Two- and four-body correlations in nuclei

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Short-Range Correlations in Nuclei

Pair correlations with realistic interactions
Short-range Correlations

Observations

- JLab experiments found that a knocked out high-momentum proton is accompanied by a second nucleon with opposite momentum
- Cross sections for $(e, e'pn)$ and $(e, e'pp)$ reactions show strong dominance of $pn$-over $pp$-pairs

Subedi et al., Science 320, 1476 (2008), …

Theoretical interpretation

- *ab initio* calculations with Argonne interactions show high-momentum components
- dominance of $pn$-over $pp$-pairs due to the tensor force

Wiringa, Schiavilla, Pieper, Carlson, PRC 85, 021001(R) (2008)
Alvioli et al., Int. J. Mod. Phys. E 22, 1330021 (2013), …

Low-energy nuclear structure

- SRC make many-body correlations hard
Nucleon-Nucleon Interactions

- Nucleons are not point-like, complicated quark and gluon sub-structure
- Nucleon-nucleon (NN) interaction: residual interaction
- Calculation within QCD not possible yet → construct realistic NN potentials ...
- describe two-nucleon properties (scattering, deuteron) with high accuracy
- high-momentum and off-shell behavior not constrained by scattering data
### Nucleon-Nucleon Interactions

#### Argonne V18/V8’
- $\pi$-exchange, phenomenological short-range
- “as local as possible”
- fitted to phase shifts up to 350 MeV, but describes elastic phase shifts up to 1 GeV


#### $N^3$LO
- potential derived using chiral EFT
- includes full $\pi$ dynamics
- power counting
- short-range behavior given by contact-terms
- regulated by non-local cut-off (500 MeV)


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**Fig. 4.**
(a) Chiral EFT for nuclear forces. (b) Improvement in neutron–proton phase shifts shown by shaded bands from cutoff variation at NLO (dashed), N2LO (light), and N3LO (dark) compared to extractions from experiment (points) [31]. The dashed line is from the N3LO potential of Ref. [20].

**Fig. 5.**
(a) Differential cross section (in mb/sr) and vector analyzing power for elastic neutron–deuteron scattering at 10 MeV (top) and 65 MeV (bottom) at NLO (light) and N2LO (dark) from Ref. [36]. (b) Ground-state energy of $^6$Li at NLO and N2LO with bands corresponding to the variation over 500–600 MeV compared to experiment (solid line, see Ref. [36] for details).

is still considerable off-diagonal strength above $k = 2$ fm, which remains problematic for nuclear structure calculations (and the coupled $3S_1–3D_1$ channel is generally worse).

One might think the solution is to simply fit with a smaller $\Lambda$, but then the fit worsens significantly as the truncation error grows with $Q$.

Note that the cutoff associated with the potential in Fig. 6 (a) is $\Lambda = 500$ MeV, which might lead one to expect no strength above $k = \pi / 2$ (5 fm).

Universality of short-range correlations

Exact solutions for A=2,3,4 nuclei with AV8’ interaction

Two-body Coordinate Space Densities

\[ \rho_{SM_S,TM_T}^{rel}(r) = \langle \psi | \sum_{i<j} \hat{P}_{ij}^{SM_S} \hat{P}_{ij}^{TM_T} \delta^3(\hat{r}_i - \hat{r}_j - r) | \psi \rangle \]

- two-body densities calculated from exact wave functions (Correlated Gaussian Method) for AV8' interaction
- coordinate space two-body densities reflect correlation hole and tensor correlations
- normalize two-body density in coordinate space at \( r=1.0 \) fm
- normalized two-body densities in coordinate space are identical at short distances for all nuclei
- also true for angular dependence in the deuteron channel

Two-body Momentum Space Densities

\[ n_{SM, TM}^{rel}(k) = \langle \psi | \sum_{i<j} A \hat{\rho}_{ij}^{SM} \hat{\rho}_{ij}^{TM} \delta^3 \left( \frac{1}{2} (\hat{k}_i - \hat{k}_j - k) \right) | \psi \rangle \]

- use normalization factors fixed in coordinate space
- two-body densities in momentum space agree for momenta \( k > 3 \text{ fm}^{-1} \)
- moderate nucleus dependence in momentum region \( 1.5 \text{ fm}^{-1} < k < 3 \text{ fm}^{-1} \)

Two-Body Densities and Many-Body Correlations

<table>
<thead>
<tr>
<th></th>
<th>(00)</th>
<th>(01)</th>
<th>(10)</th>
<th>(11)</th>
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<td>(d)</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
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<tr>
<td>(t)</td>
<td>0.010</td>
<td>1.361</td>
<td>1.490</td>
<td>0.139</td>
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<tr>
<td>(h)</td>
<td>0.011</td>
<td>1.361</td>
<td>1.489</td>
<td>0.139</td>
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<tr>
<td>(\alpha)</td>
<td>0.008</td>
<td>2.572</td>
<td>2.992</td>
<td>0.428</td>
</tr>
<tr>
<td>(\alpha^*)</td>
<td>0.034</td>
<td>2.714</td>
<td>2.966</td>
<td>0.286</td>
</tr>
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</table>

- occupation in \((ST)\)=(10) channel almost exactly as in IPM
- \((ST)\)=(01) significantly depopulated in favor of \((ST)\)=(11) channel
- three-body correlations induced by the two-body tensor force: depopulation of \((ST)\)=(01) channel is the price one has to pay for getting the full binding from the tensor force in the \((ST)\)=(10) channel

\[S=0, T=1, L=0\]
\[S=1, T=1, L=1\]
\[S=1, T=0, L=0\] uncorrelated
\[S=1, T=0, L=2\] correlated

Short-range correlations in nuclei using No-Core Shell Model and SRG

Short-range correlations with “soft” interactions

Unitary Transformations

- Many-body problem very hard to solve for bare interaction
- Use unitary transformations to obtain a “soft” realistic interaction ($V_{\text{low-k}}$, UCOM, SRG, …)

$$\hat{H}_{\text{eff}} = \hat{U}^\dagger \hat{H} \hat{U}$$

- The transformation is done in $N$-body approximation

$$\hat{H}_{\text{eff}} = \hat{T} + \hat{V}_{\text{eff}}^{[2]} + \ldots \hat{V}_{\text{eff}}^{[N]}$$

and is therefore unitary only up to the $N$-body level
- Deuteron binding energy and $NN$ phase shifts are conserved
- Not only the Hamiltonian, all operators, including density operators, have to be transformed

$$\hat{B}_{\text{eff}} = \hat{U}^\dagger \hat{B} \hat{U}$$

- SRG operator evolution studied for Deuteron
- SRG operator evolution for radius and Gaussian two-body operator on 3-body level
Similarity Renormalization Group

- SRG provides a family of similarity transformations depending on a flow parameter $\alpha$
- Evolve Hamiltonian and unitary transformation matrix (momentum space)
  \[ \frac{d\hat{H}_\alpha}{d\alpha} = [\hat{\eta}_\alpha, \hat{H}_\alpha], \quad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha \]
- Intrinsic kinetic energy as metagenerator
  \[ \hat{\eta}_\alpha = (2\mu)^2 \left[ \hat{T}_{\text{int}}, \hat{H}_\alpha \right] \]
- Evolution is done here on the **2-body level** – $\alpha$-dependence can be used to investigate the role of missing higher-order contributions
- Hamiltonian evolution can nowadays be done on the 3-body level
  (Jurgenson, Roth, Hebeler, . . .)


$\alpha=0$: **fully correlated** wave function

$\alpha$ large: mean-field like wave function with **pairwise correlations**
$\alpha = 0.00 \text{ fm}^4$

$V_{(LL'S)}(k, k') = \langle k(LS)|\hat{V}|k'(L'S)\rangle$

Thomas Neff | Sep 22, 2016 | ECT*, Trento, Italy
Contributions to the binding energy

- Energy depends slightly on flow parameter — indicates missing three-body terms in effective Hamiltonian
- Binding energy dominated by (ST)=(10) channel, contribution from tensor part of effective Hamiltonian decreases with flow parameter
- Sizeable repulsive contribution from odd (ST)=(11) channel related to many-body correlations — decreases with flow parameter

$^4\text{He}: \rho_{\text{rel}}(r)$ and $n_{\text{rel}}(k)$

- SRG softens interaction - suppression at short distances and high-momentum components removed in wave function
- these features are recovered with SRG transformed density operators
- small but noticeable dependence on flow parameter $\alpha$

\( ^4\text{He}: n_{ST}^{\text{rel}}(k) \)

- high-momentum components much stronger in \((ST)=(10)\) channel
- flow dependence is weak in \((ST)=(10)\) channel
- flow dependence is strong in \((ST)=(01)\) and \((11)\) channels, especially for momenta above Fermi momentum — signal of many-body correlations

Relative momentum distributions for $K=0$ pairs show a very weak dependence on flow parameter and therefore on many-body correlations — ideal to study two-body correlations.

Momentum distribution vanishes for relative momenta around $1.8 \text{ fm}^{-1}$ in the $(ST)=(01)$ channel.

In (ST)=(10) channel momentum distributions above Fermi momentum dominated by pairs with orbital angular momentum $L=2$

- For $K=0$ pairs only $L=0,2$ relevant, for all pairs also higher orbital angular momenta contribute
- The $^4\text{He} K=0$ momentum distributions above 1.5 fm$^{-1}$ look like Deuteron momentum distributions

4He: Relative Probabilities

- Relative probabilities for K=0 pairs similar for AV8' and N3LO interactions, AV8' in good agreement with JLab data
- For K=0 pairs ratio of pp/pn pairs goes to zero for relative momenta of about 1.8 fm⁻¹
- This is not the case if we look at all pairs, here many-body correlations generate many pairs in the (ST)=(11) channel


JLab Hall A data
Fermionic Molecular Dynamics

Nuclear structure calculations with a Gaussian wave-packet basis
Fermionic Molecular Dynamics

**Fermionic**

Intrinsic many-body states

$$|Q\rangle = \hat{A}\{|q_1\rangle \otimes \cdots \otimes |q_A\rangle\}$$

are antisymmetrized $A$-body states

**Molecular**

Single-particle states

$$\langle x|q\rangle = \sum_i c_i \exp\left\{-\frac{(x-b_i)^2}{2a_i}\right\} \otimes |X_{\uparrow, \downarrow}, X_{\uparrow, \downarrow}\rangle \otimes |\xi\rangle$$

- Gaussian wave-packets in phase-space (complex parameter $b_i$ encodes mean position and mean momentum), spin is free, isospin is fixed
- width $a_i$ is an independent variational parameter for each wave packet
- use one or two wave packets for each single particle state

FMD basis contains **harmonic oscillator shell model** and **Brink-type cluster** configurations as limiting cases
Variation and Projection

- minimize the energy of the intrinsic state
- intrinsic state may break symmetries of Hamiltonian
- restore inversion, translational and rotational symmetry by projection on parity, angular (and linear) momentum

\[
\hat{P}^\pi = \frac{1}{2} (1 + \pi \hat{T})
\]

\[
\hat{P}^{JM}_M = \frac{2J + 1}{8\pi^2} \int d^3\Omega \, D^{JM}_M(\Omega) \hat{R}(\Omega)
\]

\[
\hat{P}^P = \frac{1}{(2\pi)^3} \int d^3X \, \exp\{-i(\hat{P} - P) \cdot X\}
\]

Generator coordinates

- use generator coordinates (radii, quadrupole or octupole deformation, strength of spin-orbit force) to create additional basis states
**Variation after Projection**

- Correlation energies can be quite large for well deformed and/or clustered states
- For light nuclei it is possible to perform real variation after projection
- Can be combined with generator coordinate method

**Multiconfiguration Mixing**

- Set of $N$ intrinsic states optimized for different spins and parities and for different values of generator coordinates are used as basis states
- Diagonalize in set of projected basis states

### Generalized Eigenvalue Problem

$$\sum_{K' b} \langle Q^{(a)} | \hat{H} \hat{P}^{\pi} \hat{P}_{K K'}^{\pi=0} | Q^{(b)} \rangle c_{K' b} = E^{\pi} \sum_{K' b} \langle Q^{(a)} | \hat{P}_{K K'}^{\pi=0} | Q^{(b)} \rangle c_{K' b}$$

**Hamiltonian kernel**

- $\sum_{K' b} \langle Q^{(a)} | \hat{H} \hat{P}^{\pi} \hat{P}_{K K'}^{\pi=0} | Q^{(b)} \rangle c_{K' b}$

**Norm kernel**

- $\sum_{K' b} \langle Q^{(a)} | \hat{P}_{K K'}^{\pi=0} | Q^{(b)} \rangle c_{K' b}$
$^8\text{Be}$: PAV/VAP/Multiconfiguration Mixing

<table>
<thead>
<tr>
<th></th>
<th>$E_b$ [MeV]</th>
<th>$r_{ch}$ [fm]</th>
<th>$B(E2)$ [$e^2$ fm$^4$]</th>
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<td>PAV</td>
<td>52.7</td>
<td>2.39</td>
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<td>VAP</td>
<td>54.8</td>
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<td>Exp.</td>
<td>56.5</td>
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$^{12}\text{C}$: Oscillator vs. Cluster Configurations

- $(0s_{1/2})^4(0p_{3/2})^8$
- Spin-orbit strength as generator coord.
- $\alpha$-cluster

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<tr>
<th></th>
<th>intr.</th>
<th>$0^+$</th>
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<th>intr.</th>
<th>$0^+$</th>
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<td>212.1</td>
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<td>$\langle T \rangle$</td>
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<td>$\langle V_{ls} \rangle$</td>
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<td>$r_{rms}$</td>
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<td>$\alpha$-cluster</td>
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Projection has a strong effect for clustered states.
**8Be: GCM/RGM and Antisymmetrization**

- Describe $^8$Be by superposition of $^4$He clusters at distances $R_i$

$$|\psi_{\text{GCM}}\rangle = \sum_i c_i \hat{A}\{|\psi_\alpha(-R_i/2)\rangle \otimes |\psi_\alpha(+R_i/2)\rangle\}$$

- This can be rewritten using intrinsic $^4$He wavefunctions and the relative motion given by a RGM wavefunction

$$\langle \rho, \xi_a, \xi_b, X|\psi_{\text{GCM}}\rangle = \int d^3 r \Phi_{\text{GCM}}(r) \hat{A}\{\delta(\rho - r)\Phi_\alpha(\xi_a)\Phi_\alpha(\xi_b)\} \psi_{\text{cm}}(X)$$

with

$$\Phi_{\text{GCM}}(r) = \sum_i c_i \left(\frac{\mu_A}{\pi a}\right)^{3/4} \exp\left\{-\mu_A \frac{(r - R_i)^2}{2a}\right\}$$

- Choose $R = 0.5$ fm, $2.5$ fm, $4.5$ fm

---

Compare with **THSR** wavefunction

$$\Phi_{\text{THSR}}(r) = \exp\left\{-\frac{r_x^2 + r_y^2}{b^2 + \beta_x^2} - \frac{r_z^2}{b^2 + \beta_z^2}\right\}$$
\( ^8\text{Be}: \text{From RGM to } \alpha-\alpha \text{ Wavefunction} \)

- RGM basis states
  \[
  \langle \mathbf{r}, \xi_a, \xi_b | \Phi_{\alpha \alpha}(\mathbf{r}) \rangle = \hat{A} \{ \delta(\mathbf{r} - \mathbf{r}) \Phi_{\alpha}(\xi_a) \Phi_{\alpha}(\xi_b) \}
  \]

- RGM norm kernel
  \[
  N(\mathbf{r}, \mathbf{r}') = \langle \Phi_{\alpha \alpha}(\mathbf{r}) | \Phi_{\alpha \alpha}(\mathbf{r}') \rangle
  \]

- Wavefunction for relative motion of two point-like \( \alpha \)'s
  \[
  \phi_{\alpha \alpha}(\mathbf{r}) = \int d^3 \mathbf{r}' \ N^{-1/2}(\mathbf{r}, \mathbf{r}') \ \Phi_{\text{GCM/THSR}}(\mathbf{r}')
  \]
Neon Isotopes

Charge Radii as Signatures of Clustering in Ground States

Neon Isotopes: Calculation

- UCOM(var) + phenomenological correction for saturation and spin-orbit
- Variation after parity projection on positive and negative parity
- Create basis states by cranking strength of spin-orbit force
- $^{15,16}\text{O}^-\text{s}^2$ and $^{15,16}\text{O}^-\text{d}^2$ minima in $^{17,18}\text{Ne}$
- Add explicit cluster configurations:
  
  $^{17}\text{Ne}$: $^{14}\text{O}^-\text{He}$
  $^{18}\text{Ne}$: $^{14}\text{O}^-\text{He}$
  $^{19}\text{Ne}$: $^{16}\text{O}^-\text{He}$ and $^{15}\text{O}^-\text{He}$
  $^{20}\text{Ne}$: $^{16}\text{O}^-\text{He}$
  $^{21}\text{Ne}$: "$^{17}\text{O}^-\text{He}$
  $^{22}\text{Ne}$: "$^{18}\text{O}^-\text{He}$

Proton/neutron densities of dominant intrinsic FMD basis states
Neon Isotopes: Energies and Radii

Separation Energies

- $S_n$ [MeV] vs. Isotope
- Experiment (•)
- FMD (△)

Charge Radii

- $r_{charge}$ [fm] vs. Isotope
- COLLAPS (•)
- FMD (△)

- $^{17,18}$Ne: $s^2/d^2$ admixture
- $^{19}$Ne: $\alpha$, $^3$He clustering
- $^{20,21,22}$Ne: $\alpha$ clustering
$^{17}\text{Ne}$: two-proton halo?

- Proton skin $r_p - r_n = 0.45$ fm
- 40% probability to find a proton at distance $r > 5$ fm
- Similar results are obtained in a three-body model


<table>
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<tr>
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<th>FMD</th>
<th>Experiment</th>
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<tr>
<td>$r_{ch}$ [fm]</td>
<td>3.04</td>
<td>3.042(21)</td>
</tr>
<tr>
<td>$r_{mat}$ [fm]</td>
<td>2.75</td>
<td>2.75(7)</td>
</tr>
<tr>
<td>$B(E2; 1/2^- \rightarrow 3/2^-)$ [e$^2$ fm$^4$]</td>
<td>76.7</td>
<td>66$^{+18}_{-25}$</td>
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<tr>
<td>$B(E2; 1/2^- \rightarrow 5/2^-)$ [e$^2$ fm$^4$]</td>
<td>119.8</td>
<td>124(18)</td>
</tr>
<tr>
<td>$s^2$ occupancy</td>
<td>42%</td>
<td></td>
</tr>
<tr>
<td>$d^2$ occupancy</td>
<td>55%</td>
<td></td>
</tr>
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Cluster States in $^{12}\text{C}$

FMD and Cluster Model Calculations
$^{12}$C: Microscopic $\alpha$-Cluster Model

- $^{12}$C is described as a system of three $\alpha$-particles
- $\alpha$-particles are given by HO (0s)$^4$ wave functions
- wave function is fully antisymmetrized
- effective Volkov nucleon-nucleon interaction adjusted to reproduce $\alpha$-$\alpha$ and $^{12}$C ground state properties

- Internal region: $\alpha$'s on triangular grid
- External region: $^8$Be$(0^+, 2^+, 4^+)$-$\alpha$ configurations

$$|\psi^{3\alpha}_{jMK\pi}(R_1, R_2, R_3)\rangle = \hat{\rho}_\pi \hat{\rho}_M \hat{A} \{ |\psi_\alpha(R_1)\rangle \otimes |\psi_\alpha(R_2)\rangle \otimes |\psi_\alpha(R_3)\rangle \}$$

Double Projection

$$|\psi^{^8\text{Be}}_{IK}\rangle = \sum_i \hat{\rho}_k^I \hat{A} \{ |\psi_\alpha(-\frac{r_i}{2}e_z)\rangle \otimes |\psi_\alpha(+\frac{r_i}{2}e_z)\rangle \} c_i^I$$

$$|\psi^{^8\text{Be},\alpha}_{IK;JM\pi}(R_j)\rangle = \hat{\rho}_\pi \hat{\rho}_M \hat{A} \{ |\psi^{^8\text{Be}}_{IK}(-\frac{R_j}{3}e_z)\rangle \otimes |\psi_\alpha(+\frac{2R_j}{3}e_z)\rangle \}$$
**12C: FMD**

- **AV18 UCOM(SRG) ($\alpha = 0.20$ fm$^4$) interaction** — Increase strength of spin-orbit force by a factor of two to partially account for omitted three-body forces.

- **Internal region**: FMD basis states obtained by VAP with radius as generator coordinate for first $0^+$, $1^+$, $2^+$, ..., perform VAP for second $0^+$, $1^+$, $2^+$, ... with radius as generator coordinate.

- **External region**: $^8$Be($0^+, 2^+, 4^+)$-$\alpha$ configurations, polarization effects in $^8$Be are important.

Basis states are not orthogonal!

$0^+_2$ and $2^+_2$ states have no rigid intrinsic structure.

\[
\begin{align*}
\langle 0_1^+ | 0_1^+ \rangle &= 0.94 \\
\langle 0_2^+ | 0_2^+ \rangle &= 0.64 \\
\langle 1_1^+ | 1_1^+ \rangle &= 0.58 \\
\langle 2_1^+ | 2_1^+ \rangle &= 0.57 \\
\langle 3_1^- | 3_1^- \rangle &= 0.91 \\
\langle 2_2^+ | 2_2^+ \rangle &= 0.50 \\
\langle 2_2^+ | 2_2^+ \rangle &= 0.49 \\
\langle 2_2^+ | 2_2^+ \rangle &= 0.44 \\
\langle 2_2^+ | 2_2^+ \rangle &= 0.41 \\
\end{align*}
\]
$^{12}\text{C}$: Matching to Coulomb Asymptotics

- asymptotically only Coulomb interaction between $^8\text{Be}$ and $\alpha$
- calculate spectroscopic amplitudes with RGM wavefunction
- use microscopic $R$-matrix method to match logarithmic derivative of spectroscopic amplitudes to Coulomb solutions

**Bound states (Whittaker)**

$$\psi_c(r) = A_c \frac{1}{r} W_{-\eta_c, L_c + 1/2} (2\kappa_c r), \quad \kappa_c = \sqrt{-2\mu(E - E_c)}$$

**Resonances (purely outgoing Coulomb - complex energy)**

$$\psi_c(r) = A_c \frac{1}{r} O_{L_c} (\eta_c, \kappa_c r), \quad \kappa_c = \sqrt{2\mu(E - E_c)}$$

**Scattering States (incoming + outgoing Coulomb)**

$$\psi_c(r) = \frac{1}{r} \{ \delta_{L_c, L_0} I_{L_c} (\eta_c, \kappa_c r) - S_{c,c_0} O_{L_c} (\eta_c, \kappa_c r) \}, \quad \kappa_c = \sqrt{2\mu(E - E_c)}$$
$^{12}\text{C}$: Spectrum

- FMD provides a consistent description of $p$-shell states, negative parity states and cluster states
$^{12}\text{C}$: $^8\text{Be}$-$\alpha$ Spectroscopic Amplitudes

- Ground state overlap with $^8\text{Be}(0^+)+\alpha$ and $^8\text{Be}(2^+)+\alpha$ configurations of similar magnitude
- Hoyle state overlap dominated by $^8\text{Be}(0^+)+\alpha$ configurations, large spatial extension
$^{12}\text{C}$: Transitions into the Continuum

- E1 transition isospin-forbidden in cluster model

$^{12}$C: Cluster States in the Oscillator Basis?

State of the art NCSM calculation with chiral NN + NNN forces

Hoyle state and other cluster states missing!

\( ^{12}\text{C}: N\hbar\Omega \text{ Decomposition} \)

\[
\text{Occ}(N) = \langle \psi | \delta(\sum_i (\hat{H}_i^{HO}/\hbar\Omega - 3/2) - N) | \psi \rangle
\]
Summary and Conclusions

Short-Range Correlations

- Repulsive core and tensor force induce strong two-body correlations
- Universal behavior of two-body densities at small distances/large momenta
- Dominance of deuteron-like pairs at high relative momenta
- SRC in the wavefunction can be “eliminated” by unitary transformations of the interaction (SRG, UCOM), high-momentum information can be recovered by corresponding transformation of operators

Clustering in Fermionic Molecular Dynamics

- Gaussian wave-packet basis has the flexibility to describe clustering and harmonic oscillator configurations
- Clustering (quartetting) naturally emerges in VAP calculations
- Spin-orbit force “breaks” clustering
- If the energy of a state is close to a cluster threshold clustering becomes important — especially in the surface of the nucleus
- Cluster and harmonic oscillator wave functions are not orthogonal - can be identical in the SU(3) limit
- Description of spatially extended cluster states would require extremely large model spaces in oscillator basis