Quartet
Quarteting and clustering and their symmetry-based connection

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I. Quarteting
II. Clustering
III. Connection
IV. Relation
V. Conclusion
I. Quartetiting

Important in different branches of culture.
Quartet excitations; shell-like models

A. Arima, V. Gillet, J. Ginocchio 1970:
Quartet-excitations

L. Satpathy, K.W. Schmid, A. Faessler 1972:
Intrashell excitations

M. Harvey 1973:
Quartet-symmetry: permut. [4], ST [1,1,1,1]
Interacting boson models

J. Dukelsky et al 1982:
U(6) spectrum generation: + parity

F. Iachello, A.D. Jackson 1982:
Both parities
Shell-like models: semi-algebraic: only S and T.

Interacting boson models:
Fully algebraic, without shell-model connection.

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Fully algebraic description (including spectrum generation) with well-defined shell content?
Algebraic models for shell-like quarteting

Quartet: as defined by
Arima et al, Satpathy et al, and Harvey.
Formalism: U(3) as introduced by Elliott.

Phenomenologic: no internal structure
Phenomenologic Algebraic Quartet Model (PAQM).
Semimicroscopic: nucleonic degrees of freedom
Semimicroscopic Algebraic Quartet Model (SAQM).
Phenomenologic Algebraic Quartet Model

Quartet: no internal structure
   (Arima et al, Satpathy et al).
Quartet-shells, each of + parity,

\[(\hbar \omega)_Q \approx 4(\hbar \omega)\].

Pauli-principle is appreciated.
SU(3) formalism

Building blocks:

\[ \hat{A}_{\alpha\beta} = \frac{1}{2} \left( \hat{a}_\alpha^+ \hat{a}_\beta + \hat{a}_\beta \hat{a}_\alpha^+ \right), \quad \alpha, \beta = x, y, z \]

\[ \hat{a}_\alpha = \sum_j \hat{a}_\alpha(j), \quad \hat{a}_\alpha^+ = \sum_j \hat{a}_\alpha^+(j), \quad j = 1, \ldots, N \]

9 operators; another linear combination:
(spherical) scalar, vector and 2nd rank tensor:

\[ \hat{n}, \quad \hat{L}_m, \quad \hat{Q}_m. \]
Algebra-chain:

\[ U(3) \supset SU(3) \supset SO(3) \supset SO(2) \]
\[ |[n_1, n_2, n_3], (\lambda, \mu), K, L, M \rangle \]

1. Complete set of basis states
2. Dynamical symmetry:
   H in terms of invariant operators
   Eigenvalue-problem: analytical solution
Physical operators: in terms of U(3) generators

\( \hat{T}^{(E_0)} = e^{(0)} \hat{n}, \quad \hat{T}^{(E_2)} = e^{(2)} \hat{Q}^{(2)}_m, \quad \hat{T}^{(M_1)} = m^{(1)} \hat{L}^{(1)}_m, \)

Hamiltonian: spherical scalar, e.g.:

\( \hat{n}, \quad \hat{Q}^{(2)} \cdot \hat{Q}^{(2)}, \quad \hat{L}^{(1)} \cdot \hat{L}^{(1)}, \)
Semimicroscopic Algebraic Quartet Model

Quartet: two protons and two neutrons with [4] permutational and [1,1,1,1] spin-isospin symmetry (as defined by Harvey).

Quartet-shells, + and - parities,

\[(\hbar \omega)_Q = (\hbar \omega).\]

Model space: symmetry-governed truncation of the no-core SU(3) shell model.
\[ E = (h\omega)n + a(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu) + d \frac{1}{2\theta} \hat{L}^2, \]

\[ B(E2, I_i \rightarrow I_f) = \frac{2I_f + 1}{2I_i + 1} \alpha^2 \left\langle (\lambda, \mu)KI_i, (1,1)2 \parallel (\lambda, \mu)KI_f \right\rangle \left\langle C^{(2)}(\lambda, \mu) \right\rangle \]
SAQM

Model space: symmetry-governed truncation of the no-core SU(3) shell model.

Effective model: lowast-grade U(3), renormalization
II. Clustering

Semimicroscopic Algebraic Cluster Model


Internal structure of clusters:

Elliott-model with $U_{C_i}^{ST}(4) \otimes U_{c_i}(3)$ algebraic structure.


Relative motion:

(truncated) vibron model with $U_R(4)$ algebraic structure

Binary clusterization:

\[ U_{C_1}^{ST}(4) \otimes U_{c_1}(3) \otimes U_{C_2}^{ST}(4) \otimes U_{c_2}(3) \otimes U_R(4). \]

Operators: group generators, matrix elements, algebraic.

Model space: only Pauli-allowed states, as in the Microscopic Cluster Model with U(3) basis.

(H. Horiuchi, T. Hecht, Y. Suzuki, K. Kato, …)
III. Connection

Strong \((U^{ST}(4) \otimes U(3))\) coupled basis;
intersection with the quartet (shell model).

Even more: identical parts of their spectra:
Multichannel Dynamical Symmetry (MUSY).
Multichannel Dynamical Symmetry (MUSY)

Unified description of different cluster configurations
(including the shell or quartet limit).

Channel: reaction channel, which defines the clusterization.

Simplest case: two-channel symmetry.
Composite symmetry:
- Simple SU(3) dynamical symmetry in both configurations.
- A further symmetry connects them.
  The latter one is a symmetry of the pseudo-space of the particle-indexes.

Similar logical structure like SUSY in nuclear spectroscopy.
Simplest case

Two-channel dynamical symm. of binary configurations


Geometrically: a symmetry with respect
to the Talmi-Moshinsky transformations that connects the
Jacobi coordinate systems of the two configurations.
The two-channel symmetry is a consequence of a usual
dynamical symmetry of an underlying three-cluster
configuration.
The quartet (shell) model limit is a special cluster configuration.

Quartet spectrum > MUSY > cluster spectra

$0^+ \text{ states in } ^{28}\text{Si}$

P Adsley et al, in preparation

Inelastic alpha-scattering at very forward angles
$E_{\text{exct}}: 6 - 14 \text{ MeV}$

$4\, (+2) \, 0^+ \text{ states.}$
P. Adsley et al, arXiv 1609.00296 [nucl-ex]
The experimental observation of the predicted cluster spectra is a strong indication of the validity of the MUSY.
IV. Relation to other models

Microscopic quartet model


Excitation spectrum:
Core + 4 nucleons > shell model > quartet as input.

SAQM: no core, no input, multi shell; not fully microsc.
<table>
<thead>
<tr>
<th>Feature</th>
<th>MQM</th>
<th>SAQM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupling</td>
<td>j-j</td>
<td>L-S</td>
</tr>
<tr>
<td>Shell</td>
<td>valence</td>
<td>no-core</td>
</tr>
<tr>
<td>Quartet</td>
<td>4 nucleons</td>
<td>2P+2n</td>
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<tr>
<td>Q-state</td>
<td>cl.core+4nucl.</td>
<td>T=S=0</td>
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<td>Parameters</td>
<td>wavefunction</td>
<td>operators</td>
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<td>$C_{i_1,j_1,J_1,T_1_2,j_2,J_2,T_2}$</td>
<td>a, b,d</td>
</tr>
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<td>Input</td>
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<td>experiment</td>
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<tr>
<td>Application</td>
<td>low-energy</td>
<td>low, high energy</td>
</tr>
<tr>
<td>Clusters</td>
<td>?</td>
<td>MUSY</td>
</tr>
<tr>
<td>Mass</td>
<td>light, heavy</td>
<td>light</td>
</tr>
</tbody>
</table>
Physical content

of the quartets are very similar.

SAQM: $T=0, S=0, L=0, \ldots$ ($J=0, \ldots$)

MQM: analysis of the correlation energy, spectrum
(of $N=Z$ even-even nuclei)
as well as the overlap with the shell-model wavefunction:
leading role of the
$T=0, J=0, 2, \ldots$ quartets.
Shell, collective and cluster models:
(sympl., contr.sp., semi- or fully algebr.)
SU(3) dynamical symmetry.

No-core shell models (NCSM), L-S coupling, SU(3)

from ab initio to semimicroscopic:

Symmetry-adapted > Symplectic   SAQM   SACM
Ab initio    > fully            > semimicroscopic
             > microscopic         quartet cluster
V. Summary and conclusions

The semimicroscopic algebraic approach to quarteting and clustering (SAQM & SACM) seems to be promising for light nuclei.

Their connecting symmetry (MUSY) has a great predictive power ($^{28}$Si: cluster spectra from quartet description as a projection).

Further connections to the shell, collective and cluster models is well-defined via its obvious symmetry properties. The model space of the SAQM is a symmetry-governed truncation of the NCSM.
Thank you for your attention!
Thank you for your attention!
\[ \hat{H} = (h\omega)\hat{n} + a\hat{C}^{(2)}_{SU3} + b\hat{C}^{(3)}_{SU3} + d \frac{1}{2\theta} \hat{L}^2, \]

\[ B(E2, I_i \rightarrow I_f) = \frac{2I_f + 1}{2I_i + 1} \alpha^2 \langle \langle (\lambda, \mu)KI_i, (1,1)2 \parallel (\lambda, \mu)KI_f \rangle \rangle C^{(2)}(\lambda, \mu) \]
Binary configurations: 3 dynamical symmetries

\[ SU_{c_1}(3) \otimes SU_{c_2}(3) \otimes U_R(4) \supset SU_c(3) \otimes SU_R(3) \supset SU(3) \supset SO(3) \]

\[ SU_{c_1}(3) \otimes SU_{c_2}(3) \otimes U_R(4) \supset SU_c(3) \otimes O_R(4) \supset SO_c(3) \otimes SO_R(3) \supset SO(3) \]

\[ SU_{c_1}(3) \otimes SU_{c_2}(3) \otimes U_R(4) \supset SU_c(3) \otimes SU_R(3) \supset SO_c(3) \otimes SO_R(3) \supset SO(3) \]
Phases and clusters

Rel. motion: vibron model.
Vibron model: U(3) – O(4).
Cluster model
- Coupling to int. d. f.
- Pauli-principle.

U(3) shell-like clusters,
O(4) rigid molecules.

4 nucleons full sd shell model

Symplectic no-core shell model
   (G.s) overlap (cluster space) > 65%

Δ SACM

///// Quasi-cluster: fr rigid mol. to strong LS
   N. Itagaki et al., *PRC 83* (2011) 014302.
\[ E = 37.8 \, n - 2.4879 \, C_2(SU(3)) + 0.0454 \, C_3(SU(3)) - 0.9837 \, K^2 + 1.1661 \, \frac{1}{(2\Theta)L(L+1)} \]