Double beta decay and n-p pairing interaction

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Double $\beta\beta$ decay is observable because even-even nuclei are more bound than the odd-odd ones (due to the pairing interaction). There are two modes, $2\nu\beta\beta$ in which 2 electrons and 2 $\bar{\nu}$ are emitted and the hypothetical (so far) $0\nu\beta\beta$ with just 2 electrons.

$^{136}\text{Xe}$ and $^{136}\text{Ce}$ are stable against $\beta$ decay (they exist in nature), but unstable against $\beta\beta$ decay ($\beta^-\beta^-$ for $^{136}\text{Xe}$ and $\beta^+\beta^+$ for $^{136}\text{Ce}$).
Observing $0\nu\beta\beta$ decay would mean that the lepton number is not a conserved quantity. It would be discovery of fundamental significance. Therefore a worldwide effort is devoted to this problem.

The physics involved is most simply described by the relation

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0,Z) (g_A^{\text{eff}})^4 (M^{0\nu})^2 |m_{\beta\beta}|^2,$$

Where $m_{\beta\beta}$ is (complex) effective neutrino mass and $M^{0\nu}$ is the nuclear matrix element that characterizes the transition $Z,A \rightarrow Z+2,A$. ($M^{0\nu}$ subdominantly depends on $(g_A^{\text{eff}})^2$.)

Since $T_{1/2}$ or its limit is the observable, and $m_{\beta\beta}$ (which is constrained by other physics, e.g. kinematic tests of the neutrino mass, neutrino oscillation phenomenology, cosmology/astrophysics) is a measure of the experimental reach, it is obvious that the knowledge of $M^{0\nu}$ and the related issue of the magnitude of $g_A^{\text{eff}}$ (i.e. quenching of the axial current) are crucial for interpreting and planning of the existing and future $0\nu\beta\beta$ decay experiments.
Nuclear matrix elements $M^{0\nu\beta\beta}$ evaluated using different approximations

Graph showing the variation of $M^{0\nu\beta\beta}$ with atomic mass number $A$ for different approximations:

- R-EDF
- NR-EDF
- QRPA Tu
- QRPA Jy
- IBM-2
- SM Mi
- SM St-Md+Tk
How well can we evaluate halflives of ordinary $\beta$ decay?

Ratios of the calculated and experimental halflives for nuclei with $22 < Z < 36$ and $T_{1/2} < 1$ day. The brown band is for $\pm 5$ for this ratio. The blue vertical band is for halflives $> 1$ sec. Left panels are for even-even nuclei, right for odd-A ones. Filled symbols are for nuclei used in the fit of the $T=0$ proton-neutron pairing. Bottom (blue) panel are FRDM results Moeller et al, 2003.

Moeller et al. (2003)
FIG. 1. Predicted $B(GT)$ values for the EC/β$^+$ decay of two semimagic nuclei as a function of $\alpha'_1$. The crosses denote the experimental values.

From Engel, Vogel, Zirnbauer; PRC 37, 731 (1988). In that paper we used the δ-function interaction. $\alpha'_1$ is the coupling constant of the T=0, S=1 part of the particle-particle interaction component.
Calculated halflives in Vogel and Zirnbauer, PRL 57, 3148 (1986). Only $^{130}$Te halflife was known at that time. We used the $\delta$-force interaction and adjusted the n-p isoscalar coupling to the rate of the $\beta^+$ decay in semimagic nuclei. No $g_A$ quenching.
Toward understanding the dependence on the isoscalar n-p interaction:

Consider one or more degenerate s.p. levels and only $L = 0$ pairs. The model hamiltonian is

$$H = -\frac{g(1+x)}{2} \sum_v S^+_v S_v - \frac{g(1-x)}{2} \sum_\mu P^+_\mu P_\mu + g_{ph} F^\mu_+ F^\mu_v.$$

where $S^+$ creates a pair with $S=0,T=1$ and $T^+$ a pair with $S=1,T=0$. $F$ is the GT operator $\sigma \tau$.

Thus $x = 1$ corresponds to the pure standard isovector pairing and $x = -1$ to the pure isoscalar pairing. $x = 0$ corresponds to the SU(4) limit of Wigner.

The model can be solved exactly using the SO(8) algebra for any $x$. (see Engel et al. Phys. Rev. C55, 1781(1997).
$G-T \beta^+ \text{ strength } \mathcal{B}(GT)$ as a function of $-x$. This is for $\Omega = 12$, $N = 10$, $g_{ph} = 0$. Note the decrease of $\mathcal{B}(GT)$ from $x=1$ to $x=0$ where $\mathcal{B}(GT) = 0$. For positive $x$, $\mathcal{B}(GT)$ decreases with increasing isospin (i.e. neutron excess).
Matrix elements $M^{2\nu}$ for the $2\nu\beta\beta$ decay. Exact $M^{2\nu}$ varies smoothly, but in the QRPA one encounters a singularity near $g_{pp}/g_{pair} = 1$. Beyond that point there are no physical solutions in QRPA.

$g_{pair} = g$, and $g_{pp} = g(1-x)$, here also $g_{ph} = 1.5g$ and thus vanishing $M^{2\nu}$ is above 1.0. This is for $\Omega = 12$, $N = 12$, $T=4$. 
The allowed $2\nu\beta\beta$ decay has been observed now in almost all important nuclei. The matrix elements can be extracted from the half-life. $1/T_{1/2} = G^{2\nu}(E_{\text{tot}}, Z) (M^{2\nu})^2$.

$$M^{2\nu}_{\text{GT}} = \sum_m \frac{\left\langle 0^+_f \mid \sum_l \sigma(l) \tau^+(l) \mid 1^+_m \right\rangle \cdot \left\langle 1^+_m \mid \sum_k \sigma(k) \tau^+(k) \mid 0^+_i \right\rangle}{E_m - (M_i + M_f)/2}.$$ 

In the shell model the interaction is fixed. Limited s.p. space is used (fp shell for $A=48$, spin-orbit partners are skipped for the heavier nuclei. In order to describe the decay, $g_A$ is quenched. The quenching ratio $q (q = g_{A\text{eff}}/g_A)$ needed is similar to the one used in the ordinary $\beta$ decay.

**Table 2**

The ISM predictions for the matrix element of several $2\nu$ double beta decays (in MeV$^{-1}$). See text for the definitions of the valence spaces and interactions.

<table>
<thead>
<tr>
<th>Nuclear Transitions</th>
<th>$M^{2\nu}$ (exp)</th>
<th>$q$</th>
<th>$M^{2\nu}$ (th)</th>
<th>INT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$</td>
<td>$0.047 \pm 0.003$</td>
<td>0.74</td>
<td>0.047</td>
<td>kb3</td>
</tr>
<tr>
<td>$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$</td>
<td>$0.047 \pm 0.003$</td>
<td>0.74</td>
<td>0.048</td>
<td>kb3g</td>
</tr>
<tr>
<td>$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$</td>
<td>$0.047 \pm 0.003$</td>
<td>0.74</td>
<td>0.065</td>
<td>gxfp1</td>
</tr>
<tr>
<td>$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$</td>
<td>$0.140 \pm 0.005$</td>
<td>0.60</td>
<td>0.116</td>
<td>gcn28:50</td>
</tr>
<tr>
<td>$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$</td>
<td>$0.140 \pm 0.005$</td>
<td>0.60</td>
<td>0.120</td>
<td>jun45</td>
</tr>
<tr>
<td>$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$</td>
<td>$0.098 \pm 0.004$</td>
<td>0.60</td>
<td>0.126</td>
<td>gcn28:50</td>
</tr>
<tr>
<td>$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$</td>
<td>$0.098 \pm 0.004$</td>
<td>0.60</td>
<td>0.124</td>
<td>jun45</td>
</tr>
<tr>
<td>$^{128}\text{Te} \rightarrow ^{128}\text{Xe}$</td>
<td>$0.049 \pm 0.006$</td>
<td>0.57</td>
<td>0.059</td>
<td>gcn50:82</td>
</tr>
<tr>
<td>$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$</td>
<td>$0.034 \pm 0.003$</td>
<td>0.57</td>
<td>0.043</td>
<td>gcn50:82</td>
</tr>
<tr>
<td>$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$</td>
<td>$0.019 \pm 0.002$</td>
<td>0.45</td>
<td>0.025</td>
<td>gcn50:82</td>
</tr>
</tbody>
</table>

In QRPA a different procedure is typically used:

Instead of making adjustments in order to describe the energy spectra, the weak interaction rates are used. In particular,
1) one begins with the $G$-matrix interaction, and adjusts first the isovector pairing strength in order to describe correctly the experimental gaps, i.e. the odd-even mass differences.
2) After the transformation to quasiparticles the block of interaction matrix elements corresponding to the isoscalar particle-particle channel is multiplied by a common factor $g_{pp}$. Thus $g_{pp} = 1$ means no adjustment.

The parameter $g_{pp}$ is determined so that the experimental value of $M^{2\nu}$ is correctly reproduced. In practice, the resulting $g_{pp}$ depends somewhat on $G$-matrix used, and decreases by $\sim 10\%$ going from $2h\omega$ to $4h\omega$ s.p. space. Typical values of $g_{pp}$ are in the range $0.8 - 1.2$.

By doing this it is not necessary to quench $g_A$. The $2\nu\beta\beta$ decay is automatically reproduced. Note, that the assumption is made that $M^{2\nu}$ is always positive.

Note that such procedure is possible only when the model can describe the $1^+$ states in the intermediate odd-odd nucleus. That is impossible (so far) in IBM-2 or in the EDF methods.
The Fermi $2\nu\beta\beta$ matrix elements must vanish when isospin is conserved. In QRPA this can be achieved when the isovector pairing strength is consistently applied.

$$M_{F(\text{cl})}^{2\nu} \equiv \langle f | \sum_k \tau_k^+ \tau_k^+ | i \rangle \quad M_{F(\text{cl})}^{2\nu} = M_F^{2\nu} \times \left( \bar{E}_{2\nu-F} - \frac{M_i + M_f}{2} \right)$$

Note that if, as in some older papers, one would use $g_{pp}^{T=1} = g_{pp}^{T=0}$ then $M_F^{2\nu} \approx M_{GT}^{2\nu}$, which is obviously incorrect.

See Simkovic et al., PRC 87, 045501(2013)
For the $0\nu\beta\beta$ decay, the transition operator contains the `neutrino propagator' or, in the coordinate representation the `neutrino potential. It is no longer true that one can consider the decay as a product of two single nucleon transitions. On the other hand, the closure approximation is acceptable, no need to describe the states in the intermediate odd-odd nucleus.

$$M_{0\nu GT}^{0\nu} = \langle 0^+_f | \mathcal{R} \sum_{kl} \Phi(E, r_{kl}) \tau^+(k)\sigma(k)\tau^+(l)\sigma(l) | 0^+_i \rangle$$

$$M_{0\nu F}^{0\nu} = \langle 0^+_f | \mathcal{R} \sum_{kl} \Phi(E, r_{kl}) \tau^+(k)\tau^+(l) | 0^+_i \rangle$$

$$\Phi(E, r) \approx e^{-1.5Er}/r, \ R = |r_k - r_l|$$ this is the neutrino potential.

Since the momentum of the intermediate virtual neutrino is $100 - 200$ MeV, the allowed approximation is no longer valid, $qR \geq 1$, all intermediate multipoles can contribute, not just $1^+$ as in the $2\nu\beta\beta$ decay.
In QRPA the adjustment of the $g_{pp}$ value to the $2\nu\beta\beta$ matrix element does not affect the $M^{0\nu}$ when the size of the s.p. basis is changed.

In the figure the full lines represent $M^{0\nu}$ and the steeper dashed lines $M^{2\nu}$.

Figure from Rodin et al., PRC 68, 044302 (2003).
When using the generator coordinate method one avoids the singularity. However, when $g^{T=0}$ is adjusted to the $\beta^+$ strength in $^{76}$Se, the CGM agrees with QRPA.

<table>
<thead>
<tr>
<th>Skyrme</th>
<th>No $g_{ph}, g^{T=0}$</th>
<th>No $g^{T=0}$</th>
<th>Full</th>
<th>QRPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>SkO'</td>
<td>14.0</td>
<td>9.5</td>
<td>5.4 (5.4)</td>
<td>5.6 (5.0)</td>
</tr>
<tr>
<td>SkM*</td>
<td>11.8</td>
<td>9.4</td>
<td>4.1 (2.8)</td>
<td>3.5 (2.5)</td>
</tr>
</tbody>
</table>

Quenching $q=1/1.27$ indicated by parentheses.

Figure from Hinohara & Engel, PRC 90, 031301 (2014)
Summary:

1) Proper treatment of the isoscalar n-p pairing is a necessary ingredient of the $M_{2\nu}$ and $M_{0\nu}$ evaluation.

2) When the strength $g^{T=0}$ is increased the magnitudes of the $M_{2\nu}$ and $M_{0\nu}$ are reduced.

3) By a proper choice of $g^{T=0}$ it is possible to reproduce the experimental values of $M_{2\nu}$.

4) Related but separate issue of the axial vector coupling constant $g_A$ quenching remains the most outstanding unsolved problem.
spares
Dependence of the 0ν and 2ν matrix element on the $g_{pp}$ parameter

QRPA for $^{76}$Ge

$M^{2\nu}(g_{pp})/M^{2\nu}(\text{exp})$

$M^{0\nu}(g_{pp})/M^{0\nu}(g_{pp} = 0.897)$
without two-body currents

$M^{0\nu}(g_{pp})/M^{0\nu}(g_{pp} = 0.870)$
with two-body currents

Relative matrix element

$g_{pp}$
By choosing the strength of the isoscalar (T=0) pairing one can modify the $\beta$ decay half-life substantially. Solid lines are for non-magic and dashed lines for semi-magic nuclei. The blue band is a factor of two in the ratio.
Figure 6. (Color online) Performance of fit functionals in even-even rare-earth nuclei. Filled symbols mark nuclei used to fit the $T = 0$ pairing. The experimental data are from Ref. [77].