Strong to weak coupling transition in large N gauge theories

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Wilson loop operator

- Unitary operator for SU(N) gauge theories.
- A probe of the transition from strong coupling to weak coupling.
- Large (area) Wilson loops are non-perturbative and correspond to strong coupling.
- Small (area) Wilson loops are perturbative and correspond to weak coupling.

't Hooft model – Durhuus-Olesen transition

- The distribution of the eigenvalues of the Wilson loop operator is a gauge invariant observable:

$$p(\theta; t); \quad \theta \in [-\pi, \pi];$$

  t is the dimensional area of the loop.
- In the large N limit:
  - Support is restricted to $|\theta| \leq \theta_0(t) < \pi$ for $t < 4$.
  - Support extends over the full unit circle for $t > 4$.
- Transition at $t = 4$ but traces of arbitrary powers of the Wilson loop operator are smooth functions of t.
- Behavior in the double scaling limit, $t \to 4$ and $\theta \to \pi$, described by universal scaling functions.

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Large N QCD in three an four dimensions

- Wilson loop operator undergoes a transition like in two dimensions as the area is changed.
- The behavior in the double scaling limit is in the same universality class as the two dimensional model.

Large N universality hypothesis

Let $\mathcal{C}$ be a closed non-intersecting loop: $x_\mu(s), s \in [0, 1]$.
Let $\mathcal{C}(m)$ be a whole family of loops obtained by dilation:
$x_\mu(s, m) = \frac{1}{m} x_\mu(s)$,with $m > 0$.
Let $W(m, \mathcal{C}(*)) = W(\mathcal{C}(m))$ be the family of operators associated with the family of loops denoted by $\mathcal{C}(*)$ where m labels one member in the family.

Large N universality hypothesis – Continued

Define

$$O_N(y, m, \mathcal{C}(*)) = \langle \det(e^{\frac{y}{2}} + e^{-\frac{y}{2}} W(m, \mathcal{C}(*))\rangle$$

$$\lim_{N\to\infty} \mathcal{N}(N, b, \mathcal{C}(*)) O_N\left(y = \left(\frac{4}{3N^3}\right)^{\frac{1}{4}} \frac{\xi}{a_1(\mathcal{C}(*))}, m = m_c\left[1 + \frac{\alpha}{\sqrt{3N}a_2(\mathcal{C}(*))}\right]\right)$$

$$= \int_{-\infty}^{\infty} du e^{-u^4 - \alpha u^2 + \xi u} \equiv \zeta(\xi, \alpha)$$

Tests of the hypothesis

The hypothesis holds true in

- Large N gauge theory in four dimensions.
- Large N gauge theory in three dimensions.
- Large N $SU(N) \times SU(N)$ principal chiral model in two dimensions – Wilson loop operator is replaced by the operator associated with the two point function and one finds a critical separation and a double scaling limit around it.

But $\cdots$

One needs to use smeared operators to define physical operators (avoid perimeter divergence in wilson loops, for example).

A new observable – I

$$\det(z + W)$$

has a fermionic representation:
$X_1, X_2 ... X_n$ are n arbitrary $N \times N$ matrices.

$$X = X_1 X_2 ... X_n$$

$$\int \prod_{j=1}^{n} [d\bar{\psi}_j d\psi_j] e^{\sum_{j=1}^{n} [\bar{\psi}_j X_j \psi_{j+1} - \bar{\psi}_j \psi_j]} = \det(1 + X)$$

A new observable – II

We can take a formal continuum limit of the fermionic path integral. The fermions interact with a one dimensional vector potential

$$a(\sigma) = A_\mu(x(\sigma))\frac{\partial x_\mu(\sigma)}{d\sigma}$$

where A_μ is the d dimensional vector potential and $x(\sigma)$ describes the one-dimensional closed loop.
The spectrum of

$$D_1(\mathcal{C}) \equiv \partial_\sigma - ia(\sigma)$$

- will have a gap for small loops,
- and will be gap-less for large loops.

A new observable – III

Two dimensional fermions
coupled to
four dimensional gauge fields

Let σ_1 and σ_2 define the coordinates on a two dimensional $l_1 \times l_2$ torus, Σ, embedded in four dimensions:

$$x_1(\sigma) = \frac{l_1}{2\pi} \cos \frac{2\pi\sigma_1}{l_1}; \qquad x_2(\sigma) = \frac{l_1}{2\pi} \sin \frac{2\pi\sigma_1}{l_1};$$
$$x_3(\sigma) = \frac{l_2}{2\pi} \cos \frac{2\pi\sigma_2}{l_2}; \qquad x_4(\sigma) = \frac{l_2}{2\pi} \sin \frac{2\pi\sigma_2}{l_2}$$

The induced metric is

$$d\sigma^2 = d\sigma_1^2 + d\sigma_2^2$$

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A new observable – IV

We define a two component gauge potential a_α on the torus by

$$a_\alpha = A_\mu(x(\sigma))\frac{\partial x_\mu}{\partial \sigma_\alpha}$$

The two dimensional massless Dirac operator is:

$$D_2(\Sigma) = \gamma_\alpha \partial_{\sigma_\alpha} - i\gamma_\alpha a_\alpha(\sigma)$$

$$\gamma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

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Ultraviolet divergences

- The two dimensional fermions are quenched in the large N limit.
- Nonabelian current:

$$J^j_\alpha(\sigma) = \bar{\psi}(\sigma)\gamma_\alpha T^j \psi(\sigma)$$

- Abelian current:

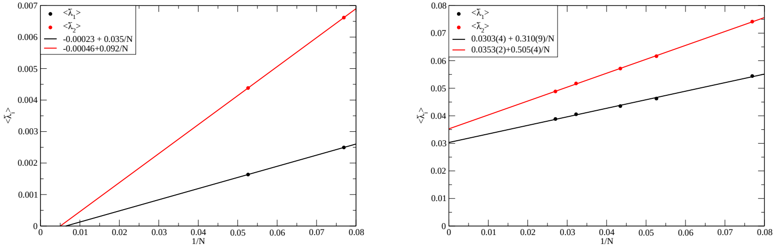
$$J_\alpha = \bar{\psi}(\sigma)\gamma_\alpha \psi(\sigma)$$

- Two counter terms that preserve chiral symmetry:

$$\mathcal{L}_1 = J^j_\alpha J^j_\alpha, \quad \mathcal{L}_2 = J_\alpha J_\alpha$$

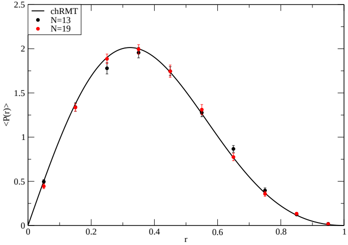
- Counter-terms are dimensionless and we have only logarithmic divergences.

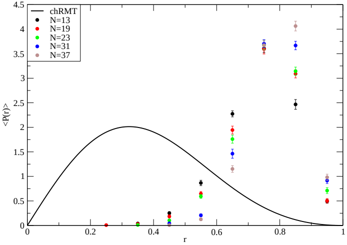
Chiral symmetry is broken for large torus but not for small torus



Left panel is a torus of size 2.7×2.7 and right panel is a torus of size 1.4×1.4 with lengths measured in units of inverse deconfinement temperature.

Chiral symmetry is broken for large torus but not for small torus





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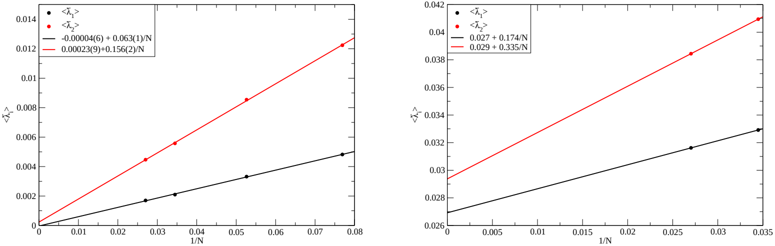
Hamiltonian version – Cylinder

Let σ_1 and σ_2 define the coordinates on a cylinder of dimensions $l_1 \times \infty$:

$$x_1(\sigma) = \frac{l_1}{2\pi}\cos\frac{2\pi\sigma_1}{l_1}; \qquad x_2(\sigma) = \frac{l_1}{2\pi}\sin\frac{2\pi\sigma_1}{l_1};$$
$$x_3(\sigma) = 0; \qquad x_4(\sigma) = \sigma_2.$$

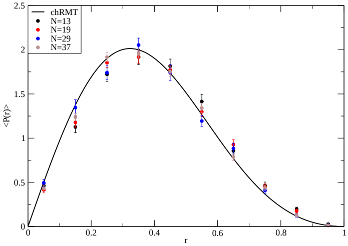
The gauge field in the σ_2 direction seen by the two dimensional fermions have an associated Polyakov loop that is unbroken in the confined phase.

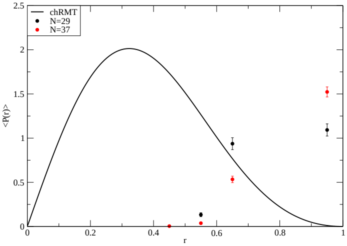
Chiral symmetry is broken for large cylinder but not for small cylinder



Left panel is a torus of size 1.8×1.8 and right panel is a torus of size 0.9×0.9 with lengths measured in units of inverse deconfinement temperature.

Chiral symmetry is broken for large cylinder but not for small cylinder





Left panel is a torus of size 1.8×1.8 and right panel is a torus of size 0.9×0.9 with lengths measured in units of inverse deconfinement temperature.

Infinite torus

- We can consider two dimensional fermions that live on the (x_1, x_2) plane at $x_3 = x_4 = 0$.
- This corresponds to an infinite torus and the two associated Polyakov loops will be unbroken in the confined phase.
- Chiral symmetry will be broken. What is the chiral condensate?
- What is the β function associated with the two dimensional fermions?

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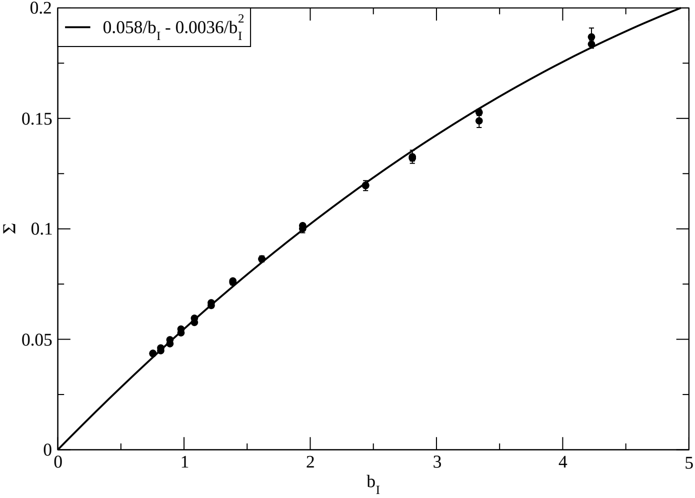
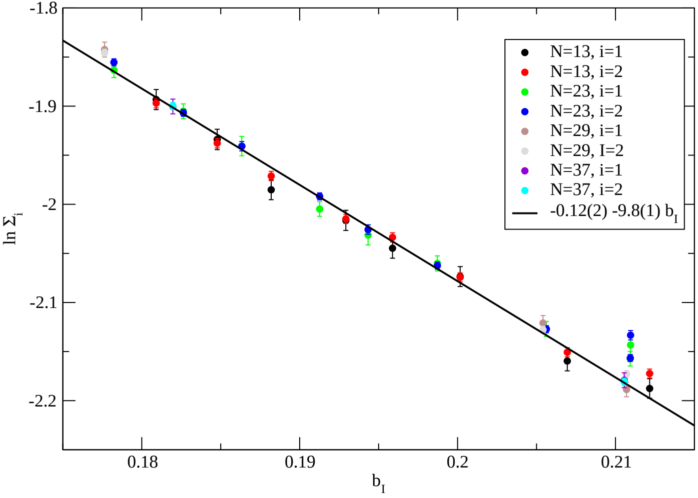
Chiral symmetry is broken for an infinite torus – Three D gauge fields



Chiral symmetry is broken for an infinite torus – Four D gauge fields



Theoretical Exercise: Abelian case

- The abelian case is Gaussian, so solvable.
- Two dimensional photon exchange is finite for three D gauge fields:

$$\int_{p_\parallel^2+p_\perp^2\le\Lambda^2}\frac{d^2p_\parallel dp_\perp}{(2\pi)^3}\frac{f(p_\parallel^2)}{p_\parallel^2+p_\perp^2}$$
$$=\int_{p_\parallel^2\le\Lambda^2}\frac{d^2p_\parallel}{4\pi^3}f(p_\parallel^2)\frac{1}{\sqrt{p_\parallel^2}}\tan^{-1}\sqrt{\frac{\Lambda^2}{p_\parallel^2}-1}$$

- Two dimensional photon exchange is UV divergent for four D gauge fields:

$$\int_{p_\parallel^2+p_\perp^2\le\Lambda^2}\frac{d^2p_\parallel d^2p_\perp}{(2\pi)^4}\frac{f(p_\parallel^2)}{p_\parallel^2+p_\perp^2}$$
$$=\int_{p_\parallel^2\le\Lambda^2}\frac{d^2p_\parallel}{(2\pi)^3}f(p_\parallel^2)\log[\Lambda/|p_\parallel|]$$

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Abelian chiral condensate

- We are interested in $V = \bar{\psi}^L \psi^R, \; \bar{V} = \bar{\psi}^R \psi^L, \; \bar{\psi}\psi = V + \bar{V}$
- Three dimensional gauge fields:

$$G(z) = \frac{(2\pi)^2}{e_0^4} \langle V(\sigma)\bar{V}(0)\rangle = \frac{1}{(2\pi z)^2} e^{4F(z)} = \left[\frac{e^\gamma}{4\pi} e^{\int_0^\infty dt \frac{e^{-zt}}{\sqrt{1+t^2}}}\right]^2$$

$$\lim_{z\to\infty} G(z) = \left[\frac{e^\gamma}{4\pi}\right]^2$$

- Four dimensional gauge fields: For

$$\Lambda|\sigma| \to \infty \quad \Lambda^{-2}\langle V(\sigma)\bar{V}(0)\rangle \sim \left[\frac{e^\gamma}{4\pi} \left(\frac{e_0^2}{2\pi^2} \log(\Lambda|\sigma|)\right)^{-\frac{2\pi^2}{e_0^2}}\right]^2$$

  No finite condensate (there is no symmetry here).
- Can add Thirring coupling g^2: produces slight change. For $\Lambda|\sigma| \to \infty \quad \Lambda^{-2}\langle V(\sigma)\bar{V}(0)\rangle \sim$ $\left[\frac{e^\gamma}{4\pi} \left(\frac{e_0^2}{2\pi(\pi+g^2)} \log(\Lambda|\sigma|)\right)^{-\frac{2\pi^2}{e_0^2}}\right]^2$ No nontrivial cont limit, as e_0^2 does not run.