A quarkyonic phase in 2-colour QCD?

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NUI Maynooth

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Outline

Background

- QC_2D vs QCD
- Lattice formulation

Bulk thermodynamics results

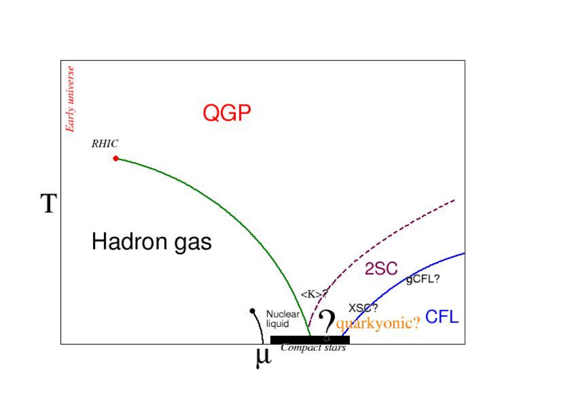
- Scaling and phase structure
- Towards j=0
- Renormalising the energy density

Gluon and quark propagation

- Tensor structures
- Gluon propagator results
- Quark propagator results

Summary

Background



- A plethora of phases at high μ, low T
- Based on models and perturbation theory
- Details depend on diquark gaps and strange quark mass
- Diquark condensation a generic feature

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$$\gamma_5 \mathcal{M}(\mu) \gamma_5 = \mathcal{M}^\dagger(-\mu) \quad \Longrightarrow \quad \det \mathcal{M} \text{ may be complex}$$

So standard Monte Carlo importance sampling can not be used!

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Indirect approach

Study QCD-like theories without a sign problem

- Generic features of strongly interacting systems at $\mu \neq 0$
- Check on model calculations

Global symmetries of QC_2D

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$m = \mu = 0$:
global $SU(2N_f)$ symmetry $\longrightarrow$ $Sp(2N_f)$ by $\langle\overline{\psi}\psi\rangle \neq 0$.

$$\Rightarrow N_f(2N_f - 1) - 1 \text{Goldstone modes}$$

$N_f = 2$: 5 modes

$\overline{\psi}\vec{\sigma}\gamma_5\psi$ pion $\psi^T\epsilon\tau_2 C\gamma_5\psi$, $\overline{\psi}\epsilon\tau_2 C\gamma_5\overline{\psi}^T$ scalar diquark

Diquark condensation

Diquarks are colour singlets in QC_2D

$\rightarrow$ superfluidity rather than colour superconductivity

$\rightarrow$ exact Goldstone mode from breaking of $U(1)_B$ symmetry

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Condensation of tightly bound diquarks (Goldstone baryons)
↔ Chiral perturbation theory

$$\langle \psi\psi \rangle \propto \sqrt{1-(\mu/\mu_o)^4}$$

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Bose–Einstein Condensation:
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$\leftrightarrow$ Chiral perturbation theory

$$\langle\psi\psi\rangle \propto \sqrt{1-(\mu/\mu_o)^4}$$

Bardeen–Cooper–Schrieffer:
Pairing of quarks near the Fermi surface

$$\langle\psi\psi\rangle \propto \Delta\mu^2$$

Lattice formulation

We use Wilson fermions:

- Correct symmetry breaking pattern, Goldstone spectrum
- $N_f < 4$ needed to guarantee continuum limit
- No problems with locality, fourth root trick
- Chiral symmetry buried at bottom of Fermi sea

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$$S = \bar{\psi}_1 M(\mu)\psi_1 + \bar{\psi}_2 M(\mu)\psi_2 - J\bar{\psi}_1(C\gamma_5)\tau_2\bar{\psi}_2^T + \bar{J}\psi_2^T(C\gamma_5)\tau_2\psi_1$$

$$\gamma_5 M(\mu)\gamma_5 = M^\dagger(-\mu), \quad C\gamma_5\tau_2 M(\mu) C\gamma_5\tau_2 = -M^*(\mu)$$

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Diquark source J introduced to

- lift low-lying eigenmodes in the superfluid phase
- study diquark condensation without uncontrolled approximations

Simulation Parameters

We work on two lattices, 'coarse' and 'fine'.

Name	β	κ	Volume	a	am_π	m_π/m_ρ
coarse	1.7	0.178	$8^3 \times 16$	0.23fm	0.79	0.80
fine	1.9	0.168	$12^3 \times 24$	0.18fm	0.65	0.80

- Simulations performed with $j = J/\kappa = 0.04$ for $\mu = 0.3 - 1.0$
- 300–500 trajectories for each μ.
- Simulations with $j = 0.02, 0.06$ for $a\mu = 0.3, 0.5, 0.7, 0.9$ (coarse lattice) $\rightarrow$ enable extrapolation to $j = 0$.

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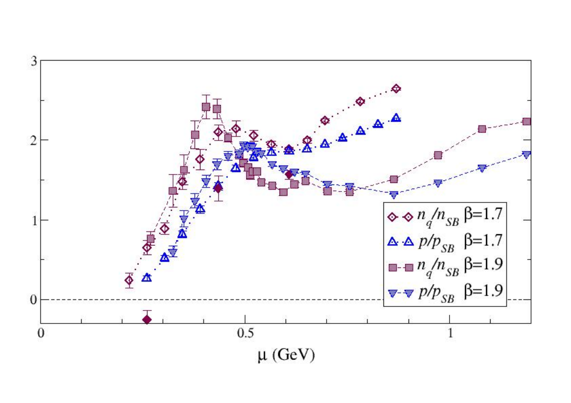
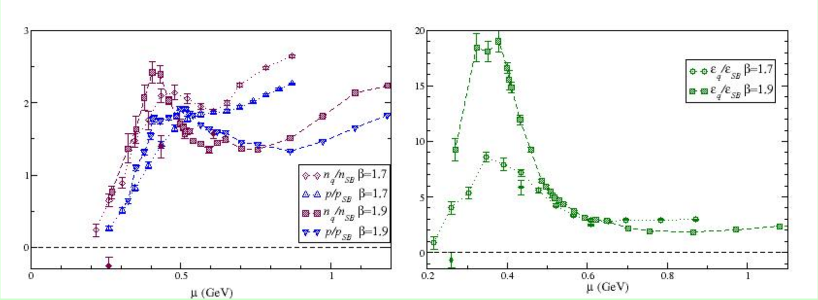
- Simulations with $j = 0.02$ for $a\mu = 0.30, 0.35, 0.40$ (fine lattice) — in progress!

Thermodynamics results



- Close to SB scaling for $\mu \gtrsim 600\text{MeV}$
- Good scaling between coarse and fine lattice for $\mu \lesssim 500\text{MeV}$
- Difference partly due to $j_f > j_c$.

Thermodynamics results



- Big peak in ε_q in intermediate region?
- $\varepsilon_q \sim 2\varepsilon_{SB} \rightarrow k_F > E_F \implies$ binding energy?
- 30–40% of total energy from gluons!?

Conformal anomaly

Conformal anomaly $\Theta = T_{\mu\mu} = \varepsilon - 3p$ is given by

$$\Theta = \Theta_g + \Theta_q = -a\frac{\partial\beta}{\partial a}\frac{3\beta}{N_c}\,\mathrm{Tr}\langle\Box\rangle + a\frac{\partial\kappa}{\partial a}\kappa^{-1}(4N_f N_c - \langle\overline{\psi}\psi\rangle)$$

Beta functions $a\frac{\partial\beta}{\partial a}, a\frac{\partial\kappa}{\partial a}$ can be estimated from parameters of the two matched lattices (coarse and fine)

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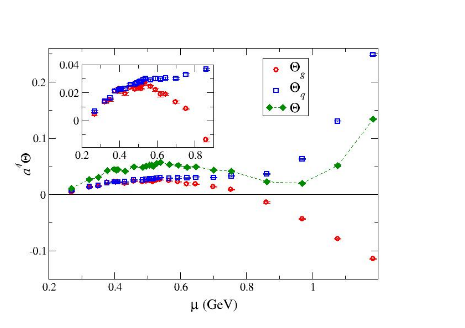
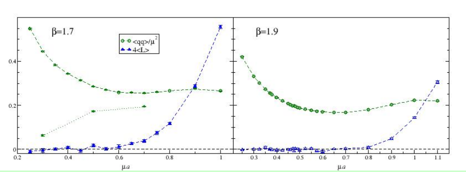


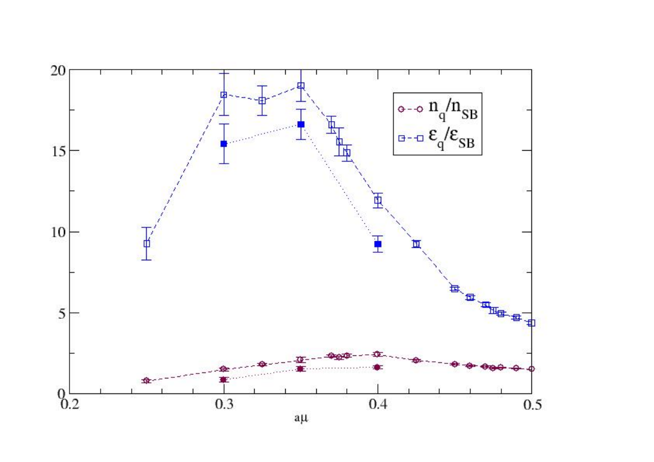
- $\Theta_q \approx \Theta_g$ up to $\mu = 500$MeV
- Θ_g becomes negative for $\mu \gtrsim 800$MeV
- Θ_q increases rapidly for $\mu \gtrsim 800$MeV
- $\Theta > 0$ for all μ

Phase transitions



- BCS scaling in $\langle qq \rangle$ for $\mu \gtrsim 0.6$GeV.
- Deconfining transition at $a\mu_d \sim 0.65$ on both lattices?!
- Still very far from saturation at μ_d
- $T_f = 44(2)$MeV, $T_c = 54(1)$MeV $\rightarrow$ temperature effect?
- BEC (superfluid nuclear matter) $\rightarrow$ BCS (Quarkyonic superfluid) crossover?

Towards $j = 0$



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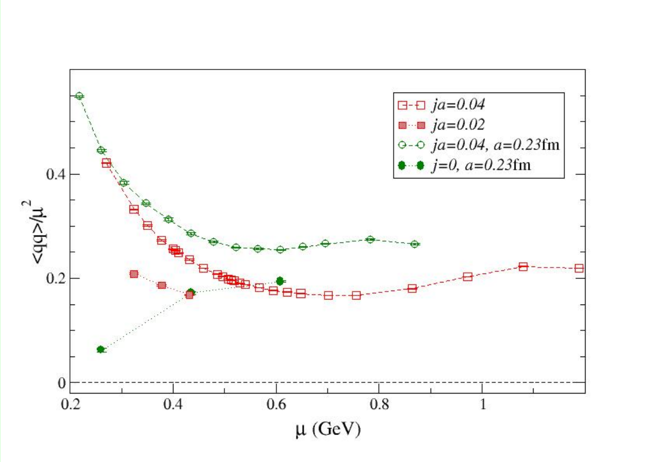


Renormalising energy densities

[With Seamus Cotter]

For an anisotropic lattice with $a_s = \xi a_t$, the renormalised energy density is

$$\varepsilon = -\frac{1}{V}\frac{\partial Z}{\partial T^{-1}} = \frac{T}{V}\left\langle a_t \frac{\partial S}{\partial a_t}\Big|_{a_s}\right\rangle = -\frac{T}{V}\left\langle \xi \frac{\partial S}{\partial \xi}\Big|_{a_s}\right\rangle$$

With the anisotropic Wilson action (and $j = 0$),

$$\varepsilon = \varepsilon_g + \varepsilon_q = \frac{T}{V}\Bigg[\left(\frac{1}{\gamma_g}\frac{\partial \beta}{\partial \xi} - \frac{\beta}{\gamma_g^2}\frac{\partial \gamma_g}{\partial \xi}\right)\langle \Box_s\rangle + \left(\gamma_g \frac{\partial \beta}{\partial \xi} + \beta \frac{\partial \gamma_g}{\partial \xi}\right)\langle \Box_t\rangle + \frac{\partial \kappa}{\partial \xi}\Big\langle \sum_i \overline{\psi} D_i \psi\Big\rangle + \left(\gamma_q \frac{\partial \kappa}{\partial \xi} + \kappa \frac{\partial \gamma_q}{\partial \xi}\right)\langle \overline{\psi} D_0 \psi\rangle\Bigg],$$

where γ_g, γ_q are the bare anisotropies.

Karsch coefficients

We need the anisotropy (Karsch) coefficients

$$\frac{\partial b_i}{\partial \xi}\,, \quad b_i = \beta, \gamma_g, \kappa, \gamma_q\,.$$

We determine these by computing the 4x4 matrix

$$\begin{pmatrix} da_s \\ d\xi \\ dM \\ d\Delta\xi \end{pmatrix} = \begin{pmatrix} \frac{\partial a_s}{\partial \beta} & \frac{\partial a_s}{\partial \gamma_g} & \frac{\partial a_s}{\partial \kappa} & \frac{\partial a_s}{\partial \gamma_q} \\ \frac{\partial \xi}{\partial \beta} & \frac{\partial \xi}{\partial \gamma_g} & \frac{\partial \xi}{\partial \kappa} & \frac{\partial \xi}{\partial \gamma_q} \\ \frac{\partial M}{\partial \beta} & \frac{\partial M}{\partial \gamma_g} & \frac{\partial M}{\partial \kappa} & \frac{\partial M}{\partial \gamma_q} \\ \frac{\partial \Delta\xi}{\partial \beta} & \frac{\partial \Delta\xi}{\partial \gamma_g} & \frac{\partial \Delta\xi}{\partial \kappa} & \frac{\partial \Delta\xi}{\partial \gamma_q} \end{pmatrix} \begin{pmatrix} d\beta \\ d\gamma_g \\ d\kappa \\ d\gamma_q \end{pmatrix}$$

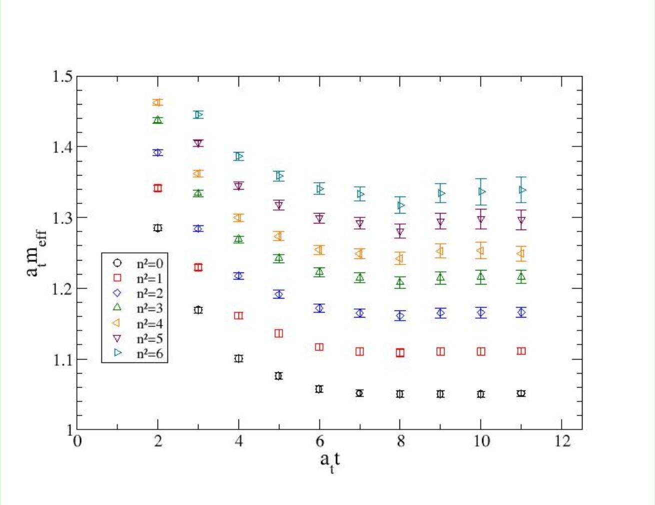
where we have defined

$$\xi = \frac{1}{2}(\xi_g + \xi_q)\,, \quad M = (m_\pi/m_\rho)^2, \quad \Delta\xi = \xi_g - \xi_q\,.$$

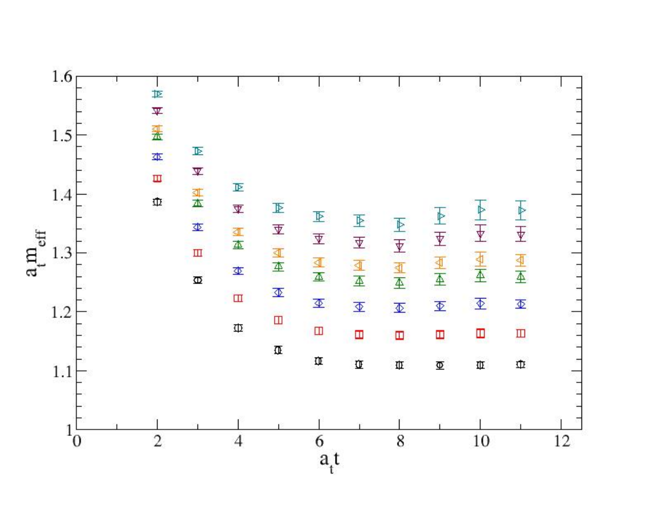
Computing Karsch coefficients

β	γ_g	κ	γ_q	a_s (fm)	m_π/m_ρ	ξ_q
1.8	1.0	0.174	1.0	0.178(8)	0.778(6)	
1.9	1.0	0.168	1.0	0.186(7)	0.80	
1.9	1.0	0.1685	1.0	0.154(15)	0.780(9)	
2.0	1.0	0.162	1.0	0.164(5)	0.830(9)	
2.0	1.0	0.163	1.0	0.145(3)	0.76(1)	
1.9	1.0	0.147	1.31		0.940(6)	1.6
1.9	1.0	0.168	0.80		0.947(4)	1.56 ???
1.9	1.0	0.180	0.87		0.75(3)	0.73
1.5	0.89	0.168	1.0	0.253(2)		
1.9	1.50	0.168	1.0		0.646(6)	1.3
1.9	0.75	0.168	1.0			

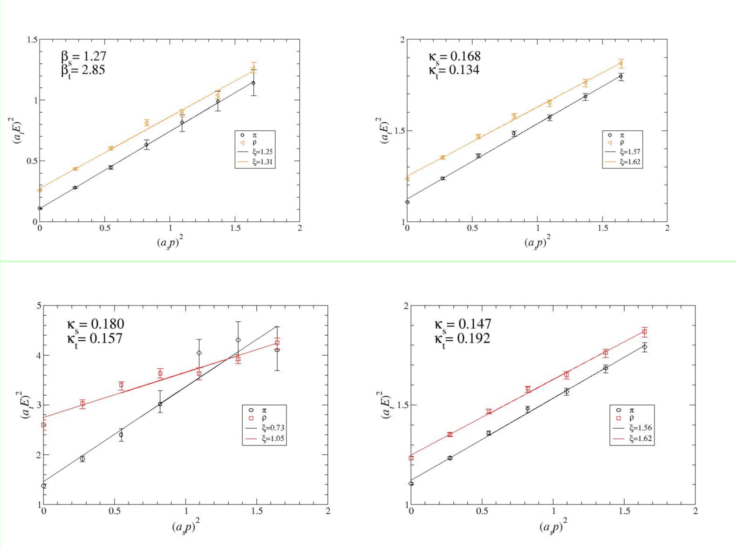
Pion dispersion relation



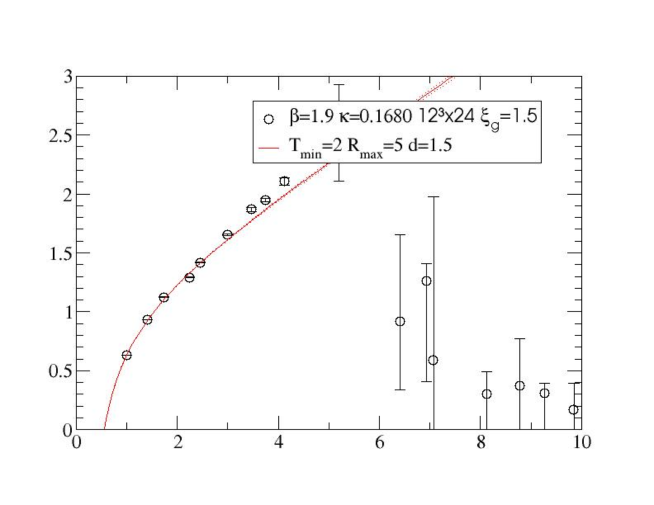
Rho dispersion relation



Dispersion relations



Static quark potential



Gluon and quark propagators in medium

The medium breaks Lorentz (Euclidean) symmetry to O(3)
$\Longrightarrow$ 1 $\to$ 2 scalar functions in gluon, 2 $\to$ 4 in quark:

$$D_{\mu\nu}(\vec{q}, q_t) = P^T_{\mu\nu} D_M(\vec{q}^{\,2}, q_t^2) + P^E_{\mu\nu} D_E(\vec{q}^{\,2}, q_t^2) + \xi \frac{q_\mu q_\nu}{q^4}$$

$$S^{-1}(\vec{p}, \tilde{\omega}) = i\vec{\not{p}} A(\vec{p}^{\,2}, \tilde{\omega}^2) + i\gamma_4 \tilde{\omega} C(\vec{p}^{\,2}, \tilde{\omega}^2) + B(\vec{p}^{\,2}, \tilde{\omega}^2) + i\gamma_4 \vec{\not{p}} D(\vec{p}^{\,2}, \tilde{\omega}^2)$$

$$S(\vec{p}, \tilde{\omega}) = i\vec{\not{p}} S_a + i\gamma_4 \tilde{\omega} S_c + S_b + i\gamma_4 \vec{\not{p}} S_d$$

where $\tilde{\omega} \equiv p_4 - i\mu$.

Gor'kov formalism

Quarks and antiquarks are in the same representation.

Construct Gor'kov spinor

$$\Psi = \begin{pmatrix} \psi \\ \overline{\psi}^T \end{pmatrix} \quad \Longrightarrow \quad \langle \Psi(x)\bar{\Psi}(y) \rangle \equiv \mathcal{G}(x,y) = \begin{pmatrix} S_N & -S_A \\ \bar{S}_A & \bar{S}_N \end{pmatrix}$$

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S_A contains information about anomalous propagation
Self-energies are diquark gaps Δ (superfluid/superconducting)
General tensor structure is the same as for normal components.

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We find that

- the vector, scalar and temporal components of the normal propagator S_a, S_b, S_c and
- the scalar and tensor components A_b, A_d of the anomalous propagator are nonzero
- all other components are zero

Fermi surface and Cooper pairs

Fermi surface

In a Fermi liquid the Fermi surface is given by

$$\det S^{-1}(\overrightarrow{p_F}, p_4 = 0) = 0 \quad \Longleftrightarrow \quad \overrightarrow{p}^2 A^2 + \tilde{\omega}^2 C^2 + B^2 = 0$$

Pole in propagator

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Pole in propagator

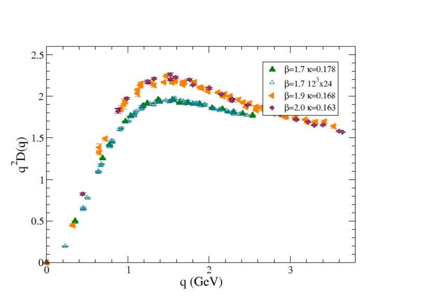
In gapped phase: zero crossing!

Size of Cooper pair

If we know the anomalous propagator $S_A(x)$ we can compute the size of the Cooper pairs:

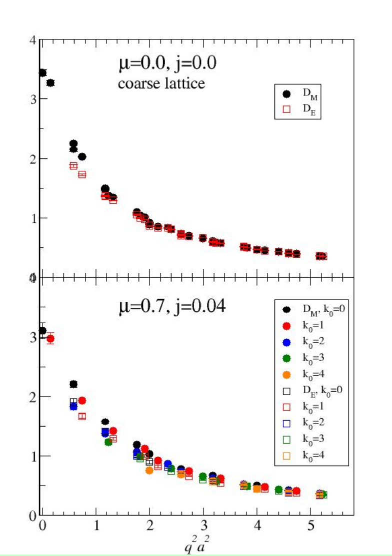
$$\xi^2 = \frac{\int d^3x \, \vec{x}^{\,2} |\frac{1}{2}\,\mathrm{Tr}(S_A(x)\Lambda^+)|^2}{\int d^3x \, |\frac{1}{2}\,\mathrm{Tr}(S_A(x)\Lambda^+)|^2}$$

Gluon propagator results

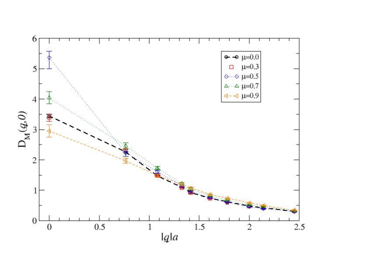


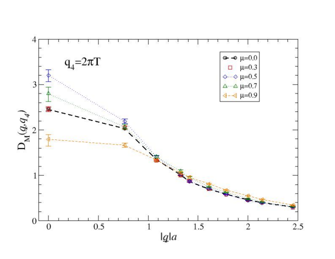
Some finite volume and lattice spacing effects at $\mu = 0$

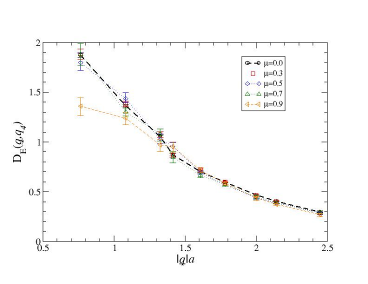
In-medium modifications, incl. violations of Lorentz symmetry, visible in magnetic gluon at $\mu = 0.7$



Coarse lattice results







- Magnetic gluon enhanced in BEC phase
- Electric and magnetic gluon suppressed in deconfined phase
- Static magnetic gluon also suppressed!

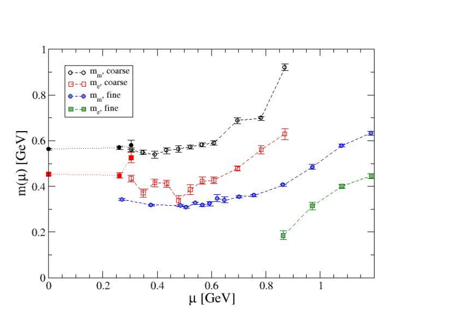
In-medium gluon mass

Crude fit to 'massive' form

$$D_{E,M}(\vec{q}, q_4) = \frac{Z}{\vec{q}^{\,2} + q_4^2 + m_{e,m}^2}$$

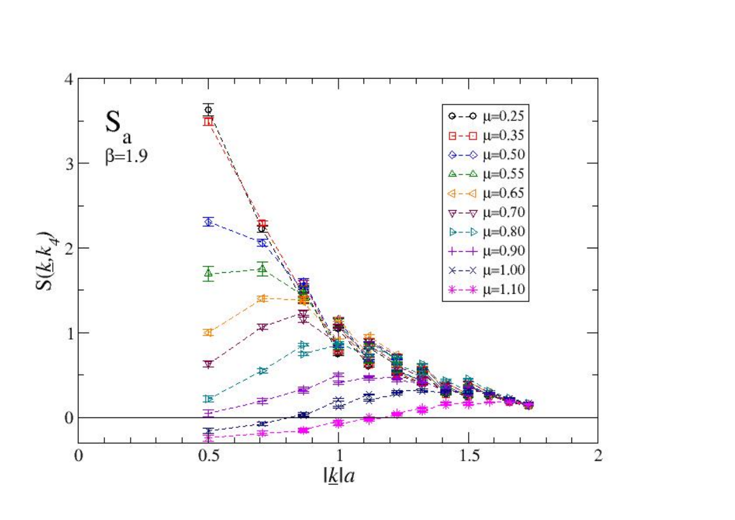
not a good fit!

Improvement:
Try Hard Dense Loop-inspired form?



Fit gives $m_e = 0$ for $a\mu < 0.7$ on fine lattice

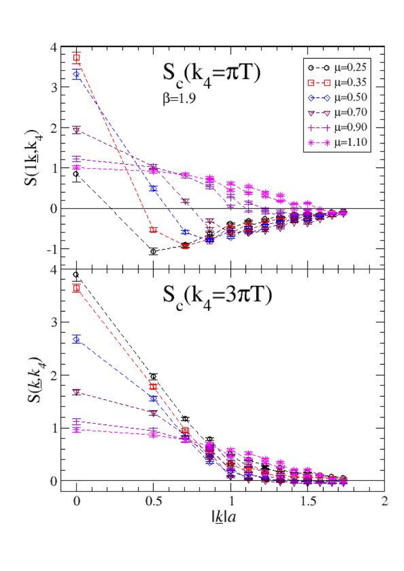
Normal quark propagator: spatial vector part



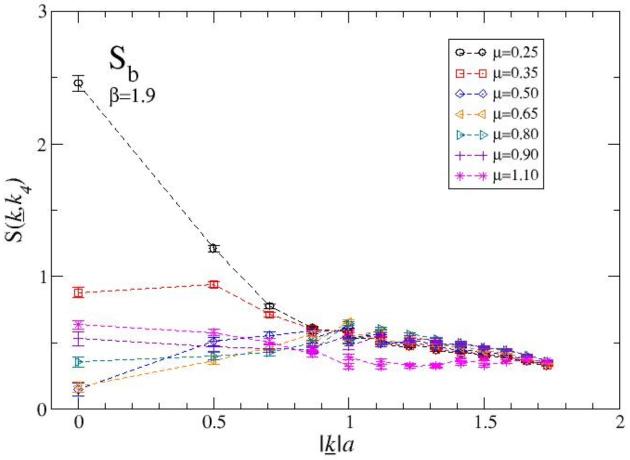
Normal quark propagator: temporal vector part

All data are for $aj = 0.04$!

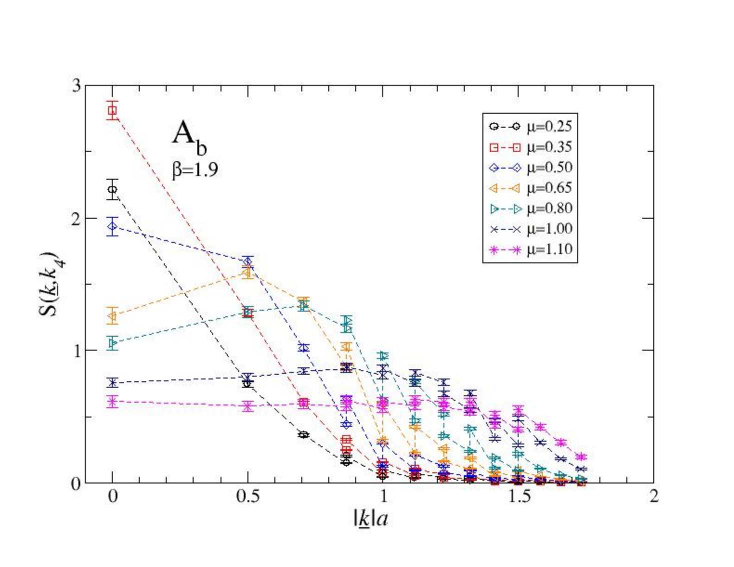
Fermi momentum may be found by extrapolating zero crossing to $k_4 = 0$?



Normal quark propagator: scalar part



Anomalous quark propagator: scalar part



Summary

Evidence for four phases/regions

- Vacuum phase below $\mu_o = m_\pi/2$
- BEC for $\mu_o < \mu < \mu_q \approx 600\text{MeV}$
  - → Approximately described by χPT
  - → Substantial peak in $\varepsilon_q/\varepsilon_{SB}$, even after $j \to 0$?
  - → Quark and gluon contributions to conformal anomaly equal
- BCS/quarkyonic for $\mu_q < \mu < \mu_d$
  - → BCS scaling of $n_q, \langle qq \rangle \to$ quark degrees of freedom?
  - → Evidence of Fermi surface in quark propagator
  - → Quark conformal anomaly constant
- Deconfined quark matter above μ_d
  - → Location of μ_d strongly temperature-dependent?
  - → Conformal anomaly increasing, dominated by quark contribution

Outlook

- Extrapolate all results to zero diquark source in progress
- Simulate at different temperatures and volumes in progress
- Renormalise energy densities in progress
- Finer lattices $\rightarrow$ continuum extrapolation?