Weak mixing angle at low energies.

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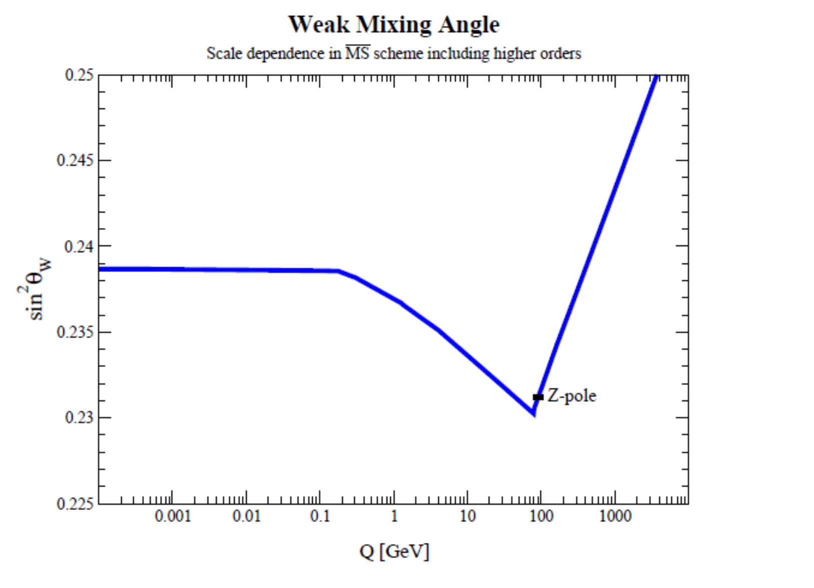
Based on the paper: "Weak mixing angle at low energies" by Jens Erler and Michael J. Ramsey-Musolf.

Main results of the paper.

1. Leading order results in the extrapolation of the weak mixing angle from the Z pole using the RGE(Renormalization Group Equation).
2. Higher order corrections to the weak mixing angle using the RGE.
3. Threshold trick to solve the RGE.
4. Use of physical restrictions to constrain the value of the weak mixing angles at low energies.

Main result of the paper.



Definitions of the Weak mixing angle.

We can define the weak mixing angle in terms of the $\mathrm{SU}(2)_{\mathrm{L}}$ and $\mathrm{U}(1)_{\mathrm{Y}}$ gauge couplings or in terms of the fine structure constant:

$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2}$$

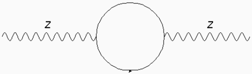
In the On-Shell Scheme one uses the relation between the weak mixing angle , the W and Z boson to all orders in perturbation theory.

Definitions of the Weak mixing angle (Effective).

In an alternative way we can define the flavor dependent mixing angles

$$v_f = T_f - 2Q_f \sin^2 \theta_f^{eff}$$

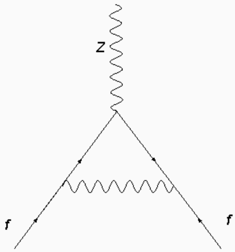
The corrections from boson self energies:



are absorbed into a (Scheme dependent) form factor and are corrections to the overall coupling strengths.

Definitions of the Weak mixing angle (Effective).

On the other hand the corrections to the $Zf\bar{f} - vertex$:



are absorbed in an another form factor k_f which is defined by

$$\sin^2\theta_f^{eff} \equiv \hat{k}_f \sin^2\hat{\theta}_W\,(M_Z)$$

the caret means minimal subtraction scheme.

The $\overline{MS}$ scheme.

With the $\overline{MS}$ **Scheme** it is possible to get a small truncation error.

- Anyway, for processes off the Z pole we can have enhancement factors coming from contributions of the form

$$\ln \frac{M_Z^2}{m_f^2}$$

- This logarithms represent a problem because we have for the running coupling constant in QCD

$$\alpha_s(\mu) = \frac{\alpha_s(\bar{\mu})}{1 + \alpha_s(\bar{\mu})\, b_1 \ln\left(\frac{\mu}{\bar{\mu}}\right)}$$

We should avoid these Logarithms!.

The logarithms mentioned before **can be a threat** to the validity of the perturbation series, that is why they should be re-summed.

- The $\overline{MS}$ scheme has a well defined to subtraction the singular terms, giving expressions with a logarithm dependence on the t´-Hooft scale.
- Using these scale as the **momentum transfer of the process** will **avoid** these **spurious logarithms**.

The fine structure constant can help us.

"*One of the main (and new) idea of the paper is to use the renormalization group equation for the fine structure constant and relate it to the RGE of the weak mixing angle.*"

Leading order as an introduction.

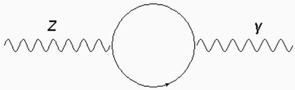
The weak mixing angle at low energies is

$$\sin^2 \hat{\theta}_W (0) = \left(1 + \Delta \hat{k} (0)\right) \sin^2 \theta_W (M_Z)$$

the Z vector coupling is changed at first order and acquires a scale dependence:

$$\hat{v}_f (\mu') = \hat{v}_f (\mu) \frac{\hat{\alpha} (\mu)}{24\pi} Q_f \sum_i [N_i^c \gamma_i v_i (\mu) Q_i] \ln \left(\frac{\mu'^2}{\mu^2} \right)$$

these corrections come from contributions of the form



Renormalization group equations (LO).

With the previous scale dependence for the vector coupling we get the following renormalization group equation

$$\mu^2 \frac{d\hat{v}_f(\mu)}{d\mu^2} = \frac{\hat{\alpha}(\mu)}{24\pi} Q_f \sum_i \left[N_i^c \gamma_i v_i(\mu) Q_i\right]$$

on the other hand for the fine structure constant we have

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{24\pi} \sum_i N_i^c \gamma_i Q_i^2$$

Renormalization group equations (LO).

After some algebra (using the solution for $\alpha(\mu_0)$):

$$\sin^2\hat{\theta}_W(\mu) = \sin^2\hat{\theta}_W(\mu_0)\left[1 + \frac{\alpha(\mu)}{24\pi\sin^2\hat{\theta}_W(\mu_0)}\sum_i N_i^c\gamma_i Q_i \ln\left(\frac{\mu_0^2}{\mu^2}\right)\left[T_i - Q_i\sin^2\hat{\theta}_W(\mu_0)\right]\right]$$

This implies

$$\Delta\hat{k}(0) = \frac{\alpha}{\pi\hat{s}^2}\left\{\frac{1}{6}\sum_f N_f^c Q_i \ln\left(\frac{M_Z^2}{m_f^2}\right)v_f - \ln\left(\frac{M_Z^2}{m_W^2}\right)\left[\frac{43}{24} - \frac{7}{4}\hat{s}^2\right]\right\}$$

coincides with previous results of Marciano and Sirlin in 1980.

Higher order RGE.

This time the renormalization group equation is more complicated because we have higher order corrections (Gorishny and others 1988)

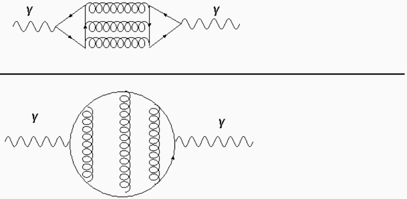
$$\mu^2 \frac{d\hat{v}_f}{d\mu^2} = \frac{\hat{\alpha}}{24\pi} Q_f \left[\sum_i K_i \gamma_i \hat{v}_i Q_i + 12\sigma \frac{\hat{\alpha}_s^3}{\pi^3} \left(\sum_q \hat{v}_q \right) \left(\sum_q Q_q \right) \right]$$

where

$$K_i = \begin{cases} N_i^c \left\{ 1 + \frac{3}{4} Q_i^2 \frac{\hat{\alpha}}{\pi} + \frac{\hat{\alpha}_s}{\pi} + \frac{\hat{\alpha}_s^2}{\pi^2} \left(\frac{125}{48} - \frac{11}{72} n_q \right) + \frac{\hat{\alpha}_s^3}{\pi^3} A\left(n_q\right) \right\} & \textit{quarks} \\ N_i^c \left\{ \frac{3}{4} Q_i^2 \frac{\hat{\alpha}}{\pi} \right\} & \textit{leptons} \\ \textit{leading beta function} & \textit{bosons} \end{cases}$$

Relative magnitude of the diagrams.

The diagrams that contain information about the OZI violation Rule and its relative magnitude are (for 4 quarks)



$$\sigma = \left[\frac{55}{216} - \frac{5}{9}\varsigma(3)\right] = 0.41$$

$$2A(n_q) = 2\left[\frac{10487}{1728} + \frac{55}{18}\varsigma(3) - \left(\frac{707}{864} + \frac{55}{54}\varsigma(3)\right)4 - \frac{77}{3888}16\right] = 2.50$$

the quotient between σ and $2A$ **is around** 16%.

Higher order RGE.

It is possible to rewrite the renormalization group equation for $\hat{s}^2$ using the one for v

$$\mu^2 \frac{d\hat{s}^2}{d\mu^2} = \frac{\hat{\alpha}}{24\pi}\left[\sum_i K_i \gamma_i Q_i A_i - 12\sigma \tilde{Q}\tilde{T} + 24\sigma \hat{s}^2 \tilde{Q}^2\right]$$

where $A_i = Q_i \sin^2\hat{\theta}_W - T_i$, $\tilde{Q} \equiv \sum_q Q_q$ and $\sum_q T_q = \tilde{T}$. On the other hand we have the RGE for the fine structure constant

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi}\left[\frac{1}{24}\sum_i K_i \gamma_i Q_i^2 + \sigma \tilde{Q}^2\right]$$

in the same spirit as we have done for the leading order, we define several constants λ_i to solve the renormalization group equation for $\hat{s}^2$.

Higher order RGE results.

After some lengthy algebra they get the following result for the weak mixing angle

$$\hat{s}^2_\mu = \hat{s}^2_{\mu_0}\frac{\hat{\alpha}_\mu}{\hat{\alpha}_{\mu_0}} + \lambda_1\left[1 - \frac{\hat{\alpha}_\mu}{\hat{\alpha}_{\mu_0}}\right] + \frac{\hat{\alpha}_\mu}{\pi}\left(\frac{\lambda_2}{3}\ln\left(\frac{\mu^2}{\mu_0^2}\right) + \frac{3\lambda_3}{4} ln\frac{\hat{\alpha}_\mu}{\hat{\alpha}_{\mu_0}} + \bar{\sigma}_{\mu_0} - \bar{\sigma}_\mu\right)$$

where the constants are defined by

$$\lambda_1 = \frac{\sum_q T_q Q_q}{2\sum_q Q_q^2} \qquad \lambda_2 = \tfrac{1}{8}\sum_i N_i^c \gamma_i\left(\lambda_1 Q_i^2 - T_i Q_i\right)$$

$$\lambda_3 = \frac{\sum_i N_i^c \gamma_i\left[\lambda_1 Q_i^4 - T_i Q_i^3\right]}{\sum_i N_i^c \gamma_i Q_i^2} \qquad \lambda_4 = \left[\lambda_1 \tilde{Q}^2 - \frac{1}{2}\tilde{Q}\tilde{T}\right]$$

and

$$\bar{\sigma} = \lambda_4 \frac{5}{36}\frac{11 - 24\varsigma(3)}{33 - 2n_q}\frac{\hat{\alpha}_s{}^2}{\pi}$$

they **depend on the number of effective quarks** of the theory.

Values of the λ

energy range	λ_1	λ_2	λ_3	λ_4
$\bar{m}_t \leq \mu$	9/20	289/80	14/55	9/20
$M_W \leq \mu < \bar{m}_t$	21/44	625/176	6/11	3/22
$\bar{m}_b \leq \mu < M_W$	21/44	15/22	51/440	3/22
$m_\tau \leq \mu < \bar{m}_b$	9/20	3/5	2/19	1/5
$\bar{m}_c \leq \mu < m_\tau$	9/20	2/5	7/80	1/5
$\bar{m}_s \leq \mu < \bar{m}_c$	1/2	1/2	5/36	0
$\bar{m}_d \leq \mu < \bar{m}_s$	9/20	2/5	13/110	1/20
$\bar{m}_u \leq \mu < \bar{m}_d$	3/8	1/4	3/40	0
$m_\mu \leq \mu < \bar{m}_u$	1/4	0	0	0
$m_e \leq \mu < m_\mu$	1/4	0	0	0
$\mu < m_e$	0	0	0	0

Threshold masses.

We define the **threshold masses** $\bar{m}_s$ $\bar{m}_u$ $\bar{m}_d$ in such a way that the matching conditions remain trivial at higher order $\alpha_i^+ = \alpha_i^-$. Their values define our range of energy where one or another effective theory is valid.

Uncertainties .

There are four sources of uncertainties:

- The value of $\hat{\alpha}(M_z)$. $\delta_\alpha \sin^2 \hat{\theta}_W(0)$
- Separation of strange and first generation quark effects. $\delta_s \sin^2 \hat{\theta}_W(0)$.
- Deviations from isospin symmetry $\delta_{CVC} \sin^2 \hat{\theta}_W(0)$.
- Zweig rule deviations $\delta_{OZI} \sin^2 \hat{\theta}_W(0)$.

Perturbative calculations.

We can calculate perturbatively the threshold masses for the heavy quarks, where we get the condition (J. Erler, 1999.)

$$\ln\frac{\mu^2}{\hat{m}^2(\mu)}+\frac{\hat{\alpha}_s(\mu)}{\pi}\left[\frac{13}{12}-\ln\frac{\mu^2}{\hat{m}(\mu)}\right]+\frac{\hat{\alpha}_s^2}{\pi^2}F=0$$

$$F\equiv\frac{655}{144}\varsigma(3)-\frac{3847}{864}+n_q\frac{361}{1296}+\frac{295}{1296}\frac{\sum_{q\neq f}Q_q^2}{Q_f^2}$$

which implies

$$\bar{m}=\hat{m}\exp\left\{-\frac{13}{24}\frac{\hat{\alpha}_s(\hat{m})}{(\pi+\hat{\alpha}_s)}-\frac{\hat{\alpha}_s^2}{2\pi^2}F\right\} \qquad (1)$$

the input values **given by experiment** $\hat{m}_c(\hat{m}_c)=1.285\,GeV$ $\alpha_s(M_z)=0.1214$ give $\bar{m}_c=1.176\,GeV$ and in the same way $\bar{m}_b=3.995\,GeV$.

Heavy $\bar{m}_s$ limit.

To get a lower limit on the strange quark contribution we use the **case** where it behaves **as a heavy quark**.

- In this case $\bar{m}_s$ **is related to** M_ϕ in the same way **as** $\bar{m}_c$ **is related to** $M_{J/\psi}$.

We define

$$\xi_q = \frac{2\bar{m}_q}{M_{1S}} \tag{2}$$

we expect the behavior $\bar{m}_1 > \bar{m}_2 \rightarrow \xi_1 > \xi_2$

- Example: for the bottom quark we have $\xi_b = 0.845$ and because $\bar{m}_b > \bar{m}_c$ thus we expect $\xi_b > \xi_c$. Which is indeed true:$\xi_c = 0.759$
- In the same way we expect $\bar{m}_s = \frac{\xi_s M_\phi}{2} < \frac{\xi_c M_\phi}{2} = 387\mathrm{MeV}$

Heavy $\bar{m}_s$ limit.

We introduce the average QCD correction to the beta function running between the scales μ_1 and μ_2 $K(\mu_1, \mu_2)$.

- We expect the behavior $\bar{m}_1 < \bar{m}_2 \to K(\bar{m}_1, \mu) > K(\bar{m}_2, \mu)$

we choose $\mu = \bar{m}_c$, then

$$\Delta_s \hat{\alpha}(\bar{m}_c) = Q_s^2 \frac{\alpha}{\pi} K(\bar{m}_s, \bar{m}_c) \ln \frac{\bar{m}_c^2}{\bar{m}_s^2} > Q_s^2 \frac{2\alpha}{\pi} K(\bar{m}_c, \bar{m}_c) \ln \frac{\xi_c M_{J/\Psi}}{\xi_s M_\phi}$$

$$> Q_s^2 \frac{2\alpha}{\pi} K(\bar{m}_c, \bar{m}_c) \ln \frac{M_{J/\Psi}}{M_\phi} = 6.9 \times 10^{-4}$$

The $SU(3)$ limit.

The **symmetric** case **implies an upper limit** to the contribution of the strange quark to $\hat{\alpha}(\mu)$. The spirit is the same although with a little more algebra in some steps.

- We have the phenomenological constraint

$$\triangle\hat{\alpha}(\bar{\mu})^{(3)} = \frac{2\hat{\alpha}^2}{\pi}\left[\left(Q_u^2 + Q_d^2\right) K_{QCD}^u ln\left(\frac{2\bar{\mu}}{\xi_{u,d} M_\omega}\right) + Q_s^2 K_{QCD}^c ln\left(\frac{2\bar{\mu}}{\xi_s M_\phi}\right)\right]$$

- If we impose the $SU(3)$ and we use the fact that this constriction maximizes the ratio between the strange and light quarks we get

$$\xi_s > \frac{2\bar{\mu}}{M_\omega^{\frac{5}{6}} M_\phi^{\frac{1}{6}}} e^{-\frac{3\pi}{4}\frac{\Delta\hat{\alpha}^{(3)}}{\alpha K}}$$

The $SU(3)$ limit.

- On the other hand we have as an experimental input the contribution of the on shell definition of $\alpha(M_z)$ from the energy range up to $1.8\,\mathrm{GeV}$, we convert this to the $\overline{MS}$ scheme and use the RGE to get

$$\Delta\hat{\alpha}^{(3)}(\bar{m}_c) = 0.00678 \pm 0.00010$$

which implies

$$\xi_s > 0.470 \text{ and } \bar{m}_s > 240\mathrm{MeV}$$

- By noting that the SU(3) limit maximizes the ratio $\frac{\Delta_s\alpha}{\Delta_{u+d}\alpha}$, and with a little bit of algebra using the phenomenological constraint and the constraint on ξ_s we get

$$\Delta_s\hat{\alpha}(\bar{m}_c) < \frac{\Delta\hat{\alpha}^{(3)}(\bar{m}_c)}{6} - \frac{5}{27}\frac{\alpha}{\pi}K_{QCD}(\bar{m}_c)\ln\frac{M_\phi}{M_\omega} = 9.9\times 10^{-4}$$

The implications.

Now we have an upper and lower constraint for the contribution of the strange quark to the fine structure constant. Then

$$\Delta_s\hat{\alpha}\left(\bar{m}_c\right) = (8.4 \pm 1.5) \times 10^{-4}$$

while for the light quarks we have

$$\Delta\hat{\alpha}^{(2)}\left(\bar{m}_c\right) = \Delta\hat{\alpha}^{(3)}\left(\bar{m}_c\right) - 6\Delta_s\hat{\alpha}\left(\bar{m}_c\right) = 0.00172 \mp 0.00090$$

The Implications.

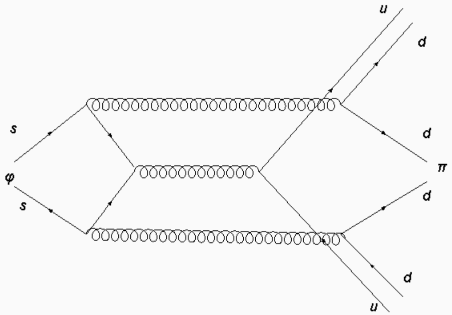
Plugging in these results into the equation for the weak mixing angle (the one we obtained using the α RGE) we see that the contribution to the uncertainty given by $\Delta\hat{\alpha}^{(3)}(\bar{m}_c)$ and $\Delta\hat{\alpha}^{(2)}(\bar{m}_c)$ are

$$\delta_\alpha \sin^2\hat{\theta}_W(0) < \delta\Delta\hat{\alpha}^{(3)}(\bar{m}_c)\left[\tfrac{1}{2} - \hat{s}^2(\bar{m}_c)\right] = \pm 3\times 10^{-5}$$

$$\delta_s \sin^2\hat{\theta}_W(0) \simeq \tfrac{1}{20}\delta\Delta\hat{\alpha}^{(2)}(\bar{m}_c)\left[\tfrac{1}{2} - \hat{s}^2(\bar{m}_c)\right] = \pm 5\times 10^{-5}$$

Deviations OZI rule.

Diagram that contributes to the OZI violation



$$\delta_{OZI} \sin^2 \hat{\theta}_W(0) = \pm 3 \times 10^{-5} \quad (3)$$

Deviations OZI rule.

The smallness of this factor is improved because

$$\delta_{OZI} \sin^2 \hat{\theta}_W(0) \sim \lambda_4 \frac{\hat{\alpha}}{\pi} \left[3 \frac{\hat{\alpha}_s}{\pi} \right]^3 \frac{5}{324} \frac{11 - 24\varsigma(3)}{33 - 2n_q}$$

where

$$\lambda_4 = \frac{1}{20}$$

for a theory with two effective quarks.

Uncertainty.

In a similar way as in the case of the strange contribution we can obtain the theoretical uncertainty corresponding to $SU(2)$ violating

$$\delta_{CVC} \sin^2 \hat{\theta}_W(0) =^{+0}_{-8} \times 10^{-6}$$

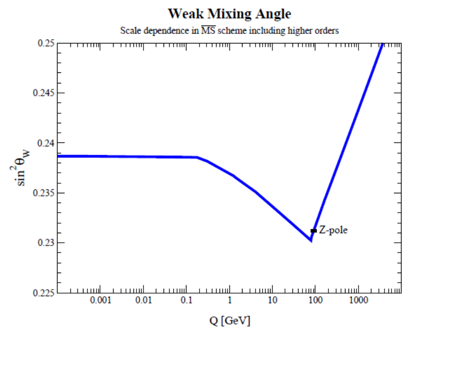
The total theoretical error is then

$$\delta_{theory} \sin^2 \hat{\theta}_W(0) = \pm 7 \times 10^{-5}$$

which was on order smaller in magnitude than the result presented by W. J Marciano in "Radiative Corrections to Neutral Current Processes". On the other hand, the experimental uncertainty at the time was

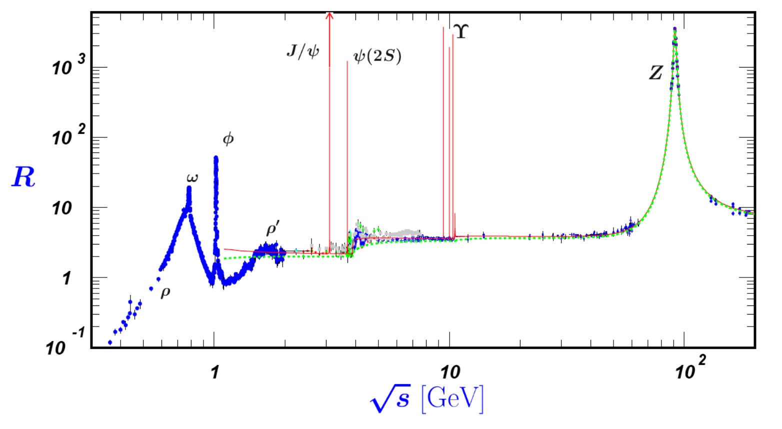
$$\delta_{exp} \sin^2 \hat{\theta}_W(M_Z) = \pm 1.5 \times 10^{-4}$$

Uncertainty.



The global fit to the precision data is $\sin^2\hat{\theta}(0) = 0.23867 \pm 0.00016$

How to improve this results.



How to improve this results.

- Paper written by Jegerlehner in 2011.
- He is able to explain the discrepancy between the τ decays and the e^+e^- anihilation.
- Reason of the discrepancy is the $\rho - \gamma$ mixing.
- Thus the error in the experimental value of $\Delta\alpha$ is smaller.

Perspectives.

- The error in m_c is smaller (lattice, QCD sum rule), we can improve the calculation using this new input.
- Include results for the value of $\Delta\alpha$ from other calculations to improve the theoretical error obtained (Jegerlehner).
- The value of the α_s is more precise, this would give us better constraints.
- Upcoming low energy experiments for the weak mixing angle, such as QWEAK or MOLLER, important to have a better theoretical uncertainty.

Recapitulation.

- A perturbatively approach was used at high energies to obtain the $\bar{m}_c$ mass.
- The threshold trick was used to get some simplified boundary conditions.
- Physical assumptions where used to constraint the values of the contributions from the s quark (using the results for the charm quark).
- A theoretical error for the weak mixing angle was obtained.

Thank you