)$ case our result:

$$\sigma w^2 = \frac{R}{4\sigma L\sqrt{1-\frac{\pi}{3\sigma L^2}}}$$

  should describe very well the Montecarlo data also near the deconfinement transition.

- It is interesting to notice that at a difference with respect to the one loop approximation this expression for the amplitude of the linear growth divereges as the deconfinement temperature is approached.