# Study of Flux tube profiles from lattice QCD

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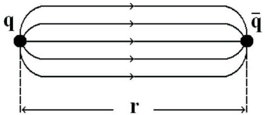
# Outline

# Dual superconductor scenario

The vacuum of QCD is a *magnetic (dual) superconductor*
[G. 'tHooft, Phys. Scripta **25** (1982) 133]

The electric field is confined into flux tubes → *QCD strings*




$$V_{q\bar{q}}(r) \longrightarrow \sigma r$$

The dual Meissner effect: formation of chromoelectric flux tubes between chromoelectric charges leading to a linear rising potential

## Aim of the work

To analyze the finger-print of the dual superconductor hypothesis, namely the Meissner effect
$\rightarrow$ analyzing the distribution of the color fields due to static quark-antiquark pair

$SU(2)$ new calculations [P. Cea and L. Cosmai, Phys.Rev.D52(1995)5152]

$SU(3)$

# Wilson loop

We are studying the distribution of the color fields due to a static quark-antiquark pair

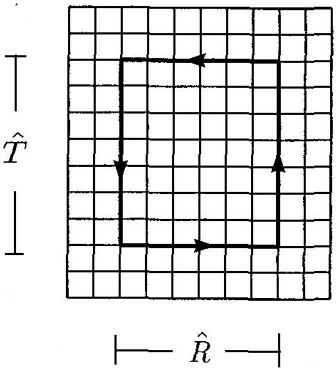
Static quark-antiquark potential

$$W(\hat{R},\hat{T}) \underset{\sim}{\hat{T}\to\infty} e^{-E(\hat{R})\hat{T}}$$

$$\underset{\sim}{\hat{R}\to\infty} e^{-\hat{\sigma}\hat{R}\hat{T}}$$

$\hat{\sigma}$ string tension

$$W_C[U] = \mathrm{Tr}\prod_{l\in C} U_l$$



[P. Cea and L. Cosmai, Phys.Rev.D52(1995)5152]

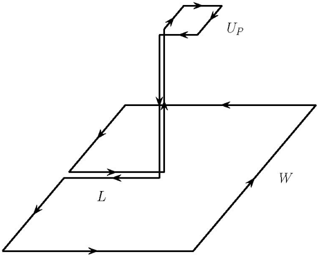
[A. DiGiacomo, M. Maggiore and S. Olejnik,Nucl.Phys.B347,441(1990)]

$$\rho_W = \frac{\left\langle \mathrm{tr}\left( W L U_P L^{\dagger} \right) \right\rangle}{\langle \mathrm{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \mathrm{tr}(U_P)\mathrm{tr}(W) \rangle}{\langle \mathrm{tr}(W) \rangle}$$

$L$ Schwinger line

$U_P$ plaquette

$W$ Wilson loop



$$L(x,y) = e^{ig\int_x^y dz_\mu A_\mu(z)} \qquad W[A] = e^{ig\oint_x^y dz_\mu A_\mu(z)}$$

$$U_{\mu\nu}(n) = e^{ig_0 a^2 F_{\mu\nu}(n)}$$

$$\rho_W \stackrel{a\to 0}{\longrightarrow} a^2 g\left[\langle F_{\mu\nu}\rangle_{q\bar{q}} - \langle F_{\mu\nu}\rangle_{\mathtt{vac}}\right]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\begin{array}{l|l} f_{12} \to E_x & f_{34} \to B_x \\ f_{13} \to E_y & f_{24} \to B_y \\ f_{14} \to E_z & f_{23} \to B_z \end{array}$$

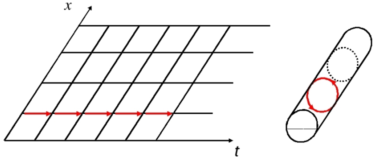
$$Z(T,V) = \mathrm{Tr}\, e^{-\frac{H}{T}} \quad (k_B = 1)$$

The path integral representation for the partition function in a quantum mechanical system is

$$Z(T,V) = \int \prod_i^N dq_i \, \langle q | e^{-\frac{H}{T}} | q \rangle = \int \prod_i^N dq_i \int_q^q Dq \; e^{-S_E[q]}$$

$$e^{-S_E[q]} = e^{-\int_o^{\frac{1}{T}} d\tau L_E(q,\dot{q})}$$

$$V = N^3 a^3$$
$$T = \frac{1}{N_\tau a}$$



# Polyakov loop

$$\rho_P = \frac{\left\langle \mathrm{Tr}\left(P(x) L U_P L^\dagger\right) \mathrm{Tr} P(y) \right\rangle}{\langle \mathrm{Tr} P(x) \mathrm{Tr} P(y) \rangle} - \frac{1}{N} \frac{\langle \mathrm{Tr}(U_P) \mathrm{Tr} P(x) \mathrm{Tr} P(y) \rangle}{\langle \mathrm{Tr} P(x) \mathrm{Tr} P(y) \rangle}$$

[A. Di Giacomo, M. Maggiore and S. Olejnik,Nucl.Phys.B347,441(1990)]

$$P(\vec{x}) = \mathrm{Tr} \prod_{x_0=1}^{N_t} U_4(\vec{x})$$

$$\langle P(\vec{0}) P^\dagger(\vec{r}) \rangle = e^{-V_{q\bar{q}}(\vec{r},T)/T}$$

# some references

[M. Fukugita and T. Niuya,'83; J.W. Flower and S.W. Otto,'85; A. Di Giacomo,*el al.*,'89; R.W. Haymaker and Y. Peng,'93]

$$\rho = \frac{\langle W U_P \rangle - \langle W \rangle \langle U_P \rangle}{\langle W \rangle} \rightarrow\sim a^4 \left( \langle F^2_{\mu\nu} \rangle_{q\bar{q}} - \langle F^2_{\mu\nu} \rangle_{\mathtt{vac}} \right)$$

## SU(2) finite Temperature

[Y. Peng and R. Haymaker,'94; S. Chagdaa and E. Laermann, 2007]

$$\rho(r,x) = \frac{\beta}{a^4} \left[ \frac{\langle P(0) P^\dagger(r) U_P(x) \rangle}{\langle P(0) P^\dagger(r) \rangle} - \langle U_P \rangle \right]$$

x denotes the distance of the plaquette from the line connecting quark sources

r is the quark separation

# Technical Informations

Standard Wilson action

$$S_G^{(SU(N))} = \beta \sum_P [1 - \frac{\mathrm{Tr}}{2N}(U_P + U_P^\dagger)]$$

| $SU(2)$ | $SU(3)$ |
|---|---|
| $20^4$ | $16^4$ |
| $24^4$ | $20^4$ |
| | $22^4$ |

1 HeatBath update followed by 4 overrelaxation steps

## Technical Informations

To disentangle the signal from the noise: Cooling

$$\frac{1}{4}\mathrm{tr}\left[\left(U^{\dagger}_{\mu}(x) - U'^{\dagger}_{\mu}(x)\right)\left(U_{\mu}(x) - U'_{\mu}(x)\right)\right] \leq \delta^2$$

with $\delta = 0.0354$
Cooling steps from 4 to 16
Statistics:

$$\begin{array}{lcl} SU(2) & \sim & 10K \\ SU(3) & \sim & 10K \end{array}$$

cooling technique

# Technical Informations

To disentangle the signal from the noise: Cooling

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Cooling steps from 4 to 16
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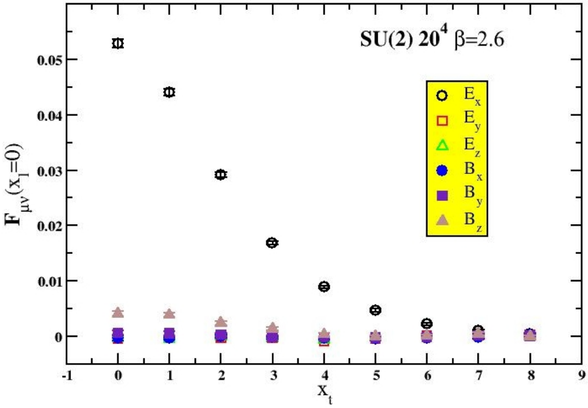
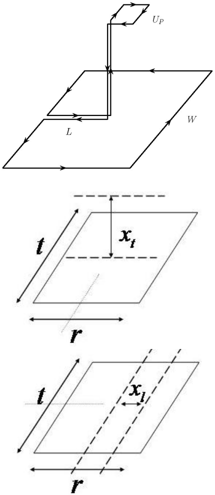
cooling technique

# London penetration length




$F_{\mu\nu}$ on $20^4$ lattice at $\beta = 2.6$, $8 \times 8$ Wilson loop

# London penetration length

$E_x$ resembles the dual version of the Abrikosov vortex field distribution

[P. Cea and L. Cosmai,Nucl.Phys.Proc.Suppl.47,318(1996)]

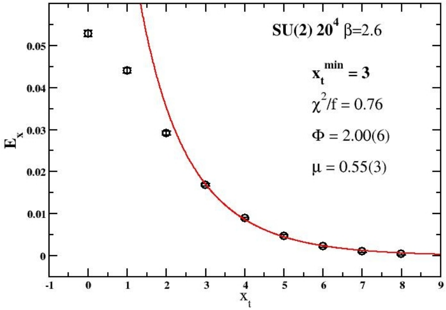
$$E_x(x_t) = \frac{\Phi}{2\pi}\mu^2 K_0(\mu x_t) \ , \ x_t > 0$$

$\lambda = \frac{1}{\mu}$ London penetration length

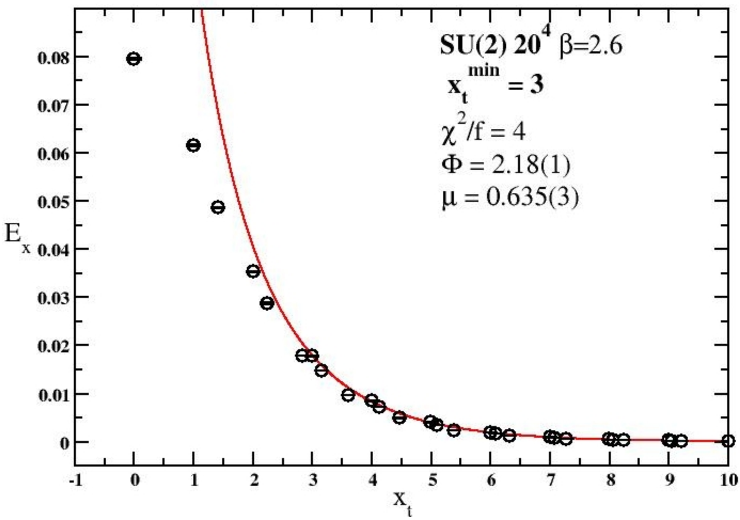
$\Phi$ the external flux

$K_0$ modified Bessel function of order zero

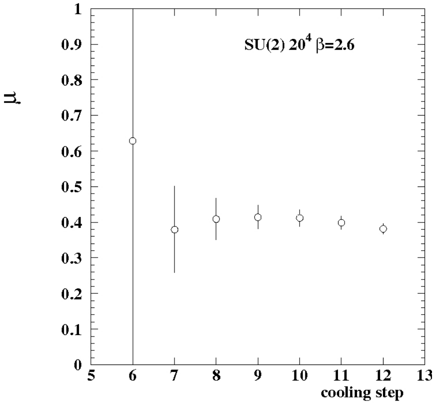
is valid if $\lambda \gg \xi$, $\xi$ being the coherence length (type-II superconductor).

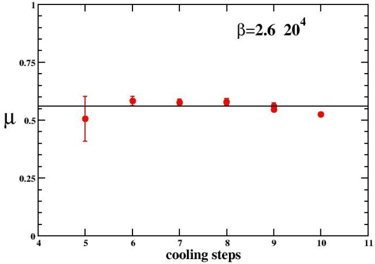


# Intermediate distances



# London penetration length - $SU(2)$




[P. Cea and L. Cosmai,Nucl.Phys.Proc.Suppl.47,318(1996)]

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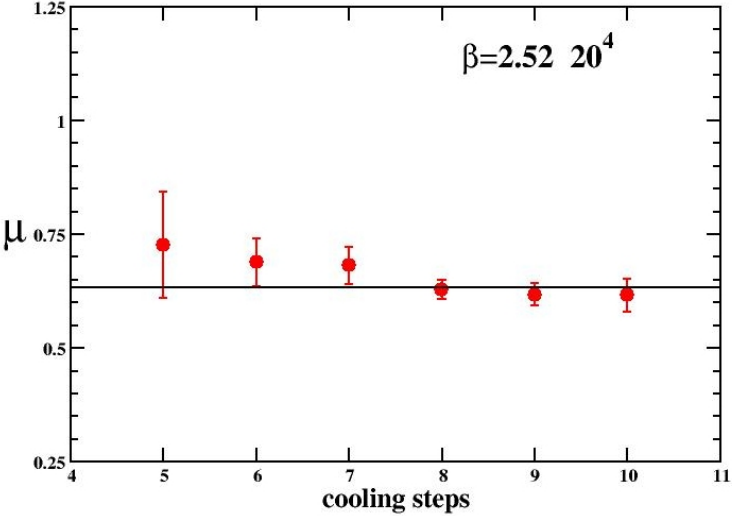

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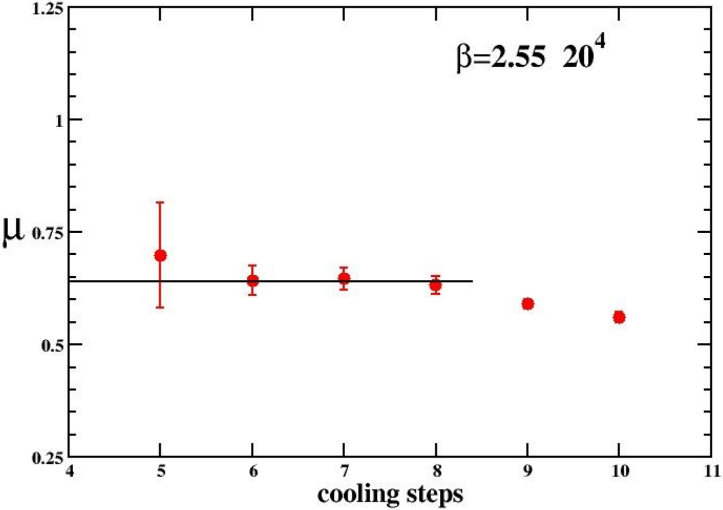

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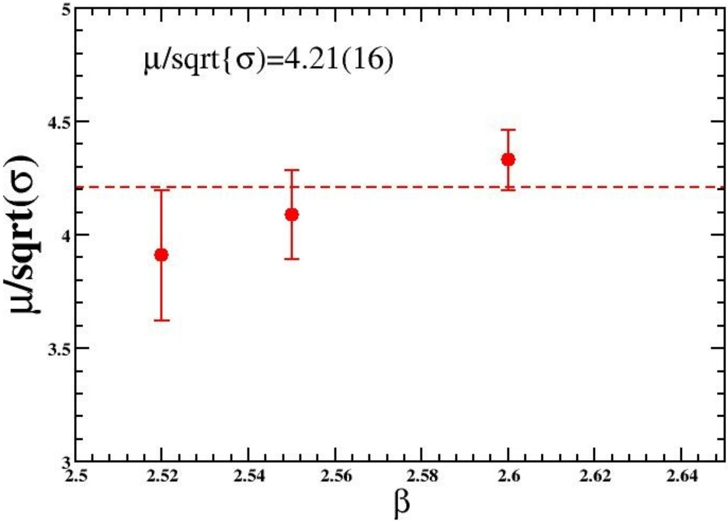
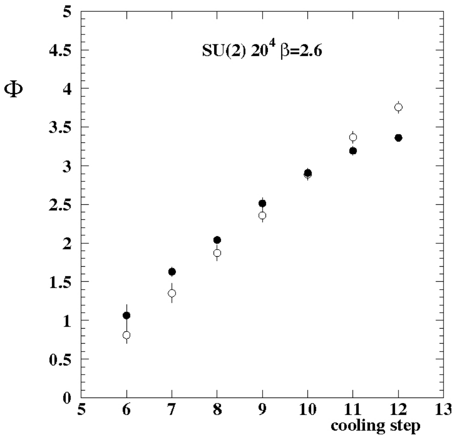
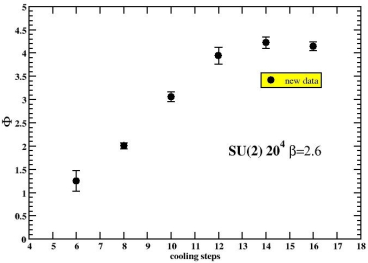


# External flux - $SU(2)$

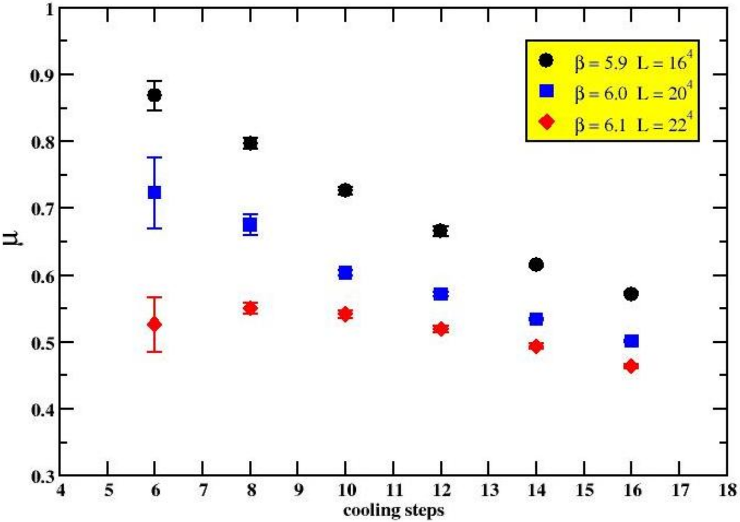



[P. Cea and L. Cosmai,Nucl.Phys.Proc.Suppl.47,318(1996)]

Full points correspond to rectangular Wilson loops ($10 \times 5$), open points to square Wilson loops ($8 \times 8$)

# London penetration length - $SU(3)$



# London penetration length - $SU(3)$

Parameterization of the SU(3) string tension
[R. G. Edwards, U. M. Heller, and T. R. Klassen, Nucl. Phys. **B517**, 377 (1998)]

$$(a\sqrt{\sigma})(g) = f_{SU(3)}(g^2)\,(\,1 + 0.2731\,\hat{a}^2(g) - 0.01545\,\hat{a}^4(g) + 0.01975\,\hat{a}^6(g)\,)/0.01364$$

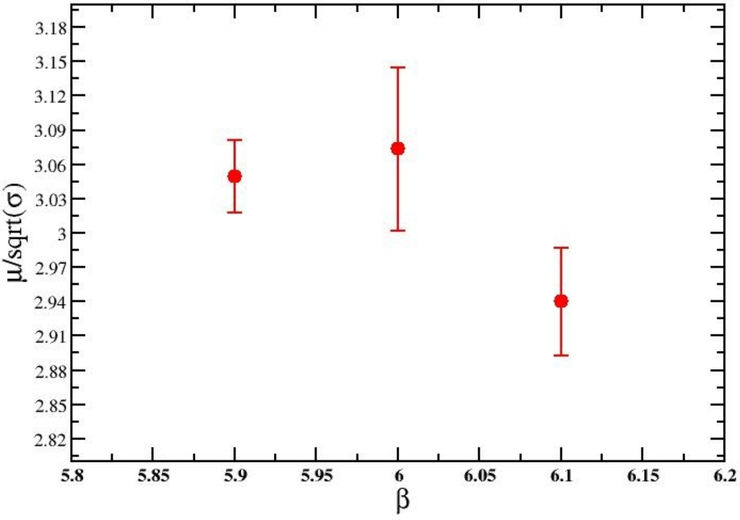
$$\hat{a}(g) = \frac{f_{SU(3)}(g^2)}{f_{SU(3)}(g^2(\beta=6))}\,; \qquad \beta = \frac{6}{g^2}$$

for $5.6 \leq \beta \leq 6.5$.

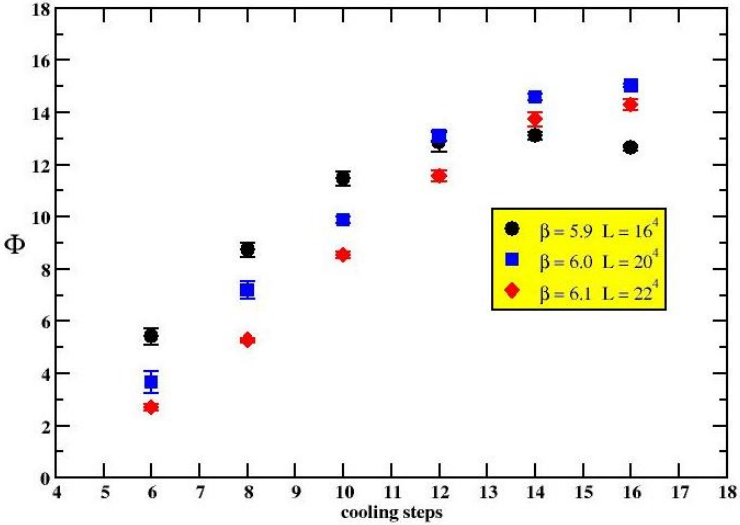
$f_{SU(3)}$ is defined as

$$f_{SU(3)}(g^2) = \left(b_0 g^2\right)^{-b_1/2b_0^2} \exp\left(-\frac{1}{2b_0 g^2}\right)$$

# London penetration length - $SU(3)$



# External flux - $SU(3)$



## Outlook

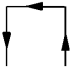
- improve scaling for $SU(3)$
- study the distribution of chromoelectric fields in the deconfined phase of $SU(2)$
- move to the deconfined phase of the $SU(3)$

Thank you!

# cooling technique

Cooling consists in freezing the quantum fluctuations of equilibrium field configurations
[Berg '81, Teper '86, Di Giacomo *et al* '90]

$$\widetilde{U}_\mu = U_\nu(x+\hat{\mu})U^\dagger_\mu(x+\hat{\nu})U^\dagger_\nu(x)$$



$$\sum_{\alpha=1}^{6} \widetilde{U}_\alpha = k\overline{U} \qquad \text{where} \qquad \overline{U} \in SU(2) \qquad \text{and} \qquad k^2 \equiv \det\left(\sum_{\alpha=1}^{6} \widetilde{U}_\alpha\right)$$

The local $SU(2)$ action is proportional to $\mathcal{R}e\,\mathrm{Tr}(1 - U\overline{U}) \Rightarrow \mathcal{R}e\,\mathrm{Tr}(U\overline{U}) = \mathcal{R}e\,\mathrm{Tr}(I)$
which requires the link to be updated as

$$U \longrightarrow U' = \overline{U}^{-1} = \overline{U}^\dagger = \frac{\left(\sum_{\alpha=1}^{6} \widetilde{U_\alpha}\right)^\dagger}{k}$$

At the $SU(3)$ level, one successively applies this algorithm to various $SU(2)$ subgroups of $SU(3)$, with $SU(2)$ subgroups selected to cover the $SU(3)$ gauge group

Return