

On the Relation of Confinement and Chiral Symmetry Breaking in Functional Approaches

Chiral Symmetry Breaking \iff Lorentz-scalar Quark Confinement

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1 How are confinement, $D\chi$ SB, and the $U_A(1)$ anomaly related?

2 Infrared Structure of Landau gauge Yang-Mills theory

- Infrared Exponents for Gluons and Ghosts
- Yang-Mills Running Coupling: Infrared fixed point
- Positivity violation for the gluon propagator
- Partial gluon confinement at any temperature

3 Quarks: Confinement vs. $D\chi$ SB & $U_A(1)$ anomaly

- Dynamically induced scalar quark confinement
- $U_A(1)$: η' mass from infrared divergent Green functions
- In the color superconducting phase ...

4 Summary and Outlook

How are confinement, D_χ SB, and $U_A(1)$ related?

“Traditional” conjecture:

- Confinement induces D_χ SB:
 - Heuristic argument \rightarrow momentum but not spin flip at “bag wall”.
 - Infrared slavery \rightarrow supercritical coupling.

However:

- D_χ SB without confinement
 - Explicit examples.
 - Structure of Dirac operator.

-
- $U_A(1)$ always anomalous in vector-like gauge theories with $\Theta = 0$
(Vafa & Witten, NPB234 (1984))
-



These arguments apply at zero T and μ !

Quarkyonic phase?

How are confinement, $D_{\chi SB}$, and $U_A(1)$ related?

CONFINEMENT

implies

- a non-perturbative **RG invariant confinement scale**

$$\Lambda = \mu \exp\left(-\int^g \frac{dg'}{\beta(g')}\right) \xrightarrow{g \rightarrow 0} \mu \exp\left(-\frac{1}{2\beta_0 g^2}\right)$$

i.e., a truly **non-perturbative** method is needed for its study!

- **infrared singularities**,

i.e., a **continuum** approach is needed (at least, in addition)!

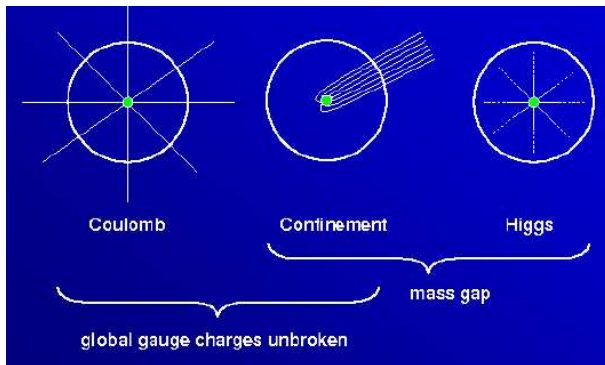
- the **suppression of long-wavelength propagation**,

i.e., a **quantum** description is needed, (semi-)classical one incompl.!

How are confinement, $D_{\chi SB}$, and $U_A(1)$ related?

NB:
Wilson loop gives only a clear criterion in the absence of quarks!

How to define *confinement* in the presence of fundamental charges:



How are confinement, $D\chi$ SB, and $U_A(1)$ related?

$D\chi$ SB and $U_A(1)$ anomaly

imply

- the existence of **topologically** non-trivial gluon field configurations and quark would-be **zero modes**,
i.e., properties of **Dirac operator** wanted.
- the existence of a **supercritical coupling** and infrared divergencies,
i.e., properties of **quark-gluon coupling** wanted.
- the dynamical generation of Lorentz-scalar couplings and **“constituent” quark masses**
i.e., properties of **quark propagator** wanted.

How are confinement, $D\chi$ SB, and $U_A(1)$ related?

Theorists' wish list: Understand

- String formation (Quark Confinement)
 - Casimir scaling at intermediate distances
 - N -ality at large distances
- Positivity violation (Gluon Confinement)
 - BRST quartet mechanism and (pseudo-)unitarity of the S -matrix
 - Oehme–Zimmermann superconvergence relations
- Rôle of gauge fixing, Gribov copies, and the configuration space
- Topological non-trivial configurations ($U_A(1)$ anomaly)
- Conformal infrared Yang-Mills sector and running coupling
- Dynamical generation of masses and Lorentz-scalar couplings ($D\chi$ SB)
- Phase structure of QCD

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Some Selected Approaches to Confinement:

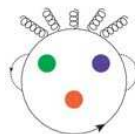
see e.g. R.A. and J. Greensite, *Quark Confinement: The Hard Problem of Hadron Physics*,
J. Phys. **G34** (Special focus issue on Hadron Physics) (2007) S3.

- ▶ **chromomagnetic monopoles**
't Hooft, diGiacomo, ...
- ▶ **calorons with non-trivial holonomy, dyons**
van Baal, Diakonov, ...
- ▶ **center vortices**
Greensite, Reinhardt, ...
- ▶ **AdS₅ / QCD correspondence**
Maldacena, Brodsky, ...
- ▶ **Coulomb confinement**
Gribov, Zwanziger, ...
- ▶ **Landau gauge Green Functions**
Smekal, Fischer, ...

How are confinement, $D\chi$ SB, and $U_A(1)$ related?

Note: Not yet understood relations between different approaches, definitely not mutually exclusive.

Also known as the *The Many Faces of QCD*



QCD correlation functions

- ★ are connected to confinement, $D\chi$ SB and the axial anomaly;
- ★ provide input into hadron phenomenology:
 - Phases of QCD
high- T phase, color superconductivity, critical end point ...
(see talk by Jan Pawłowski)
 - Bethe–Salpeter equations for mesons
masses, decays, reactions ...
 - Faddeev equations for baryons
form factors, meson production, GPDs, ...

and thus would be capable of describing **hadrons** as bound states as well as **strongly interacting matter** in terms of glue and quarks.

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and thus would be capable of describing **hadrons** as bound states as well as **strongly interacting matter** in terms of glue and quarks.

However: Infinite hierarchy of complicated equations!

??? Infrared behaviour of correlation functions ...

Restriction to lower n -point functions?

E.g. in linear covariant gauges:

7 primitively divergent Green functions in QCD,

5 primitively divergent Green functions in Yang-Mills theory.

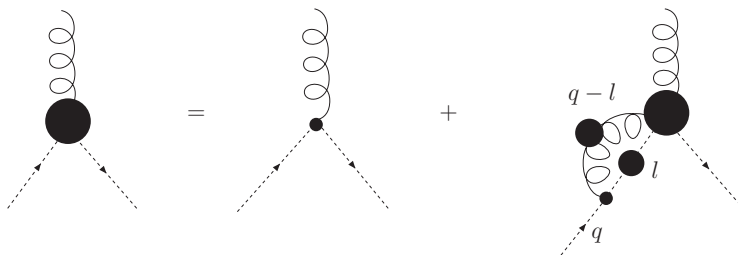
- gluon and ghost [and quark] propagators as well as
- 3-gluon, 4-gluon and gluon-ghost [and quark-gluon] vertices

R.A. *et al.*, “Confinement and Green functions in Landau-gauge QCD”

PoS(Confinement8)019 [arXiv:0812.2896 [hep-ph]]

Infrared Structure of Landau gauge Yang-Mills theory

- Starting point in gauges with transverse gluon propagator: Ghost-Gluon-Vertex fulfills Dyson-Schwinger equation



- Transversality of gluon \Rightarrow **Bare Vertex** for $q_\mu \rightarrow 0$
- No anomalous dimensions in the IR

J. C. Taylor, Nucl. Phys. B **33** (1971) 436.

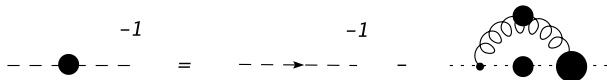
C. Lerche, L. v. Smekal, PRD **65** (2002) 125006.

A. Cucchieri, T. Mendes and A. Mihara, JHEP 0412:012 (2004).

W. Schleifenbaum, A. Maas, J. Wambach and R. A., Phys.Rev.D72 (2005) 014017.

Infrared Exponents for Gluons and Ghosts

- Dyson-Schwinger equation (DSE) for the ghost-propagator:



Ansatz for Gluon, $Z(p^2) \sim (p^2)^\alpha$,
and Ghost Ren. Fct., $G(p^2) \sim (p^2)^\beta$.

- ▶ Selfconsistency $\Rightarrow -\beta = \alpha + \beta =: \kappa$ i.e.

$$Z(p^2) \sim (p^2)^{2\kappa}, \quad G(p^2) \sim (p^2)^{-\kappa}$$

L. v. Smekal, A. Hauck, R. A., Phys. Rev. Lett. **79** (1997) 3591

- ▶ IR enhanced ghost propagator: $0.5 \leq \kappa < 1$

Kugo–Ojima confinement criterion,
Oehme–Zimmermann superconvergence relation,
and Gribov–Zwanziger horizon condition fulfilled!

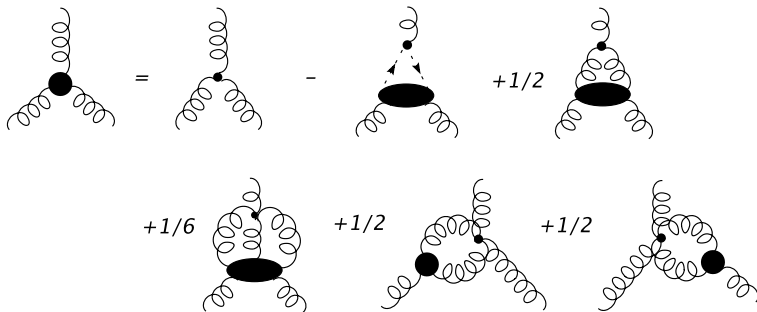
P. Watson and R.A., Phys. Rev. Lett. **86** (2001) 5239

Infrared Exponents for Gluons and Ghosts

R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Example: DSE for 3-gluon-vertex



Infrared Exponents for Gluons and Ghosts

R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

MATHEMATICA:

R. A., M. Q. Huber, K. Schwenzer, Comp. Phys. Comm. **180** (2009) 965
[arXiv:0808.2939 [hep-th]]



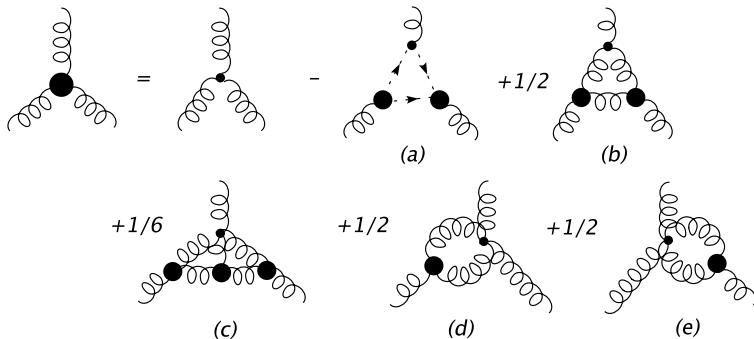
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Apply asymptotic expansion to all primitively divergent Green functions:

Skeleton expansion &

generalized formulas (neg. dim.) for Feynman integrals:

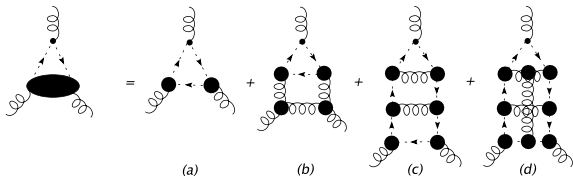


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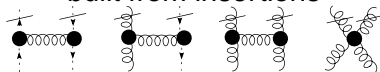
R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Three-gluon vertex: **higher order** in skeleton expansion



built from insertions



insertions have **zero** IR anomalous dimensions \Rightarrow
IR-analysis valid to all orders in skeleton expansion

Infrared Exponents for Gluons and Ghosts

Use DSEs and ERGEs:

→ **Two different** towers of equations for Green functions

E.g. ghost propagator

The image shows two equations for the ghost propagator. The left equation is a Dyson-Schwinger Equation (DSE): a dashed line with a black dot and a superscript -1 is equal to another dashed line with a black dot and a superscript -1 minus a loop diagram. The loop diagram consists of a dashed line with a black dot and a wavy line with a cross. The right equation is an Exact Renormalization Group Equation (ERGE): a dashed line with a black dot and a superscript -1 is equal to a sum of four diagrams. The first diagram is a loop with a wavy line and a cross, and a dashed line with a black dot and a superscript -1 . The second diagram is a loop with a wavy line and a cross, and a dashed line with a black dot and a superscript $-1/2$. The third diagram is a loop with a wavy line and a cross, and a dashed line with a black dot and a superscript -1 . The fourth diagram is a loop with a wavy line and a cross, and a dashed line with a black dot and a superscript -1 .

Infrared Exponents for Gluons and Ghosts

IR-Analysis of two different towers of equations \Rightarrow

Unique **scaling** solution

(C.S. Fischer and J.M. Pawłowski, PRD **75** (2007) 025012; PRD **80** (2009) 025023)

and a one-parameter family of solutions with IR trivial Green functions, (the **decoupling** solutions).

C.S. Fischer, A. Maas and J.M. Pawłowski, AoP **324** (2009) 2408.

see also

A.P. Szczepaniak and E.S. Swanson, PRD **65** (2002) 025012 (! Coulomb gauge!).

A.C. Aguilar, D. Binosi, J. Papavassiliou, PRD **78** (2008) 025010 and refs. therein.

P. Boucaud *et al.*, JHEP **0806** (2008) 099 and refs. therein.

RA, M.Q. Huber, K. Schwenzer, PRD **81** (2010) 105010.

M.Q. Huber, K. Schwenzer, RA, arXiv:0904.1873, EPJC in print (published online).



Infrared Exponents for Gluons and Ghosts

Scaling solution vs. decoupling solutions:

- Most lattice calculations of gluon propagator favor decoupling sol.
- Scaling solution respects BRST, decoupling solution breaks BRST.
- Strong coupling lattice calculations: two different IR exponents, supports existence of scaling solution.

(L.v. Smekal, A. Sternbeck, arXiv:0811.4300 [hep-lat];

A. Cucchieri, T. Mendes, Phys. Rev. **D80** (2010) 016005.)

A potential resolution of the puzzle:

IR behaviour depends on non-perturbative completion of gauge.

(A. Maas, Phys. Lett. **B689** (2010) 107.)

Assumption here: \exists at least one gauge with scaling solution.



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General Infrared Exponents for Gluons and Ghosts

Scaling solution:

n external ghost & antighost legs and m external gluon legs
(one external scale p^2 ; **solves DSEs and STIs**):

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

- Ghost propagator IR divergent
- Gluon propagator IR suppressed
- Ghost-Gluon vertex IR finite
- 3- & 4- Gluon vertex IR divergent
- ★ IR fixed point for the coupling from each vertex
- ★ Conformal nature of Infrared Yang-Mills theory!
- ★ Ghost sector of YM-theory dominates IR!

D. Zwanziger, Phys. Rev. D **69** (2004) 016002



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Yang-Mills Running Coupling: Infrared fixed point

Ghost-Gluon-Vertex UV finite:

$$\alpha_S(\mu^2) = \frac{g^2(\mu^2)}{4\pi} = \frac{1}{4\pi\beta_0} g_0^2 Z(\mu^2) G^2(\mu^2)$$

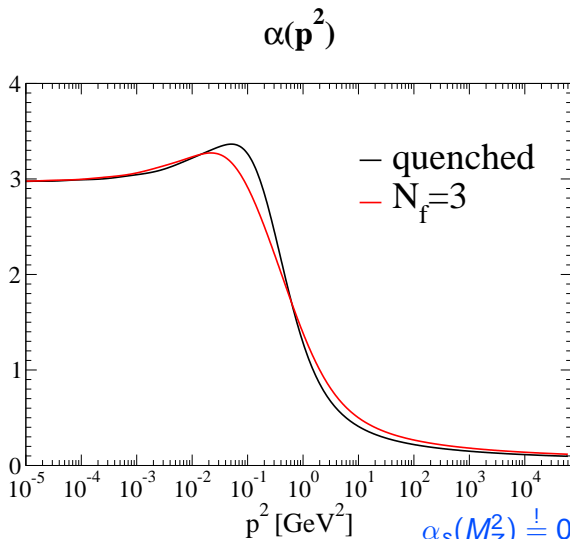
With known IR behavior of gluon (Z) and ghost (G) function:

IR fixed point

$$\alpha_c = \alpha_S(k^2 \rightarrow 0) \simeq 2.972^*$$

$$^* \alpha_S(0) = \frac{4\pi}{6N_c} \frac{\Gamma(3-2\kappa)\Gamma(3+\kappa)\Gamma(1+\kappa)}{\Gamma^2(2-\kappa)\Gamma(2\kappa)}$$

Running Coupling



Positivity violation for the gluon propagator

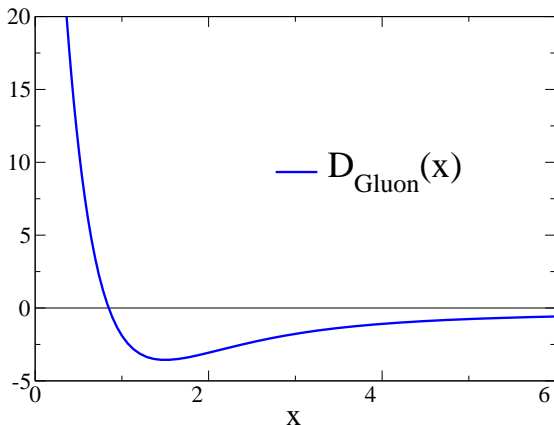
Simple argument [Zwanziger]:
IR vanishing gluon propagator implies

$$0 = D_{gluon}(k^2 = 0) = \int d^4x D_{gluon}(x)$$

$\implies D_{gluon}(x)$ has to be negative for some values of x .

Positivity violation for the gluon propagator

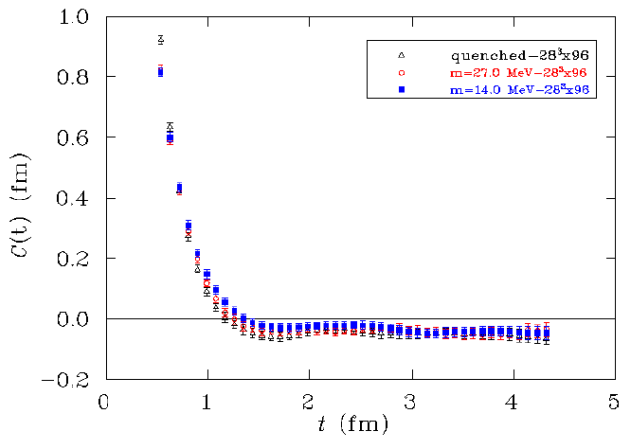
Fourier transform of DSE result:



Gluons unobservable \implies **Gluon Confinement!**

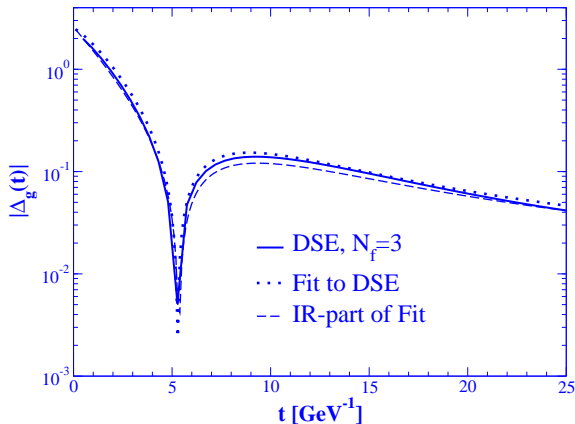
Positivity violation for the gluon propagator

Lattice (P. Bowman et al., Phys.Rev.**D76** (2007) 094505):



Positivity violation for the gluon propagator

Fourier transform of DSE result:



R.A., W. Detmold, C.S. Fischer and P. Maris, PRD70 (2004) 014014



Positivity violation for the gluon propagator

$$D_{gluon}^{\text{fit}}(p^2) = w \frac{1}{p^2} \left(\frac{p^2}{\Lambda_{\text{QCD}}^2 + p^2} \right)^{2\kappa} \left(\alpha_{\text{fit}}(p^2) \right)^{-\gamma}$$

- IR part: cut for $-\Lambda_{\text{QCD}}^2 < p^2 < 0$
- D_{gluon}^{fit} : cut along negative, i.e. timelike, half-axis!

Wick rotation possible!

- w arbitrary normalization parameter
- $\kappa = \frac{93 - \sqrt{1201}}{98}$ fixed from IR analysis
- $\gamma = \frac{-13N_c + 4N_f}{22N_c - 4N_f}$ from perturbation theory
- **Effectively one parameter[†]: $\Lambda_{\text{QCD}} = 520 \text{ MeV}$!**

from fits to lattice data: $\Lambda_{\text{QCD}} \approx 380 \text{ MeV}$



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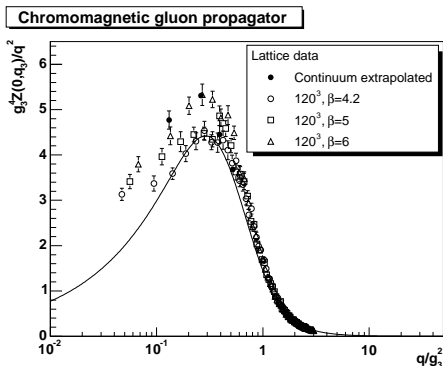
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Partial gluon confinement at any T

Gluon propagator at high T :

A. Maas, J. Wambach, RA, EPJ **C37** (2004) 335; **C42** (2005) 93.

A. Cucchieri, A. Maas and T. Mendes, PR **D75** (2007) 076003.



Gribov-Zwanziger / Kugo-Ojima scenario / positivity violation



Partial gluon confinement at any T

Gribov-Zwanziger / Kugo-Ojima scenario / positivity violation
at any T :

No infrared singularities, *c.f.* Linde (1980),
because no chromomagnetic mass of type $\omega_m(\vec{k} = 0) = m_m(T)$!

K. Lichtenegger, D. Zwanziger, Phys. Rev. D **78** (2008) 034038.

No surprise:

- three-dimensional YM theory confining
- area law for spatial Wilson loop
- Coulomb string tension $\neq 0$ at any T

Static chromomagnetic sector is never deconfined!



Picturing Gluon Confinement

DSE scaling solution of Yang-Mills theory:

- ▶ Gluon propagator vanishes on the light cone, and
- ▶ n -point gluon vertex functions diverge on the light cone!

⇒ Attempts to kick a gluon free (*i.e.* to produce a real gluon) immediately results in production of infinitely many virtual soft gluons!

⇒ perfect color charge screening

+ positivity violation (which implies BRST quartet cancelation):

Gluon confinement!



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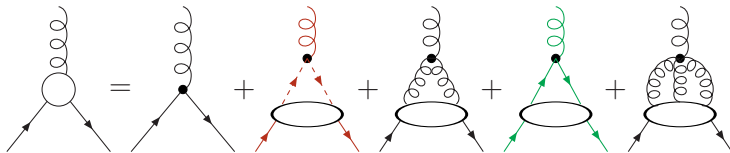
+ positivity violation (which implies BRST quartet cancelation):

Gluon confinement!

Dynamically induced scalar quark confinement

R.A., C.S. Fischer, F. Llanes-Estrada, K. Schwenzer, *Annals Phys.* **324** (2009) 106.

Quark-gluon vertex:



Quark diagram: Hadronic contributions ('unquenching')

Ghost diagram: Infrared leading!

Dynamically induced scalar quark confinement

Chiral symmetry dynamically or explicitly broken:
quark propagator infrared finite

$$S(p) = \frac{\not{p} + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \rightarrow \frac{Z_f \not{p}}{M^2} + \frac{Z_f}{M}$$

AND

$$\Gamma_\mu = ig \sum_{i=1}^{12} \lambda_i G_\mu^i, \quad G_\mu^1 = \gamma_\mu, \quad G_\mu^2 = \hat{p}_\mu, \quad G_\mu^3 = \dots$$

WITH $\lambda_{1,2,\dots} \sim (p^2)^{-1/2-\kappa}$

INFRARED DIVERGENT



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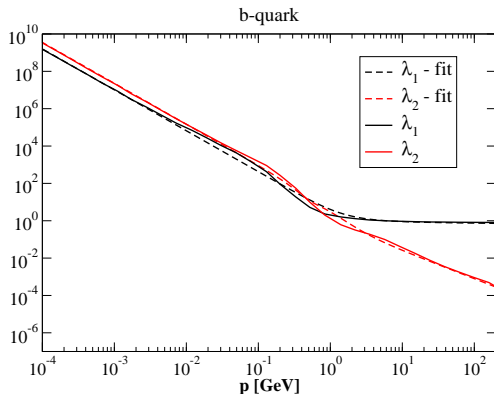
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Quark-Gluon vertex IR divergent!

Scalar component λ_2 in IR even larger than vector component λ_1 !



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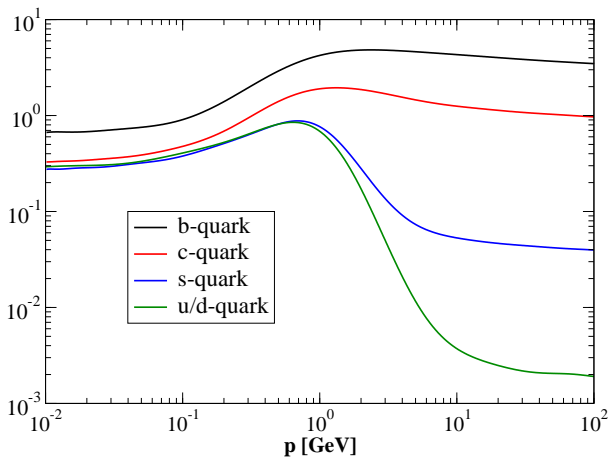
$$\Gamma^{qg}(p^2) \sim (p^2)^{-1/2-\kappa}, \quad Z_f(p^2) \sim \text{const}, \quad Z(p^2) \sim (p^2)^{2\kappa}$$

running coupling from quark-gluon is IR divergent:

$$\alpha^{qg}(p^2) = \alpha_\mu [\Gamma^{qg}(p^2)]^2 [Z_f(p^2)]^2 Z(p^2) \sim \frac{\text{const}_{qg}}{N_c} \frac{1}{p^2}$$

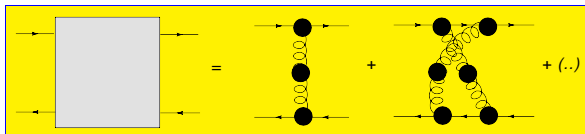
Dynamically induced scalar quark confinement

Dynamically generated quark mass function:



Dynamically induced scalar quark confinement

“Quenched” quark-antiquark potential



infrared divergent such that

$$V(\mathbf{r}) = \int \frac{d^3p}{(2\pi)^3} H(p^0 = 0, \mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{r}} \sim |\mathbf{r}|$$

i.e. linear, dominantly scalar, quark confinement!

Chiral symmetry **artificially** enforced:

$$S(p) = \left(\frac{\not{p} + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \right)_{M \rightarrow 0} \rightarrow \frac{Z_f \not{p}}{p^2}$$

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WITH $\lambda_{1,3,\dots} \sim (p^2)^{-\kappa}$ and $\lambda_{2,4,\dots} = 0$.

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Quark-antiquark potential: **No confinement**

$$\Gamma^{0,0,2}(p^2) \sim \text{const.}$$

$$V(\mathbf{r}) \sim \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2} e^{i\mathbf{p}\mathbf{r}} \sim \frac{1}{|\mathbf{r}|}$$

At vanishing T and μ : **Confinement** \Leftrightarrow **D χ SB**

Picturing quark confinement

Implication of YM-DSE scaling solution for quark sector:

- ▶ quark propagator IR trivial ($D_{\chi\text{SB}}$),
- ▶ quark-gluon vertex functions including a self-consistently generated scalar quark-gluon coupling ($D_{\chi\text{SB}}$!) diverge on the quark “mass” shell!

⇒ Attempts to kick a quark free (*i.e.* to produce a real quark) immediately results in production of infinitely many virtual soft gluons!

⇒ linearly rising potential
i.e., infrared slavery:

Quark confinement!

String formation? Properties of confining field configuration? ...? ...?

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$U_A(1)$: η' mass from IR divergent Green functions

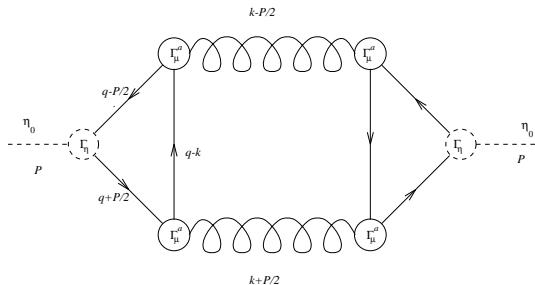
R.A., C. S. Fischer, R. Williams, Eur. Phys. J. A **38** (2008) 53.

$U_A(1)$ symmetry anomalous $\Rightarrow \eta'$ mass $\gg \pi$ mass

Where is this encoded in the Green functions?

(J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.)

E.g. in:



$$\Gamma_\mu D^{\mu\nu} \Gamma_\nu \propto 1/k^4$$

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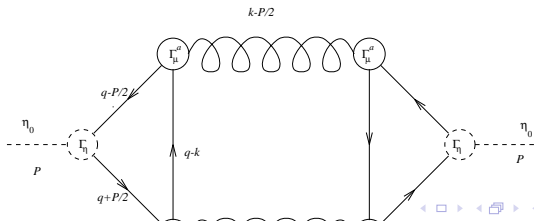
QCD vacuum: winding number spots as, e.g., instantons,

couple to chiral quark zero modes $\Rightarrow U_A(1)$ symmetry broken!

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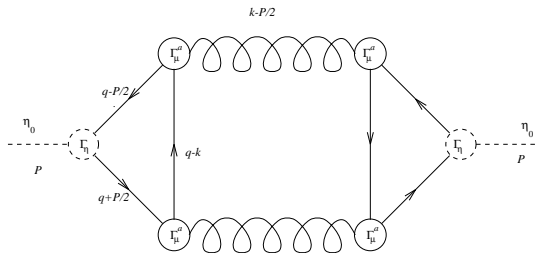
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η' mass from IR divergent Green functions

However: Infinitely many diagrams (n -gluon exchange) contribute!

Nevertheless:

Calculate contribution from **diamond diagram only** employing DSE results for the gluon and quark propagators and quark-gluon vertex (provides correct pseudoscalar and vector meson masses):

$$\chi^2 \approx (160\text{MeV})^4 \text{ vs. phenomenological value } (180\text{MeV})^4$$

$$\text{results in: } m_\eta = 479\text{MeV}, m_{\eta'} = 906\text{MeV}, \theta = -23^\circ.$$

Conclusion:

(Fluct.) topologically non-trivial fields \Leftrightarrow IR singularities of GF!

... another view to generate the Witten-Veneziano mechanism ...

Quark confinement $\Rightarrow U_A(1)$ anomaly!



In the color superconducting phase

D. Nickel, R.A., J. Wambach, Phys.Rev.**D77** (2008) 114010.

R.A., D. Horvatic, B.J. Schaefer, in preparation.

Quark Dyson–Schwinger eq. at non-vanishing chemical potential: (No sign problem!)

- so far results for an abelian-type vertex *model*

$$\Gamma_{\mu}(p, q; k) \rightarrow \gamma_{\mu} \alpha(k^2) / Z(k^2)$$

- add **medium modifications** of [bare] quarks to inverse vacuum gluon propagator
- use DSE $\alpha(k^2)$ (see above) or phenomenological / lattice fit

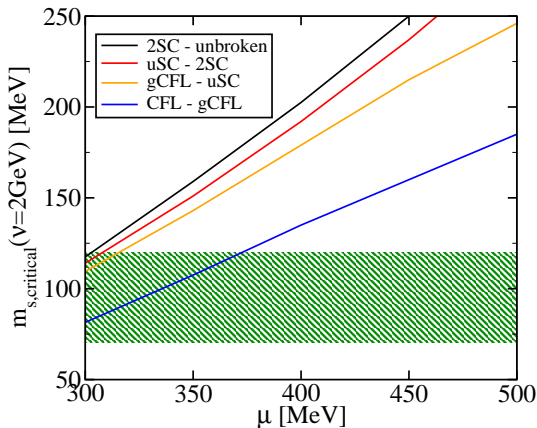
M.S. Bhagwat *et al.*, Phys. Rev. **C68** (2003) 015203 [arXiv:nucl-th/0304003]

- impose electric and color neutrality (not decisive for main result)



A Note on Color Superconductivity

Phase diagram: Renormalized strange mass vs. chemical potential



Only color-flavor-locked phase for realistic strange quark masses!

Landau gauge QCD Green functions

- ▶ Gluons confined by ghosts: Positivity violated!
Gluons removed from S -matrix!
- ▶ Infrared-finite strong running coupling in Yang-Mills theory!
Conformal Nature of Infrared Yang-Mills theory!
- ▶ Analytic structure of gluon propagator:
effectively one parameter!
- ▶ Positivity violation at any temperature!
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for cold dense quark matter:

- ▶ self-consistently calculated quark-gluon-vertex (χ SB-breaking part!)
- ▶ dual quark condensate (resp., dressed Polyakov loop) in different phases

How are confinement, $D\chi$ SB, and the $U_A(1)$ anomaly related in strongly interacting matter?

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Schladming, Styria, Austria, February 26 - March 5, 2011

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Sebastian Diehl (University of Innsbruck)	Ultracold Quantum Gases and the Functional Renormalization Group
Richard J. Furnstahl (Ohio State University)	The Renormalization Group in Nuclear Physics
Anna Hasenfratz (University of Colorado)	Exploring the Conformal Window
Daniel Litim (University of Sussex)	Gravity and the Renormalization Group
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Lorenz von Smekal (TU Darmstadt)	Universal Aspects of QCD-like Theories
Uwe C. Täuber (Virginia Tech Blacksburg)	Renormalization Group: Applications in Statistical Physics

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