Parton saturation and particle production in pA collisions

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Outline

- The partonic “phase diagram” and saturation
- High energy scattering and Wilson lines
- Dihadron production in forward region in pA collisions
- JIMWLK evolution of dipoles, quadrupoles, ...
- The Gaussian approximation
- Production at different rapidities
Partonic “phase diagram”

\[ Y = \ln \frac{1}{x} \]

\[ \ln Q_s^2(Y) = \lambda Y \]

Saturation

Dilute system

BFKL

DGLAP
Saturation momentum

- Saturation when \( \frac{xg(x, Q_s^2)}{Q_s^2 R^2} \sim \frac{1}{\alpha_s} \)

- \( Q_s^2(x, A) \sim Q_0^2 A^{1/3} \left( \frac{x_0}{x} \right) \lambda \) with \( \lambda = 0.2 \div 0.3 \)
The process

- Large-x quark from proton splits into quark-gluon pair
- Interacts with soft components of nucleus
- Quark-gluon pair “measured” in forward region
The outgoing state

- Mixed representation: transverse momenta $\rightarrow$ coordinates
- Nucleus viewed as large classical color field
- Eikonal interaction $\rightarrow$ Wilson lines: $V_x^+ = P \exp \left[ i g \int dx^- t^a A_{x^a}^+(x^-) \right]

$$|\Psi_{\text{out}}\rangle = \int DA^+ \Phi_Y [A^+] \int_{x,b} dz p^- g e^{ip \cdot b} \sum_{j,\beta, c, \lambda} \phi^\lambda_{\alpha\beta}(p, zp^-, x - b) \\
\left[ T^d V(b) \tilde{V}^{dc}(x) - V(b + z(x - b)) T^c \right]_{ij} \\
|(1 - z)p^-, b, j, \beta; zp^-, x, c, \lambda\rangle \otimes |A^+\rangle$$

Marquet '07
The cross section

□ From \( \langle \Psi_{\text{out}} | N_q(q) N_g(k) | \Psi_{\text{out}} \rangle \) calculate cross section

\[
\frac{d\sigma^{qA\rightarrow qgX}}{d^3k d^3q} = \frac{\alpha_s N_c}{2} \int_{x\dot{x}b\dot{b}} e^{ik \cdot (x-\dot{x})+i(q-p) \cdot (\dot{b}-b)} \sum_{\lambda \alpha \beta} \phi^*_{\lambda \alpha \beta}(p, zp^-, \dot{x} - \dot{b}) \phi^\lambda_{\alpha \beta}(p, zp^-, x - b)
\]

\[
\left\langle \frac{1}{N_c} \text{tr}[V^\dagger(x) V(b) V^\dagger(\dot{b}) V(\dot{x})] \frac{1}{N_c} \text{tr}[V^\dagger(\dot{x}) V(x)] + \ldots \right\rangle_Y
\]

□ QCD dynamics in \( \langle \ldots \rangle_Y = \int D\mathcal{A}^+ W_Y[\mathcal{A}^+] \ldots \)

□ \( e^{-Y} = x = \frac{|k| e^{-y_k} + |q| e^{-y_q}}{\sqrt{s}} \ll 1 \) in forward region
Di-hadron azimuthal correlations

- For $p_{1T}, p_{2T} > Q_s$, width in $\Delta \phi \sim Q_s / p_{iT}$

- For $p_{1T}, p_{2T} \ll Q_s$, IR divergent: subtracted and added as DPS in a collinear treatment
Wilson line correlators

□ Dipole $\hat{S}_{12} = \frac{1}{N_c} \text{tr}(V_1^\dagger V_2)$, quadrupole $\hat{Q}_{1234} = \frac{1}{N_c} \text{tr}(V_1^\dagger V_2 V_3^\dagger V_4)$

□ At large-$N_c$ can factorize $\langle \hat{Q} \hat{S} \rangle_Y = \langle \hat{Q} \rangle_Y \langle \hat{S} \rangle_Y$

□ S not enough for at least two particles measured. But S and Q is all we need for any number of particles.

Kovner, Lublinsky ’06 / Dominguez, Marquet, Stasto, Xiao ’12
Color Glass Condensate

- QCD, frozen sources, occupation numbers of order $1/\alpha_s$

- All orders in $\alpha_s \ln 1/x$ and classical field $A^\mu_\alpha \sim O(1/g)$

First idea: McLerran, Venugopalan ’93
JIMWLK evolution of correlators

- QCD dynamics encoded in JIMWLK Hamiltonian

\[ H = -\frac{1}{16\pi^3} \int_{uvz} \mathcal{M}_{uvz} [1 + \tilde{V}_u \tilde{V}_v - \tilde{V}_u \tilde{V}_z - \tilde{V}_z \tilde{V}_v]^{ab} \delta a \delta b \]

- Evolution of expectation value of arbitrary correlator

\[ \frac{\partial W_Y[\alpha]}{\partial Y} = HW_Y[\alpha] \implies \frac{\partial \langle \hat{O} \rangle_Y}{\partial Y} = \langle H \hat{O} \rangle_Y \]

- Easy to work out: act on end-point, use Fierz identities.

Jalilian-Marian, Iancu, McLerran, Kovner, Leonidov, Weigert ’97-'00
The Dipole

\[ \frac{\partial \langle \hat{S}_{12} \rangle_Y}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int_z M_{12z} \langle \hat{S}_{1z} \hat{S}_{z2} - \hat{S}_{12} \rangle_Y \]

\[ \begin{align*}
\text{Weak scattering: linear in } T = 1 - S \ll 1, \text{ BFKL, easy to solve} \\
\text{Strong scattering, assume large } N_c: \text{ linear in } S
\end{align*} \]

\[ \frac{\partial \langle \hat{S}_{12} \rangle_Y}{\partial Y} = -\bar{\alpha}_s \int_{1/Q_s^2}^{r_{12}^2} \frac{dz^2}{z^2} \langle \hat{S}_{12} \rangle_Y = -\bar{\alpha}_s \ln(r_{12}^2 Q_s^2) \langle \hat{S}_{12} \rangle_Y \]

\[ \text{Local in } S, \text{ trivially solved if we know } Q_s(Y) \]
The Quadrupole

\[
\frac{\partial \langle \hat{Q}_{1234} \rangle_Y}{\partial Y} = \frac{\bar{\alpha}_s}{4\pi} \int_z (M_{12z} + M_{14z} - M_{24z}) \langle \hat{S}_{1z} \hat{Q}_{z234} \rangle_Y \\
+ (M_{12z} + M_{23z} - M_{13z}) \langle \hat{S}_{z2} \hat{Q}_{1z34} \rangle_Y \\
+ (M_{23z} + M_{34z} - M_{24z}) \langle \hat{S}_{3z} \hat{Q}_{12z4} \rangle_Y \\
+ (M_{14z} + M_{34z} - M_{13z}) \langle \hat{S}_{z4} \hat{Q}_{123z} \rangle_Y \\
- (M_{12z} + M_{14z} + M_{23z} + M_{14z}) \langle \hat{Q}_{1234} \rangle_Y \\
- (M_{12z} + M_{34z} - M_{13z} - M_{24z}) \langle \hat{S}_{12} \hat{S}_{34} \rangle_Y \\
- (M_{14z} + M_{23z} - M_{13z} - M_{24z}) \langle \hat{S}_{14} \hat{S}_{23} \rangle_Y
\]

Jalilian-Marian, Kovchegov '04 (in factorized form)
Quadrupole in limiting cases

- Weak scattering, expand Wilson lines, 2-gluon exchange
  \[ \hat{Q}_{1234} \simeq 1 - T_{12} + T_{13} - T_{14} - T_{23} + T_{24} - T_{34} \]

- Evolving like “six BFKL’s”

- Strong scattering, assume large \( N_c \), keep quadratic terms, local

\[
\frac{\partial \langle \hat{Q}_{1234}\rangle_Y}{\partial Y} \sim - \frac{\bar{\alpha}_s}{2} \left[ \ln(r_{12}^2 Q_s^2) + \ln(r_{34}^2 Q_s^2) + \ln(r_{14}^2 Q_s^2) + \ln(r_{23}^2 Q_s^2) \right] \langle \hat{Q}_{1234}\rangle_Y \\
- \frac{\bar{\alpha}_s}{2} \left[ \ln(r_{12}^2 Q_s^2) + \ln(r_{34}^2 Q_s^2) - \ln(r_{13}^2 Q_s^2) - \ln(r_{24}^2 Q_s^2) \right] \langle \hat{S}_{12}\rangle_Y \langle \hat{S}_{34}\rangle_Y \\
- \frac{\bar{\alpha}_s}{2} \left[ \ln(r_{14}^2 Q_s^2) + \ln(r_{23}^2 Q_s^2) - \ln(r_{13}^2 Q_s^2) - \ln(r_{24}^2 Q_s^2) \right] \langle \hat{S}_{32}\rangle_Y \langle \hat{S}_{14}\rangle_Y,
\]

- Given \( Q_s(Y) \) and dipole, can solve for quadrupole, but better ...
Look for functional form

- Write logs in terms of log-derivative of dipole
  - Leads to functional form: Quadrupole in terms of dipole
  - Better than log-accuracy
  - Extends to running coupling
  - An, *a priori*, unexpected result

- Ordinary 1st order inhomogeneous differential equation
Solution to the quadrupole

\[
\langle \hat{Q}_{1234}\rangle_Y = \sqrt{\langle \hat{S}_{12}\rangle_Y \langle \hat{S}_{32}\rangle_Y \langle \hat{S}_{34}\rangle_Y \langle \hat{S}_{14}\rangle_Y} \\
+ \frac{1}{2} \int_{Y_0}^{Y} \frac{\langle \hat{S}_{13}\rangle_y \langle \hat{S}_{24}\rangle_y}{\sqrt{\langle \hat{S}_{12}\rangle_y \langle \hat{S}_{32}\rangle_y \langle \hat{S}_{34}\rangle_y \langle \hat{S}_{14}\rangle_y}} \frac{\partial}{\partial y} \left[ \frac{\langle \hat{Q}_{1234}\rangle_{Y_0}}{\sqrt{\langle \hat{S}_{12}\rangle_{Y_0} \langle \hat{S}_{32}\rangle_{Y_0} \langle \hat{S}_{34}\rangle_{Y_0} \langle \hat{S}_{14}\rangle_{Y_0}}} \right] dy
\]

- Expanding solution for small $T$: correct result. Linear Hamiltonian, $Q$ linear in $T$ for small $T$
- Valid in two limits, not exact at transition but cannot be bad
- Can integrate over $y$ for simple configurations
- Local in $Y$ under reasonable assumptions

Iancu, DNT ’11
The Gaussian approximation

- Same approximation at the level of the Hamiltonian:

At saturation, dropping real terms, cutoff z-integration

\[ H_{\text{sat}} \simeq -\frac{1}{8\pi^2} \int_{uv} \ln \left[ (u - v)^2 Q_s^2(Y) \right] \left( 1 + \tilde{V}_u^\dagger \tilde{V}_v \right)^{ab} \frac{\delta}{\delta \alpha^a_u} \frac{\delta}{\delta \alpha^b_v} \]

Modify the “Sudakov” kernel to extend at low density

\[ \frac{1}{4\pi^2} \ln \left[ (u - v)^2 Q_s^2(Y) \right] \rightarrow \gamma_Y(u, v) = -\frac{1}{2g^2 C_F} \frac{\partial \ln \langle \hat{S}_{uv} \rangle_Y}{\partial Y} \]

The solution is a Gaussian wavefunction
\[ \langle \hat{S}_6 \rangle_Y \]

- \text{MFA } N_c = 3
- \text{MFA large-} N_c
- \text{Factorized } N_c = 3
- \langle \hat{S} \rangle_Y^3
- \langle \hat{S} \rangle_Y
- \text{JIMWLK}

Iancu, DNT ’11

with data from Dumitru, Jalilian-Marian, Lappi, Schenke, Venugopalan ’11
Gaussian average quadrupole equation for given configuration

Solve for \( S \), compare to BK

Various running coupling scenarios

(Verified Levin-Tuchin law at saturation and extracted \( Q_s \))

Alvioli, Soyez, DNT ’12
2g production with $Y_1 >> Y_2$

Without extra gluon: Scattering of gluonic quadrupole

Fig1: Easy to systematize: “real” JIMWLK evolution of G1

Fig2: Complicated: “real” evolution of source of G1

Virtual terms: without source evolution, G1 evolution IR-divergent
Two particle production with $\gamma_1 \gg \gamma_2$

- Need to describe wavefunction squared of projectile: two types of Wilson lines

- Physical quark is math. dipole: $S_{12}(x\bar{x}) = \frac{1}{N_c} \text{tr}(\bar{V}_x V_\bar{x})$

- Generating functional. Evolve, produce gluon, set $\bar{V} = V$

$$H_p = \frac{1}{4\pi^3} \int_{uv} \frac{y-u}{(y-u)^2} \cdot \frac{\bar{y}-v}{(\bar{y}-v)^2} \left[ \bar{R}_v (\bar{W}_{\bar{y}} W_Y^\dagger)^{ba} R_u + \bar{L}_v L_u - \bar{R}_v \bar{W}_{\bar{y}}^{ba} L_u - \bar{L}_v W_Y^{\dagger ba} R_u \right]$$

- Solution?

Kovner, Lublinsky ’06
Langevin form of JIMWLK

Blaizot, Iancu, Weigert ’02

Nucleus R-mover, first approximation ~ δ(x−)

Wilson lines built stochastically adding layers in x−

\[
V_\mathbf{x}^\dagger(n\varepsilon + \varepsilon) = \exp(i\varepsilon\alpha_{L\mathbf{x}}^a t^a) V_\mathbf{x}^\dagger(n\varepsilon) \exp(-i\varepsilon\alpha_{R\mathbf{x}}^a t^a)
\]

Iancu, DNT ’11

\[
\alpha_{L\mathbf{x}}^a = \frac{\sqrt{\alpha_s}}{\pi} \int dz \, K^i_{\mathbf{x}z} \nu^{ia}_z \\
\alpha_{R\mathbf{x}}^a = \frac{\sqrt{\alpha_s}}{\pi} \int dz \, K^i_{\mathbf{x}z} \nu^{ib}_z U^{ba}_z
\]

Lappi, Mantysaari ’12

K is WW kernel, ν random color charge, U leads to cascade

Gaussian noise \[\langle \nu^{ia}_{\mathbf{x}}(m\varepsilon)\nu^{jb}_{\mathbf{y}}(n\varepsilon) \rangle = \frac{1}{\varepsilon} \delta_{mn} \delta_{\mathbf{x}y} \delta^{ij} \delta^{ab} \]
Symmetric longitudinal expansion
Langevin form of WFS

As before for $V$. In complex conjugate amplitude:

$$\tilde{\bar{V}}_x^\dagger(n\varepsilon + \varepsilon) = \exp(i\varepsilon \tilde{\bar{\alpha}}_L^a t^a) \tilde{\bar{V}}_x^\dagger(n\varepsilon) \exp(-i\varepsilon \tilde{\bar{\alpha}}_R^a t^a)$$

$$\tilde{\bar{\alpha}}_L^a = \frac{\sqrt{\alpha_s}}{\pi} \int \mathrm{d}z \mathcal{K}_{xz}^i \nu_z^{ia} \quad \tilde{\bar{\alpha}}_R^a = \frac{\sqrt{\alpha_s}}{\pi} \int \mathrm{d}z \mathcal{K}_{xz}^i \nu_z^{ib} \tilde{\bar{U}}_z^{ba}$$

Single noise, same as before

How to treat functional derivatives?
Conclusion

- Justify Gaussian approximation at finite $N_c$:
  An analytical solution to JIMWLK (as function of $S$)

- Particle production at same rapidity needs only $S$ and $Q$ at large $N_c$ (good enough)

- Particle production at different rapidities is more involved:
  color charge sources of softest gluons also scatter

- Set-up a Langevin approach