$\rho$-meson broadening in QCD at finite temperature

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Electromagnetic Probes of Strongly Interacting Matter
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Interactions with matter at finite $T$ modify particle properties.

Particles with short lifetimes, comparable to plasma lifetime like $\Sigma$, $K^*$, $\rho$, are useful probes; they form, decay and scatter both within the QGP and the hadronic phases.

These phenomena are linked to hadronization and are therefore of non-perturbative nature.
Heavy-Ion Collisions at High Energy
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Nuclear matter is compressed and heated in high energy nuclear reactions.

Fundamental degrees of freedom are liberated.
Collision energies

Relativistic Heavy Ion Collider

AGS: $\sqrt{s_{NN}} = 4.8$ GeV

SPS: $\sqrt{s_{NN}} = 17$ GeV

RHIC: pp, dA, AA
$\sqrt{s_{NN}}$: 20 to 200 GeV

LHC $\sqrt{s_{NN}} = 5.6$ TeV
QCD Phases

~197

~ 10 times normal nuclear density
Mass vs Width Change


SPS ~ 1995
Dileptons in Low Mass Region NA60 (2009)

Width Growth NA60

PHENIX low momentum pairs
Analysis Tools

Correlator of vector currents

\[ \Pi_{\mu\nu}(q^2) = i \int d^4x \, e^{iq \cdot x} \langle 0 | T(V_{\mu}(x)V_{\nu}(0)) | 0 \rangle \]

\[ = -g_{\mu\nu} \, \Pi_1(q^2) + q_\mu q_\nu \, \Pi_0(q^2) \]
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Take one of these structures free from singularities and apply Cauchy's theorem in complex energy plane
Complex Energy Plane
Since integration path does not enclose any singularities

\[ \frac{1}{\pi} \int_{0}^{s_0} ds \ s^{N-1} \text{Im}\Pi(s) \bigg|_{QCD} = -\frac{1}{2\pi i} \oint_{C(|s_0|)} ds \ s^{N-1} \Pi(s) \bigg|_{QCD} \]
Since integration path does not enclose any singularities

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Contains hadron degrees of freedom close to the positive real axis

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This object is related to the spectral density.
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QCD (pert. and non-pert) is valid on the circle

\[
\frac{1}{\pi} \int_0^{s_0} ds \ s^{N-1} \ \text{Im} \Pi(s) = -\frac{1}{2\pi i} \int_{C(|s_0|)} ds \ s^{N-1} \Pi(s) \bigg|_{\text{QCD}}
\]

This object is related to the spectral density
Quark-Hadron Duality

Operator Product Expansion

$$\Pi(Q^2) \bigg|_{QCD} = \sum_{M=0}^{\infty} \frac{C_{2M+2}}{Q^{2M+2}} \langle O_{2M+2} \rangle$$
$$\frac{1}{2\pi i} \sum_{M=1}^{N} C_{2M+2} \langle O_{2M+2} \rangle \int_{C(|s_0|)} ds \frac{s^{N-1}}{s^{M+1}}$$

$$= - \frac{1}{2\pi i} \sum_{M=1}^{N} C_{2M+2} \langle O_{2M+2} \rangle (2\pi i) \delta_{M,N-1}$$

$$= - C_{2N} \langle O_{2N} \rangle.$$
Non-Perturbative Part: OPE

\[
\frac{1}{2\pi i} \sum_{M=1}^{\infty} C_{2M+2} \langle O_{2M+2} \rangle \int_{C(|s_0|)} ds \frac{s^{N-1}}{s^{M+1}}
\]

\[
= - \frac{1}{2\pi i} \sum_{M=1}^{\infty} C_{2M+2} \langle O_{2M+2} \rangle (2\pi i) \delta_{M,N-1}
\]

\[
= - C_{2N} \langle O_{2N} \rangle.
\]

N=0,1,2...
Non-Perturbative Part: OPE

\[ \frac{1}{2\pi i} \sum_{M=1} C_{2M+2} \langle O_{2M+2} \rangle \int_{C(|s_0|)} ds \frac{s^{N-1}}{s^{M+1}} \]

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N=0,1,2,...
Leading condensates

dimension $d = 4$ gluon condensate

$$C_4 \langle \hat{O}_4 \rangle = \frac{\pi}{3} \langle \alpha_s G^2 \rangle$$

dimension $d = 6$ four-quark condensate

$$C_6 \langle \hat{O}_6 \rangle = -8\pi^3 \alpha_s \left[ \langle (\bar{q}\gamma_\mu \gamma_5 \lambda^a q)^2 \rangle + \frac{2}{9} \langle (\bar{q}\gamma_\mu \lambda^a q)^2 \rangle \right]$$

No invariant operators of $d=2$ in QCD, it is standard practice to assume $C_2 \langle O_2 \rangle = 0$
For increasing $T$ and/or $\mu_B$ the energy threshold for the continuum goes to 0.
Finite T

Two contributions:

1) Annihilation channel (available also at T=0)
2) Dispersion channel (Landau damping)
At finite $T$, spectral function has support both at space-like and time-like momenta.
Sum Rules

\[
(-1)^{N+1} C_{2N} \langle O_{2N} \rangle = \frac{1}{\pi} \int_0^{s_0} dss^{N-1} \text{Im} \Pi^{\text{HAD}}(s)
- \frac{1}{\pi} \int_0^{s_0} dss^{N-1} \text{Im} \Pi^{\text{PQCD}}(s)
\]

\[
\Pi^{\text{HAD}}(q^2) = \frac{m_{\rho}^2 f_{\rho}^{-2}}{(m_{\rho}^2 - q^2) - im_{\rho} \Gamma_{\rho}} + \text{pion loop from vector current coupled to pions}
\]

\[
\text{Im} \Pi^{\text{HAD}}(q^2) = \frac{m_{\rho}^3 f_{\rho} \Gamma_{\rho}/\pi}{(m_{\rho}^2 - q^2)^2 + m_{\rho}^2 \Gamma_{\rho}^2} + \]

Explicit Sum Rules, N=1
Explicit Sum Rules, $N=1$

\[ N = 1: \]
\[ 0 = 8\pi m^2 \rho f^2 \rho \left[ \arccot \left( \frac{\Gamma_{\rho}}{m_{\rho}} \right) - \arccot \left( \frac{\Gamma_{\rho} m_{\rho}}{m^2_{\rho} - s_0} \right) \right] - s_0 \]
\[ - \frac{4\pi^2 T^2}{9} + 2 \int_0^{s_0} ds \eta_F(\sqrt{s}/2) \]
Explicit Sum Rules, N=2
Explicit Sum Rules, N=2

\[ N = 2 : \]
\[ -C_4 \langle O_4 \rangle = 8\pi m_\rho^3 f_\rho^2 \Gamma_\rho \]
\[ \times \left[ \left( \frac{m_\rho}{\Gamma_\rho} \right) \left( \text{arccot} \left( \frac{\Gamma_\rho}{m_\rho} \right) - \text{arccot} \left( \frac{\Gamma_\rho m_\rho}{m_\rho^2 - s_0} \right) \right) \right. \]
\[ + \left. \frac{1}{2} \ln \left[ \frac{\Gamma_\rho^2 m_\rho^2 + (m_\rho^2 - s_0)^2}{m_\rho^2 (m_\rho^2 + \Gamma_\rho^2)} \right] \right] - \frac{s_0^2}{2} \]
\[ + 2 \int_0^{s_0} ds \text{sn}_F (\sqrt{s}/2) \]
Explicit Sum Rules, N = 3
Explicit Sum Rules, $N=3$

\[
N = 3 : \\
-C_6^3 O_6 = 8\pi m_\rho^3 f_\rho^2 \Gamma_\rho \\
\times \left[ \left( \frac{m_\rho}{\Gamma_\rho} \right) \left( m_\rho^2 - \Gamma_\rho^2 \right) \left( \arccot \left( \frac{\Gamma_\rho}{m_\rho} \right) - \arccot \left( \frac{\Gamma_\rho m_\rho}{m_\rho^2 - s_0} \right) \right) + s_0 \right] \\
+ m_\rho^2 \ln \left[ \frac{\Gamma_\rho^2 m_\rho^2 + (m_\rho^2 - s_0)^2}{m_\rho^2 (m_\rho^2 + \Gamma_\rho^2)} \right] - \frac{s_0^3}{3} \\
+ 2 \int_0^{s_0} ds n_F(\sqrt{s}/2)
\]
Need inputs

$C_2\langle O_2 \rangle, \ C_4\langle O_4 \rangle, \ C_6\langle O_6 \rangle, \ s_0, \ f_\rho, \ m_\rho, \ \Gamma_\rho$
Need inputs

Inputs: Finite temperature behavior of

\[ C_2\langle O_2 \rangle, \ C_4\langle O_4 \rangle, \ C_6\langle O_6 \rangle, \ s_0, \ f_\rho, \ m_\rho, \ \Gamma_\rho \]
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\[ C_2 \langle O_2 \rangle \sim 0 \]
Inputs: Finite temperature behavior of

\[ C_2\langle O_2 \rangle, \ C_4\langle O_4 \rangle, \ C_6\langle O_6 \rangle, \ s_0, \ f_\rho, \ m_\rho, \ \Gamma_\rho \]

\[ C_2\langle O_2 \rangle \sim 0 \]

Three equations, six unknowns. Need a guide to solve the problem
$T=0$

This gives

\[ f_\rho = 5, \ m_\rho = 0.776 \text{ GeV}, \ \Gamma_\rho = 0.145 \text{ GeV} \]

\[ C_4 \langle O_4 \rangle = 0.12 \text{ GeV}^4, \]
\[ C_6 \langle O_6 \rangle = -0.39 \text{ GeV}^6, \]
\[ s_0 = 1.44 \text{ GeV}^2 \]
Results: $C_4 <O_4>$ at Finite $T$

\[
C_4 \langle \hat{O}_4 \rangle(T) / C_4 \langle \hat{O}_4 \rangle(0) = 1 - 1.65 (T/T_c)^{8.735} + 0.04967 (T/T_c)^{0.7211}
\]

$T_c = 197$ MeV
Results: $C_6\langle O_6 \rangle$ at Finite $T$

\[ C_6\langle \hat{O}_6 \rangle(T) = C_6\langle \hat{O}_6 \rangle(0) \left[ 1 - \left( \frac{T}{T_q^*} \right)^b \right] \]

\[ b = 8 \]

\[ T_q^* = 187 \text{ MeV} \]
Results: $s_0$ at Finite $T$

\[ s_0(T)/s_0(0) = 1 - 0.5667(T/T_c)^{11.38} - 4.347(T/T_c)^{68.41} \]

$T_c = 197$ MeV
Results: leptonic decay constant at Finite $T$

\[ \frac{f_\rho(T)}{f_\rho(0)} = 1 - 0.3901(T/T_c)^{10.75} + 0.04155(T/T_c)^{1.269} \]

$T_c = 197$ MeV
Results: rho mass at Finite T

\[ M_\rho(T) = M_\rho(0)[1 - \left(\frac{T}{T^*_M}\right)^c] \]

\[ c = 10 \]

\[ T^*_M = 222 \text{ MeV} \]
Results: rho width at Finite T

\[ \Gamma_\rho(T) = \frac{\Gamma_\rho(0)}{1 - (T/T_c)^a} \]

\[ a = 3 \]

\[ T_c = 197 \text{ MeV} \]
Conclusions

Solution constrained by physical expectations as well as available lattice input.

Thermal width of rho meson shows a dramatic increase, roughly a factor of 20 near $T_c$. However rho mass decreases only slightly.

Next step: Explore consequences for dimuon rate in heavy ions.