Cluster correlations in deformed states

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Cluster structures coupling with deformed states

- Clustering and deformation are important for nuclear structure changes.
- Cluster components couple with deformed states.
Deformation and clustering in $^{40}$Ca
Coexistence of normal-deformed and superdeformed bands

- Normal-deformed (ND) and superdeformed (SD) bands coexists.
- Energy gap of Nilsson orbits explain existence of those bands.

Deformation and clustering in $^{40}\text{Ca}$

$\alpha-^{36}\text{Ar}$ cluster structure of the ND band

The ND states are strongly populated by $\alpha$-transfer $^{36}\text{Ar}(^6\text{Li}, d)$ reactions.

The ND states contain large amounts of $\alpha-^{36}\text{Ar}$ cluster structure components.

Deformation AND Clustering should be taken into account.

Outline

1. Method to take into account symmetries and correlations.
3. Influence of cluster structure components on deformation.
Wave function

Deformed-basis antisymmetrized molecular dynamics (AMD) wave function $|\Phi\rangle$:
Slater determinant of Gaussian wave packets that can deform.

$$|\Phi\rangle = \hat{A} |\varphi_1, \varphi_2, \cdots, \varphi_A\rangle,$$

$$\varphi_i \propto \exp \left[ - (\mathbf{r} - \mathbf{Z}_i) \cdot \mathbf{M}(\mathbf{r} - \mathbf{Z}_i) \right] \sigma_i \tau_i.$$
Energy variational calculation with a constraint potential

Parameters in wave functions are determined by energy variational calculations with a constraint potential $V_{\text{cnst}}$.

\[
\delta \left[ \langle \hat{P}^\pi \Phi \mid \hat{H} \mid \hat{P}^\pi \Phi \rangle + V_{\text{cnst}} \right] = 0
\]

- Effective interaction $\hat{H}$: Gogny D1S
- $V_{\text{cnst}}$: quadrupole deformation parameter $\beta$ (def. structure) intercluster distance (cluster structure)
- Conjugate gradient method.
Deformation parameter $\beta$

- Deformed structures tend to be obtained.

Intercluster distance

- Distance between centers of mass of groups of nucleons.
- Cluster structures are obtained.

Projection methods

Reflection: Parity projection $\hat{P}^\pi$

$$\frac{1 + \pi \hat{P}_r}{2} \left| \Phi \right\rangle$$

Rotation: Angular momentum projection $\hat{P}^J_{MK}$

$$\frac{2J + 1}{8\pi^2} \int d\Omega D^{J*}_{MK}(\Omega) \hat{R}(\Omega) \left| \Phi \right\rangle$$

$$\approx \frac{2J + 1}{8\pi^2} \sum_i W_i D^{J*}_{MK}(\Omega_i) \hat{R}(\Omega_i) \left| \Phi \right\rangle$$
Generator coordinate method (GCM)

\[ |\Phi_n^{J\pi M}\rangle = \sum_i f_n^i \hat{P}_{MK_i}^J \hat{P}_\pi |\Phi_i\rangle \]

\[ f_1 + f_2 + f_3 + \ldots \]

\[ + f'_1 + f'_2 + f'_3 + \ldots \]

\[ + f''_1 + f''_2 + f''_3 + \ldots \]
Framework
Alignment of wave functions for GCM with angular momentum projection

By minimization of variance of \( \hat{J}_z \), concentrated \( K \)-distribution is obtained.

\[
|\Phi\rangle \rightarrow |\Phi'\rangle = \hat{R} |\Phi\rangle,
\]

\[
\delta \left( \langle \hat{J}_z^2 \rangle - \langle \hat{J}_z \rangle^2 \right) = 0.
\]

ex) \( K \)-distribution of a spherical odd nucleus \(^{17}\text{O}\)

[Y. Taniguchi, PTEP, accepted, arXiv:1608.08726]
Candidates of mechanism of cluster correlations in deformed states

Threshold energy rule

Cluster structures develop in excited states whose excitation energies are similar to threshold energies of the cluster decay.

- It works well to $p$-shell nuclei.
- It does not work to heavier nuclei.

[H. Hriuchi et al, Prog. Theor. Phys. Suppl. 52, 89 (1972)]
Candidates of mechanism of cluster correlations in deformed states

Particle-hole configurations


d correlation

Nucleons excited to a higher shell correlate and form a cluster.

- It is an open problem whether this rule works well for various nuclei such as odd nuclei.
Deformed states in $^{35}$Cl

- A negative-parity deformed band was observed by a $\gamma$-spectroscopy experiment.
- It is predicted that this deformed states have $\alpha^{-31}$P cluster structure with no direct evidence.

- Structure of the $\pi = -$ deformed band.
- $\alpha^{-31}$P and $t^{-32}$S clustering.
- Mechanism of coupling of cluster structure with deformed states.

In negative-parity states, a local minimum with $3\hbar\omega$ excited configurations exists at $\beta \sim 0.4$. 
Negative-parity $\alpha^{-31}P$ and $t^{-32}S$ cluster wave functions

- In the L/S type, a smaller cluster is on the long/short axis of a larger cluster.
- Energies of L type wave functions of $\alpha^{-31}P$ and $t^{-32}S$ cluster structures are similar in small intercluster distance region.
- Energy of L type wave functions with small intercluster distance is similar to that of $3\hbar\omega$ configurations on the $\beta$-energy curve.
Various deformed bands such as a $K^{\pi} = \frac{1}{2}^-$ are obtained.

Dominant components of the $K^{\pi} = \frac{1}{2}^-$ band have $3\hbar\omega$ excited configurations on the $\beta$-energy curve.

The $K^{\pi} = \frac{1}{2}^-$ deformed band corresponds to the observed deformed band.
\( \alpha \)- and \( t \)-cluster structure components in the \( K^\pi = \frac{1}{2}^- \) deformed band

\[ J^\pi = \frac{1}{2}^- (K^\pi = \frac{1}{2}^- \text{ band}) \]

- The \( K^\pi = \frac{1}{2}^- \) deformed states contain similar amounts of \( \alpha \)-type \( \alpha^{-31}P \) and \( t \)-type \( t^{-32}S \) cluster structure components.

- Threshold energy is not important for coupling with deformed states.

\[ E_{th}(\alpha) = 7 \text{ MeV} \]
\[ E_{th}(t) = 18 \text{ MeV} \]
Mechanism of coupling of cluster structure with deformed states

- L type $\alpha$- and $t$-cluster structure become $3\hbar\omega$ excited configurations in small intercluster distance.
- $sd$-shell orbits in a $^{32}\text{S}$ cluster are fully occupied in the direction of the long axis.
- A $^{31}\text{P}$ cluster has a proton hole at a $sd$-shell orbit in the direction of the long axis.

**Mechanism of coupling of cluster structure with deformed states**

**small distance**  Similar ph-configurations of cluster structure and deformed states.

**large distance**  Structure of cluster wave functions change gradually with increasing intercluster distance.
L type cluster structure components couple with excited deformed states.

Threshold energy is not important of coupling of cluster structure with deformed states.

Particle-hole configurations of cluster structures with small intercluster distance are important for coupling with low-lying deformed states.
Deformation and Clustering

- Cluster structure components couple with deformed states.
- How do cluster structure components influence deformation?

Triaxiality and cluster structure components

- ND states in $^{40}$Ca
- SD states in $^{28}$Si
$^{40}$Ca: Triaxial deformation of ND states

- Triaxial ND bands (band 2 and 4).
  


  [E. Ideguchi et al, Phys. Rev. Lett. 87, 222501 (2001)]
40 Ca: Energy curves

(a) Energy curves for Gogny D1S and Skyrme SLy7 parametrizations, showing the quadrupole deformation parameter $\beta$ and energy [MeV].

(b) The ND local minimum state forms a triaxial shape with $4\hbar\omega$ excited configuration.

- The ND local minimum state forms a triaxial shape with $4\hbar\omega$ excited configuration.
- (a) S and (b) L types of $\alpha^{36}\text{Ar}$ cluster structure are obtained with the intercluster distance constraint.
The ND states form triaxial shapes with $4\hbar\omega$ excited configurations.
The ND states contain large amounts of L type $\alpha-^{36}\text{Ar}$ cluster structure components.

The SD local minimum state form axial symmetric shape with $4\hbar\omega$ excited configuration.

S Type: Triaxial
L Type: Axial symmetric
The SD states have axially symmetric shape with $4\hbar\omega$ excited configuration.


Candidates of the SD states were observed.

$^{28}$Si: $\alpha$–clustering correlation in SD states

SD: large $\alpha$–$^{24}$Mg cluster structure component

Clustering and Triaxiality

<table>
<thead>
<tr>
<th>nucleus</th>
<th>state</th>
<th>shape</th>
<th>configuration</th>
<th>cluster</th>
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</thead>
<tbody>
<tr>
<td>$^{40}\text{Ca}$</td>
<td>ND</td>
<td>triaxial</td>
<td>$4\hbar\omega$</td>
<td>$\alpha-^{36}\text{Ar}$ L type</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>SD</td>
<td>prolate</td>
<td>$4\hbar\omega$</td>
<td>$\alpha-^{24}\text{Mg}$ L type</td>
</tr>
</tbody>
</table>

- L type $\alpha$ cluster structure components couple with ND states in $^{40}\text{Ca}$ and SD states in $^{28}\text{Si}$.
- Owing to shapes of larger clusters $^{36}\text{Ar}$ and $^{24}\text{Mg}$, total systems of $\alpha$-cluster structures form triaxial and axial symmetric shapes, respectively.
- Cluster components that have an oblate cluster enhance triaxial deformation in excited states.
- ND states in $^{40}$Ca and SD states in $^{28}$Si have triaxial and axial symmetric shapes, respectively.
- L type $\alpha$ cluster structure couple with ND states in $^{40}$Ca and SD states in $^{28}$Si.
- Cluster components that contain an oblately deformed cluster enhance triaxiality of excited deformed states.
Conclusions

- Cluster correlations in deformed states are investigated by using the AMD and the GCM.
- Cluster structure components couple with deformed states.
- The negative-parity deformed band in $^{35}$Cl
  - The deformed states contain similar amounts of $\alpha$- and $t$-cluster structure components.
  - Particle-hole configurations of cluster structures with small intercluster distance are important for the coupling more than threshold energies.
- ND states in $^{40}$Ca and SD states in $^{28}$Si
  - ND states in $^{40}$Ca and SD states in $^{28}$Si have triaxial and axial symmetric shape, respectively.
  - Both deformed states contain $\alpha$-cluster structure components.
  - Shape of the cluster is important for triaxiality of total system.
  - Cluster structure components that contain an oblate cluster enhance triaxiality of excited deformed states.