Measures of the width of the confining string

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Intrinsic vs. fluctuation width of the string

“Width of the string” can mean two different things:

1. the confining string has a certain intrinsic thickness of order $1/\sqrt{\sigma}$
2. the amplitude of the fluctuations of the string

- notion (1) involves physics on the energy scale $\sqrt{\sigma}$.
- to study (2), the string can be treated as ‘thin’ in an effective theory; in the following, I concentrate on this notion of width.

Width of the string: effective-theory level

Effective string theory:

$$S[\boldsymbol{h}] = \frac{\sigma}{2} \int_0^{\beta} dt \int_0^{r} dx \, \partial_a \boldsymbol{h}(t,x) \cdot \partial_a \boldsymbol{h}(t,x)$$

Natural definition of the string width within the effective theory:

$$w^2(x) = \langle h^2(t,x) \rangle$$

Prediction: [Lüscher, Münster, Weisz, Nucl. Phys. B180 (1981)]

$$w^2(r/2) = \frac{d-2}{2\pi\sigma} \log(r/r_0).$$

Width of the string: fundamental theory level

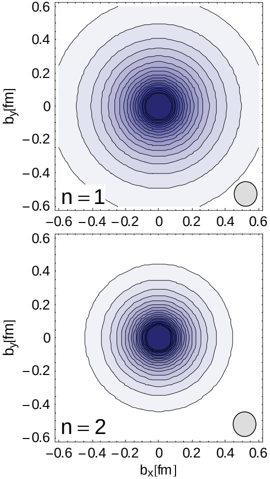
(Will consider SU(N) gauge theory, but the questions below also apply to spin systems.)

1. for an open string (i.e. with Dirichlet boundary conditions in space provided by a static $\bar{Q}Q$ pair), transverse size can be probed with a local operator as a function of transverse distance $\boldsymbol{r}_\perp$

2. for a closed string (toron), one has to work in momentum space and calculate its form factors with respect to a local operator.

- but which operator? is there a preferred, ‘canonical’ one?
- how will results from different operators differ?
- which one should be used to compare with the $\langle h^2(t,x)\rangle$ prediction of the effective theory?

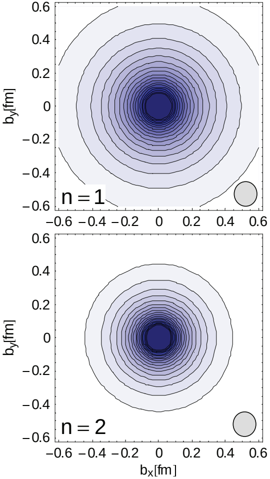
Generalized Form Factors in Hadron Structure Studies



[QCDSF, 0708.2249]

Transverse profile of the pion in the infinite-momentum frame, as probed by two isovector twist-two operators

Generalized Form Factors in Hadron Structure Studies



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Full Info on Probability Distribution contained in Generalized Parton Distribution H: [Burkardt hep-ph/0005108]

$$q(x, \boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp}{(2\pi)^2}\, e^{-i\Delta_\perp \cdot \boldsymbol{b}_\perp}\, H(x, \xi = 0, -\Delta_\perp^2)\,,$$

Now take x-moments (x=longitudinal momentum fraction)

$$\begin{aligned} H^n(\xi, t) &\equiv \int_{-1}^{1} \mathrm{d}x\, x^{n-1} H(x, \xi, t), \\ H^{n=1}(\xi, t) &= A_{10}(t), \qquad H^{n=2}(\xi, t) = A_{20}(t) + (2\xi)^2 C_{20}(t), \end{aligned}$$

- $A_{10}(t)$ is the vector form factor
- for $\mathcal{O}_{\mu_1\mu_2}(x) = \frac{1}{2}\bar{q}(\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2} + \gamma_{\mu_2}\overleftrightarrow{D}_{\mu_1})q\,,$

  $\langle P'|\mathcal{O}_{\mu_1\mu_2}(q)|P\rangle = 2\bar{P}_{\mu_1}\bar{P}_{\mu_2}A_{20}(t) + 2\Delta_{\mu_1}\Delta_{\mu_2}C_{20}(t)\,,$

  where $\bar{P} \equiv (P + P')/2$.

Measures of the size of the proton

One can think of the nucleon as having a ‘core’ surrounding by a ‘pion cloud’. As $m_\pi \to 0, \ldots$

- $\langle r^2_{\rm Dirac} \rangle \sim \log m_\pi \,,$
- $\langle r^2_{\rm Pauli} \rangle \sim m_\pi^{-1},$
- $\langle r^2_{\rm mass} \rangle \sim$finite.

The size of the ‘cloud’ depends on how one probes it.

Q: for a long flux-tube, will any probe operator see the logarithmic growth of the width?

Form Factors with respect to the Energy Momentum Tensor (I)

Concentrate on the transverse structure as measured by the lowest-dimensional twist-two operator, namely the energy-momentum tensor.

Decompose the full $(d+1)$-dimensional energy-momentum tensor into irreducible representations of d-dimensional space. In $d=2$ for simplicity, the decomposition takes the form

$$\left(\begin{array}{cc|c} T_{tt} & T_{ty} & T_{tx} \\ T_{yt} & T_{yy} & T_{yx} \\ \hline T_{xt} & T_{xy} & T_{xx} \end{array}\right)$$

In the following, we choose the normalization of states such that

$$\langle P'|P\rangle = (2\pi)^{d-1}\delta^{d-1}(\boldsymbol{P}' - \boldsymbol{P})\,\frac{2E_{\boldsymbol{P}}}{M}\,.$$

Form Factors with respect to the Energy Momentum Tensor (II)

- T_{xx} measures the stress in the x direction, it is a scalar operator from the point of view of physics within an $x =$cst slice ($P'_x = P_x = 0$):

$$\langle P'|T_{xx}(0)|P\rangle = M^2 f_4(q^2), \qquad q = P' - P. \tag{1}$$

- (T_{tx}, T_{yx}) is a conserved vector from the point of view of an $x =$cst slice, if one restricts the matrix elements to states that are translationally invariant in the x direction: $\partial_t T_{tx} + \partial_y T_{yx} + \underbrace{\partial_x T_{xx}}_{=0} = 0$.

- $\Rightarrow$ vector form factor of a scalar object, hence (cf. pion e.m. form factor),

$$\langle P'|T_{ax}(0)|P\rangle = M\bar{P}_a f_3(q^2), \qquad a = t, y. \tag{2}$$

- matrix elements of T_{ab} (conserved tensor) are parametrized by two FFs,

$$\langle P'|T_{ab}(0)|P\rangle = \bar{P}_a\bar{P}_b f_1(q^2) + (\bar{P}_a q_b + \bar{P}_b q_a) f_2(q^2). \tag{3}$$

EMT Form Factors: interpretation

The transverse stress-energy structure of the ground state of the string is characterized by four form factors $\{f_i(q^2)\}_{i=1}^{4}$.

The forward matrix elements provide their interpretation:

$$\begin{aligned} \langle P|T_{\mu\nu}(0)|P\rangle &= P_\mu P_\nu, \\ \frac{\langle P|\int d^{d-1}x T_{xx}|P\rangle}{\langle P|P\rangle} &= L\frac{\partial E}{\partial L} = \sigma_{\rm eff}(L) \end{aligned}$$

imply

$$f_1(0) = 1, \qquad f_3(0) = 0, \qquad f_4(0) = \frac{1}{\sigma L^2}.$$

- f_1 describes the energy distribution of the string
- f_2 describes the distribution of transverse energy flux
- f_3 describes the distribution of longitudinal energy flux
- f_4 describes the distribution of longitudinal stress.

Discretization of the Energy-Momentum Tensor in $d = 2$

On the lattice in $d = 2$ space dimensions with action

$$S_{\mathrm{g}} = \beta \sum_x S_{0x}(x) + S_{0y}(x) + S_{xy}(x)\,,$$

a natural discretization of the energy density and longitud. stress operators is

$$\begin{aligned} a^3 T_{00} &= S_{xy}(\frac{4}{3}\beta Z(\beta) + \frac{1}{3}\frac{\partial\beta}{\partial\log a}) + (S_{0y} + S_{0x})(-\frac{2}{3}\beta Z(\beta) + \frac{1}{3}\frac{\partial\beta}{\partial\log a}). \\ a^3 T_{xx} &= S_{0y}(\frac{4}{3}\beta Z(\beta) + \frac{1}{3}\frac{\partial\beta}{\partial\log a}) + (S_{0x} + S_{xy})(-\frac{2}{3}\beta Z(\beta) + \frac{1}{3}\frac{\partial\beta}{\partial\log a}). \end{aligned}$$

At treelevel,

$$\frac{\partial\beta}{\partial\log a} = -\beta, \qquad Z(\beta) = 1.$$

Corrections to these values are O($g^2 a$), i.e. neglecting them amounts to introducing an O(a) discretization error.

Sum Rules for Plaquette Probes [3d version of Michael sum rules]

$$\frac{a^3}{\beta}\begin{pmatrix} T_{00} \\ T_{xx} \\ T_{yy} \end{pmatrix} = \begin{pmatrix} +1 & -1 & -1 \\ -1 & -1 & +1 \\ -1 & +1 & -1 \end{pmatrix}\begin{pmatrix} S_{xy} \\ S_{0x} \\ S_{0y} \end{pmatrix} \Rightarrow \quad S_{0x} = -\frac{a^3}{2\beta}(T_{00}+T_{xx}), \ \ldots$$

$$\frac{\langle\psi|\int d^d\boldsymbol{x} T_{00}(x)|\psi\rangle}{\langle\psi|\psi\rangle} = E, \qquad \frac{\langle\psi|\int d^d\boldsymbol{x} T_{xx}(x)|\psi\rangle}{\langle\psi|\psi\rangle} = L_x\frac{\partial E}{\partial L_x}$$

$\Rightarrow$ for a string of length L along the x-direction,

$$\begin{aligned} -\frac{\langle\psi|\beta\sum_{\boldsymbol{x}} S_{xy}(x)|\psi\rangle}{\langle\psi|\psi\rangle} &= 2aL\frac{\partial E}{\partial L}, \\ -\frac{\langle\psi|\beta\sum_{\boldsymbol{x}} S_{0x}(x)|\psi\rangle}{\langle\psi|\psi\rangle} &= \tfrac{1}{2}a(E+\frac{\partial E}{\partial L}), \\ -\frac{\langle\psi|\beta\sum_{\boldsymbol{x}} S_{0y}(x)|\psi\rangle}{\langle\psi|\psi\rangle} &= \tfrac{1}{2}aE. \end{aligned}$$

For a long string $E \propto L$: $\langle S_{xy}\rangle : \langle S_{0x}\rangle : \langle S_{0y}\rangle = 2 : 1 : \frac{1}{2}$.

Interpreting the recent results of Gliozzi, Pepe, Wiese 1002.4888

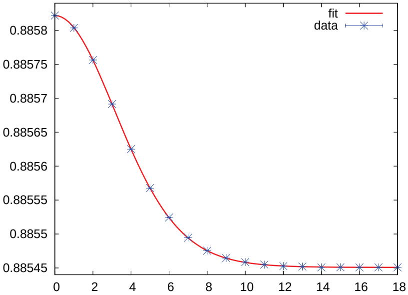

FIG. 1: *The ratio* $\langle \Phi_0 \Phi_r P_x \rangle / \langle \Phi_0 \Phi_r \rangle$ *as a function of the transverse displacement* x_3 *at fixed distance* $r = 19$ *between the external static quarks. The solid line is a fit of the numerical data using eq.(20).*

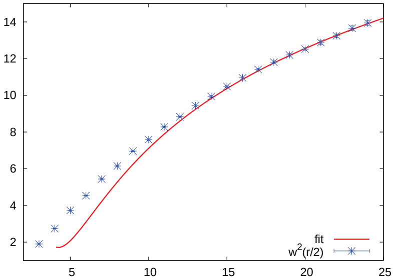

FIG. 2: *The squared width of the confining string* $w^2(r/2)$ *at its midpoint as a function of the distance* r *between the external quark charges. The solid curve is a fit to the next-to-leading order prediction of the low-energy effective field theory from eq.(13).*

Test of the Equation $-\frac{\langle\psi|\beta\sum_x S_{0x}(x)|\psi\rangle}{\langle\psi|\psi\rangle} = \frac{1}{2}a(E + \frac{\partial E}{\partial L}),$

The profile is approximately a Gaussian, with

$$\int_{-\infty}^{\infty} A\exp(-\tfrac{1}{2}x^2/R^2) = \sqrt{2\pi}\cdot A\cdot R$$

Thus the left hand side of the Eq. amounts to

$$\sqrt{2\pi}\beta ARL.$$

For $L = 19$, GLW find $R \approx \sqrt{12.1}$ and $A \approx 0.00038$. If we neglect the quark self-energies and the string corrections, $E \approx \sigma L$ and the RHS amounts to

$$\sigma L.$$

Simplifying the common factor L, we have $LHS \approx 0.030$ and $RHS \approx 0.026$. Satisfactory!

(Approximations made: neglect of the quark self-energies and the use of the treelevel renormalization factors for the plaquette)

Conclusion

- there is a tower of operators that can characterize the width of the QCD string, cf. Generalized Parton Distributions
- what is the analog of the momentum fraction x for a non-relatistivic $(1/M \ll r_\perp)$ system such as the QCD string?
- using plaquettes as a probe amounts to considering linear combinations of the stress-energy tensor
- based on this observation, the area under the Gaussian obtained by GPW can be understood.

Backup Slides

Connection between GPDs and Generalized Form Factors

Mellin-moments of the GPDs,

$$H^n(\xi,t) \equiv \int_{-1}^{1} \mathrm{d}x\, x^{n-1} H(x,\xi,t),$$

are given by polynomials in the longitudinal momentum transfer ξ,

$$\begin{aligned}
H^{n=1}(\xi,t) &= A_{10}(t), & E^{n=1}(\xi,t) &= B_{10}(t),\\
H^{n=2}(\xi,t) &= A_{20}(t) + (2\xi)^2 C_{20}(t), & E^{n=2}(\xi,t) &= B_{20}(t) - (2\xi)^2 C_{20}(t),\\
H^{n=3}(\xi,t) &= A_{30}(t) + (2\xi)^2 A_{32}(t)\,, & E^{n=3}(\xi,t) &= B_{30}(t) + (2\xi)^2 B_{32}(t)\,.
\end{aligned}$$

The coefficients are the Generalized Form Factors calculated on the lattice.

For example, if $\mathcal{O}_{\mu_1\mu_2}(x) = \frac{1}{2}\bar{q}(\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2} + \gamma_{\mu_2}\overleftrightarrow{D}_{\mu_1})q\,,$ $(\bar{P} \equiv (P+P')/2)$

$$\langle P'|\mathcal{O}_{\mu_1\mu_2}(q)|P\rangle = \bar{P}_{\{\mu_1}\langle\!\langle\gamma_{\mu_2\}}\rangle\!\rangle\, A_{20}(t) + \frac{i}{2m}\bar{P}_{\{\mu_1}\langle\!\langle\sigma_{\mu_2\}\alpha}\rangle\!\rangle\Delta_\alpha\, B_{20}(t) + \frac{1}{m}\Delta_{\{\mu_1}\Delta_{\mu_2\}}\, C_{20}(t)\,.$$

Impact-Parameter Dependent Quark Distributions

The GPDs determine in particular the distribution of partons and their helicities in impact parameter space, [Burkardt hep-ph/0005108]

$$\begin{aligned} q(x,\boldsymbol{b}_\perp) &= \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp}{(2\pi)^2}\, e^{-i\Delta_\perp\cdot\boldsymbol{b}_\perp}\, H(x,\xi=0,-\Delta_\perp^2)\,, \\ \Delta q(x,\boldsymbol{b}_\perp) &= \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp}{(2\pi)^2}\, e^{-i\Delta_\perp\cdot\boldsymbol{b}_\perp}\, \widetilde{H}(x,\xi=0,-\Delta_\perp^2)\,. \end{aligned}$$

On the lattice,

- of the x-dependence, only first few $x^{0,1,2}$ moments are accessible
- the impact-parameter dependence $\boldsymbol{b}_\perp$ can be obtained more easily, currently $(0.4\mathrm{GeV})^2 < \Delta_\perp^2 < (2.0\mathrm{GeV})^2$

The same goes through for the gluonic GFFs.

(continued)

For a string of length L along the x-direction,

$$-\frac{\langle\psi|\beta\sum_{\boldsymbol{x}} S_{xy}(x)|\psi\rangle}{\langle\psi|\psi\rangle} = 2aL\frac{\partial E}{\partial L}, \quad (4)$$

$$-\frac{\langle\psi|\beta\sum_{\boldsymbol{x}} S_{0x}(x)|\psi\rangle}{\langle\psi|\psi\rangle} = \tfrac{1}{2}a(E+\frac{\partial E}{\partial L}), \quad (5)$$

$$-\frac{\langle\psi|\beta\sum_{\boldsymbol{x}} S_{0y}(x)|\psi\rangle}{\langle\psi|\psi\rangle} = \tfrac{1}{2}aE. \quad (6)$$

For a long string $E \propto L$, we expect the various plaquettes (summed over a time-slice) to obey

$$\langle S_{xy}\rangle : \langle S_{0x}\rangle : \langle S_0 y\rangle = 2 : 1 : \tfrac{1}{2}. \quad (7)$$