Connections between dilepton data and chiral symmetry restoration

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Work in progress in collaboration with Ralf Rapp

Electromagnetic Probes Workshop

ECT*

May 22,2013

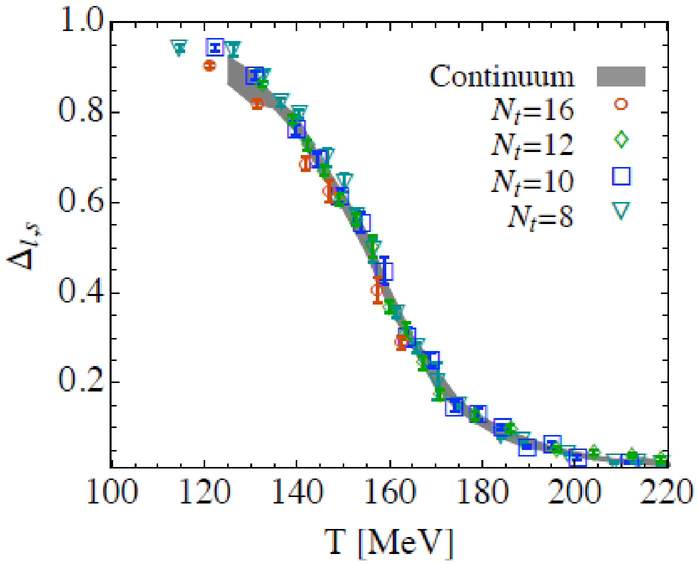
Outline

I. Introduction
II. Sum Rule Analysis
III. Hadronic Effective Field Theory –MYM
IV. Vector results
V. Axial vector results
VI. Summary

I. Introduction
IA. General motivation

On the lattice:

Wuppertal-Budapest Collaboration



- Goal: Experimentally verify chiral symmetry restoration.

- Ideal probes are meson which are chiral partners
  - i.e. ρ and a1.
  - In medium ρ can be investigated by thermal dilepton rates.
  - But a1 measurements prove difficult.

Create a frame work which the properties of the ρ and a1 are uniquely connected.

By measuring the ρ properties infer the a1 properties and deduce chiral symmetry restoration.

II. Sum Rules
IIA. Setup

QCD sum rules and Weinberg-type sum rules

Integral relations between spectral functions and chirally even/odd condensates.

- Weinberg type sum rules:
  - Studies the difference between vector and axial meson spectral functions.
  - Directly related to chiral symmetry breaking.

$$\int ds\,(\rho_V(s) - \rho_A(s))\,s^n = f_n$$

Das, Mathur, and Okubo, 1967
Weinberg, 1967
Kapusta and Shuryak, 1994

- QCD sum rules (with Borel transform, in vacuum):
  - Applicable to vector or axial vector channel individually.

$$\frac{1}{M^2}\int ds \frac{\rho_V(s)}{s} e^{-s/M^2} = \frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right) + \frac{m_q\langle\bar{q}q\rangle}{M^4} + \frac{1}{24M^4}\langle\frac{\alpha_s}{\pi}G^2_{\mu\nu}\rangle - \frac{56\pi\alpha_s}{81M^6}\langle\mathcal{O}_4^V\rangle$$

$$\frac{1}{M^2}\int ds \frac{\bar{\rho}_A(s)}{s} e^{-s/M^2} = \frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right) + \frac{m_q\langle\bar{q}q\rangle}{M^4} + \frac{1}{24M^4}\langle\frac{\alpha_s}{\pi}G^2_{\mu\nu}\rangle + \frac{88\pi\alpha_s}{81M^6}\langle\mathcal{O}_4^A\rangle$$

Shiffman, Vainshtein, Zakharov, 1979

By providing information about one side of the sum rule gain insight into the other.

IIB. Vacuum

ρ SF from effective field theory approach. Rapp and Wambach 1999

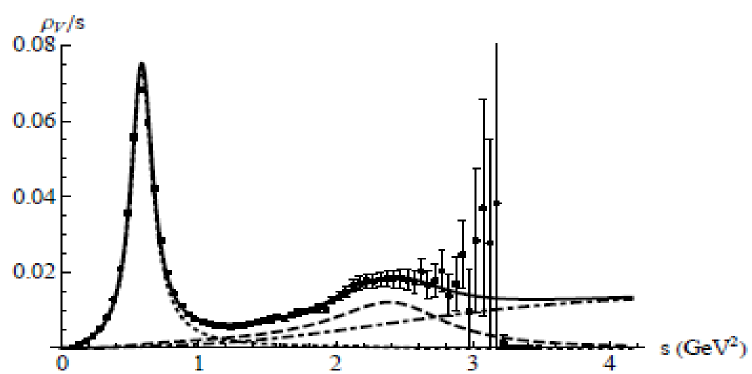
Other resonances are phenomenological (Breit-Wigner for a1 and ρ').

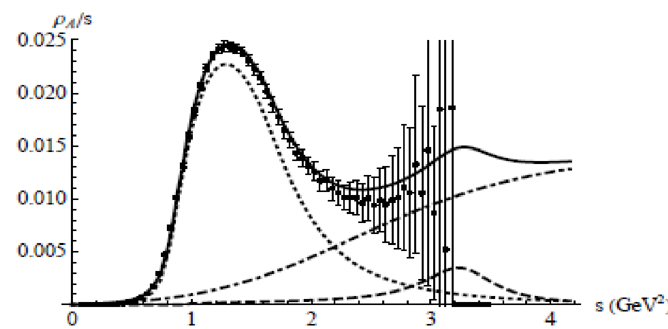
Same perturbative continuum for both channels.

Fit parameters of model to tau –decay data and WSRs.

PMH and Rapp, 2012

Data from ALEPH (Barate et al. 1998)





WSRs call for an excited axial vector state, a1'(1800)!

IIC. Low temperature limit: chiral mixing

- Spectral modification governed by interaction with thermal pions.

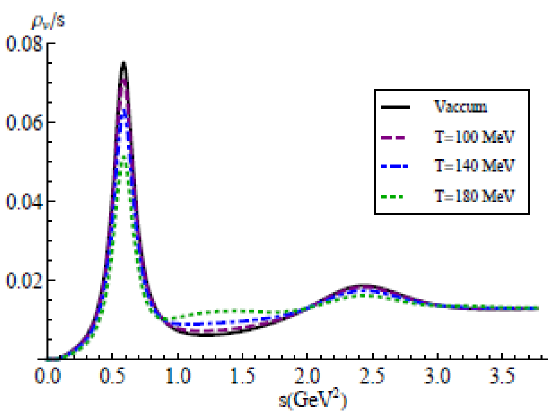
Dey, Eletsky, Ioffe, 1990

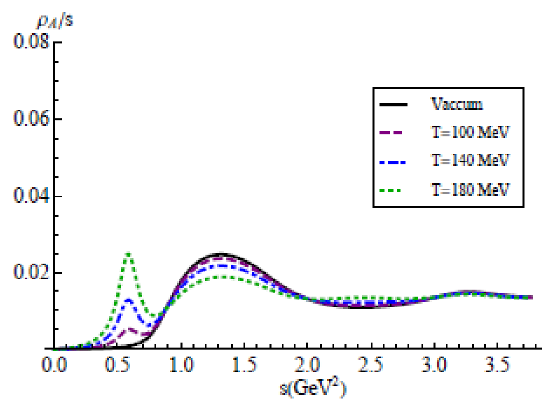
$$\rho_V(T) = \rho_V(T=0)(1-\epsilon) + \rho_A(T=0)\epsilon$$

$$\rho_A(T) = \rho_A(T=0)(1-\epsilon) + \rho_V(T=0)\epsilon$$

$$\epsilon = \frac{2}{f_\pi^2}\int \frac{d^3p}{(2\pi)^3 E_p} n_B(E_p)$$

- Temperature dependence of condensates can also determined by thermal pions.





Holt, PMH, Rapp, 2012

IID. Sum rules and chiral mixing

Weinberg-type sum rules are automatically satisfied as well as the vacuum at all temperatures.

Kapusta and Shuryak, 1993

For finite pion mass, QCD sum rules need to be numerically evaluated.

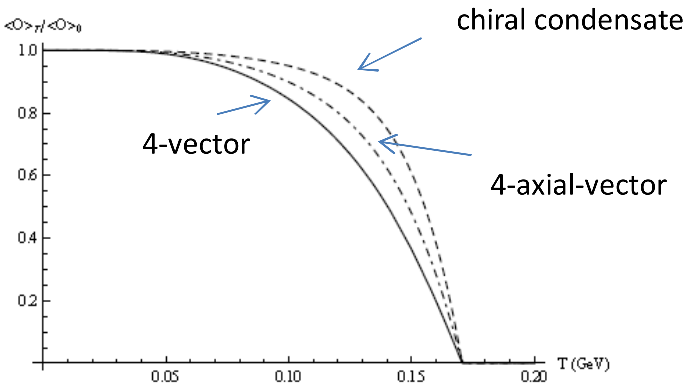
T (MeV)	0	100	120	140	160
ε	0	.06	.1	.16	.23
dV (%)	.24	.32	.48	.85	1.43
dA (%)	.56	.65	.78	1.05	1.6

QCS sum rules are reasonable satisfied at low temperatures but they break down at temperatures on the order of the pion mass

Additional physics (i.e. more resonances) needed for $T > m_\pi$.

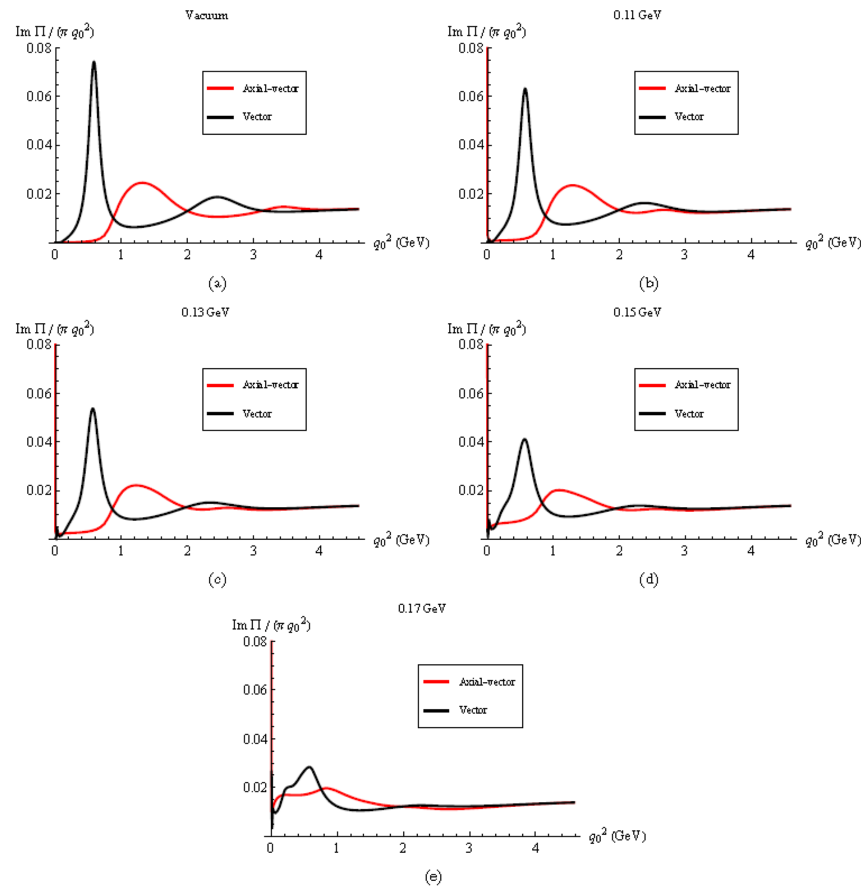
IIE. Sum rules and hadron resonance gas

Used a HRG to model temp dependence of quark condensates.



Input in medium ρ SF from EFT approach.

Deduce axial vector spectral function from simultaneously satisfying QCDSRs and WSRs



Some ambiguities remain with predictive power limited by input.

III. Hadronic Effective Field Theory
IIIA. Massive Yang-Mills

Extensive history

- Consider a chiral non-linear sigma model to describe pions.
- Group structure of theory given by

$$SU_L(2) \times SU_R(2)$$

- Used gauge principle to extend theory to a local gauge theory
  - Limits the number of free parameters in the Lagrangian
  - Introduces two gauge bosons, one left and one right
- Physical vector and axial vector mesons are identified as these corresponding gauge bosons.
- Consider the most general Lagrangian which obeys symmetries to 3 powers of covariant derivative.
  - Includes “non-minimal” terms but does not extend beyond that.
  - Perform a1 shift to remove direct a1-π interaction.
- Gauge and chiral symmetries are broken by an explicit mass term for the mesons.

Lagrangian has 4 free parameters, m0, g, σ, and ξ or m_ρ , m_a , $g_{\rho\pi\pi}$, $g_{\rho\pi\pi}^{(3)}$.

IIIB. Why MYM?

- Gauge principal is fundamental basis for constructing modern field theories.

- It is then natural to identify the gauge bosons of such a theory with light meson states
  - These are the actual chiral partners which should implement the symmetries.
  - In this setup there is no power counting which suppresses a1 interaction so one should consider an explicit a1.

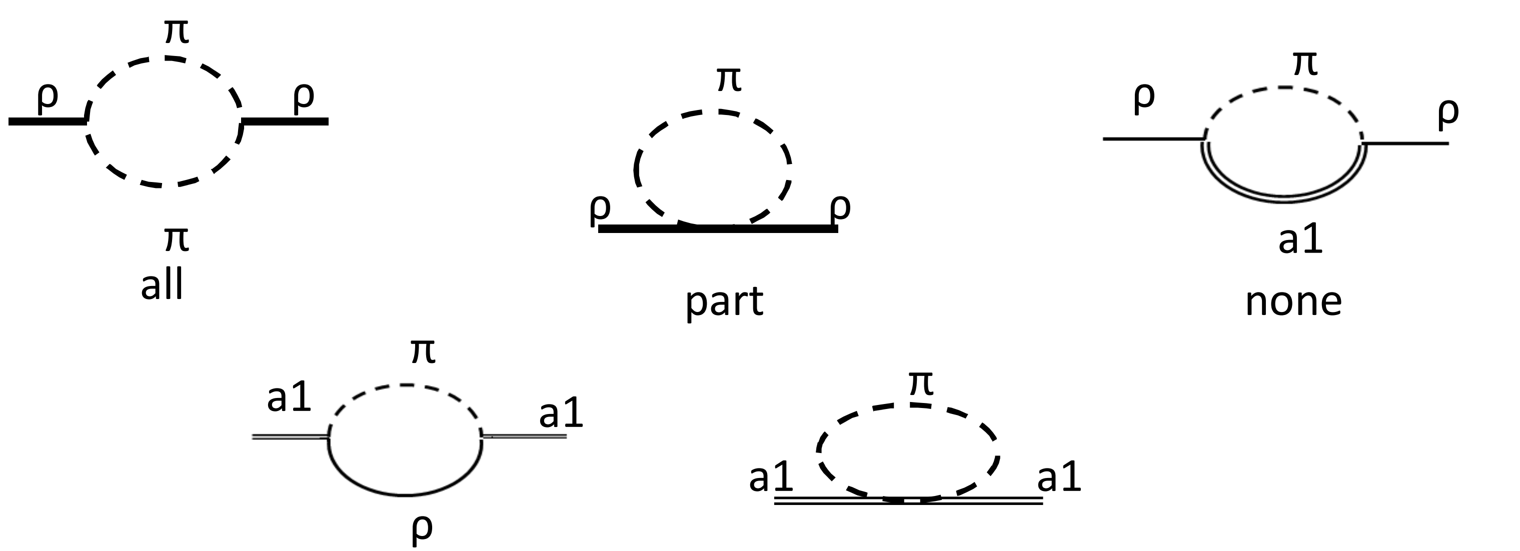
- Simplest implementation of the gauge principles
  - Can satisfy tree level phenomenology (with non-minimal terms).
  - No calculation has been made in vacuum at the 1 loop level which is consistent with spectral function data.
  - Issues of higher derivatives are addressed by regularization.

IIIC. General procedure

- Calculate vector and axial vector current-current correlators
  - Need several different self energies and transition amplitudes

$$\Pi_V = \Sigma_\gamma + \frac{\Gamma^2_{\gamma\rho}}{p^2 - m^2_\rho - \Sigma_\rho} \qquad \Pi^T_A = \Sigma_W + \frac{\Gamma^2_{W\,a}}{p^2 - m^2_a - \Sigma_a}$$

- For each calculate fewest number of diagrams need to preserve symmetry and are relevant for data we wish to describe.



When considering a broad rho, additional vertex correction diagrams will be needed

IIID. Electromagnetic coupling

- One could introduce naïve Vector Meson Dominance.

$$\mathcal{L} = \frac{M_\rho^2}{g_{\rho\pi\pi}} \rho_\mu \gamma^\mu \qquad \frac{M_{a_1}^2}{g_{\rho\pi\pi}} a_\mu W^\mu \Longrightarrow \sim \frac{M_{a_1} M_\rho}{\sqrt{2} g_{\rho\pi\pi}} a_\mu W^\mu$$

- But does this apply to the axial vector channel as well?
  - Chiral symmetry implies a weaker coupling.
  - Prevents agreement with tau-decay data with reasonable choices of other parameters.
- Choose to gauge external EM probes
  - New covariant derivatives and new interactions which are both gauge invariant and chirally symmetric
  - Gauge bosons of the same gauge group as the mesons
    - EM are external fields only, do not propagate in loops
    - Unlike HLS where these are a different gauge group.

$$\mathcal{L} = f \rho_{\mu\nu} \gamma^{\mu\nu} + f' a_{\mu\nu} W^{\mu\nu} \qquad f, f' \sim 1/g_{\rho\pi\pi}$$

Similar to Kroll interaction

IIIE. Regularization and renormalization

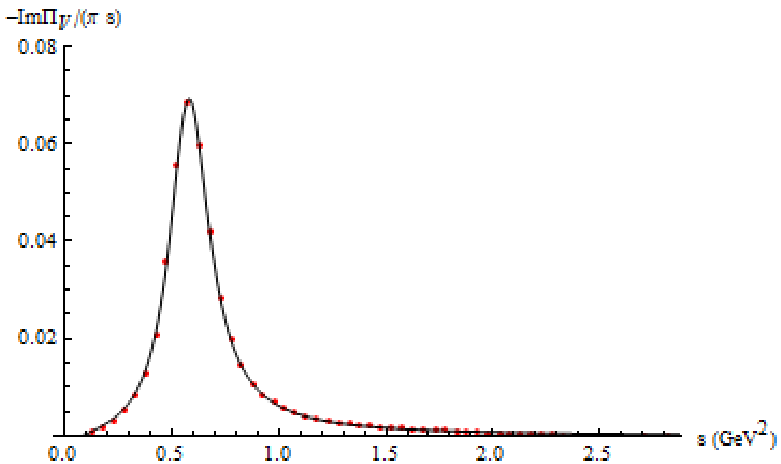
- Calculations are divergent at loop level– need to be regulated.
  - Cut-off or form factor will break gauge symmetry
  - Pauli-Villars with "heavy pion" breaks chiral symmetry
  - Need to use Dimensional Regulation
- To remove resulting infinites counter terms are added to Lagrangian
  - Counter terms have same structure as terms in Lagrangian
  - Will need additional higher derivative terms because of momentum dependence of divergences.

$$\frac{F_\pi^2}{8} D_\mu U^\dagger D^\mu U \to \delta Z_\pi^{(2)} \frac{F_\pi^2}{8} D_\mu U^\dagger D^\mu U \qquad \delta Z_\pi^{(2)} = \delta Z_\pi^{(2)\infty} + \delta Z_\pi^{(2) finite}$$

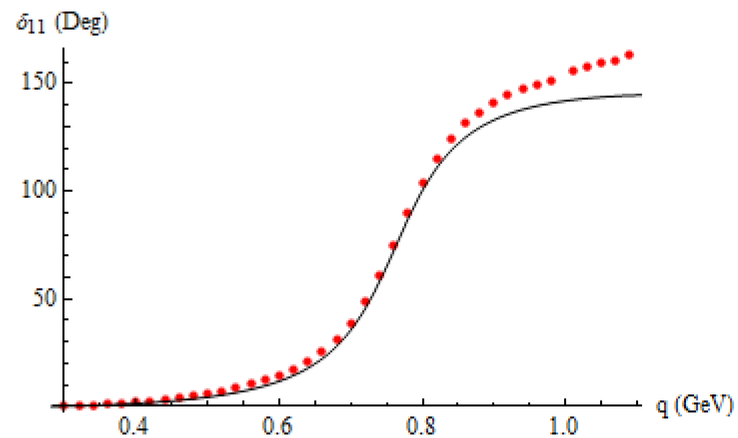
- Infinite part chosen to cancel divergences
- Finite part introduces new parameter which will be fit to data
  - Take a minimalist approach.
  - Can be thought of as renormalizing the fields and couplings

6 extra parameters are introduced: 1 for the vector channel and 5 for axial channel

IV. Vector Channel
IVA. Vacuum



2π data from ALEPH, 1998



Data Froggatt and Petersen, 1977

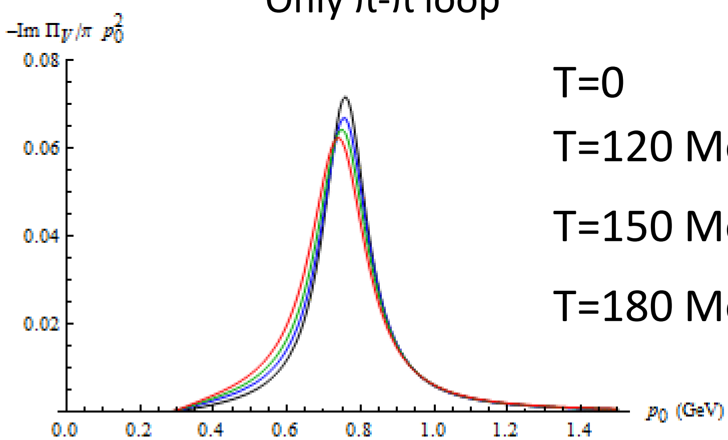
- For Vacuum fit parameters to tau-decay
  - Fair agreement with data with a moderate 3 derivative coupling, still ok with zero coupling.
  - Parameter set here is preferred once axial vector channel analysis is performed.

$$m_\rho = 767\text{MeV} \qquad m_a = 1400\text{MeV} \qquad g_{\rho\pi\pi} = 4.6 \qquad g^{(3)}_{\rho\pi\pi} = 2.75\text{GeV}^{-2}$$

IVB. In medium

- Considered medium modification of rho but not of pions.
  - Both statistical and kinematics

Only π-π loop



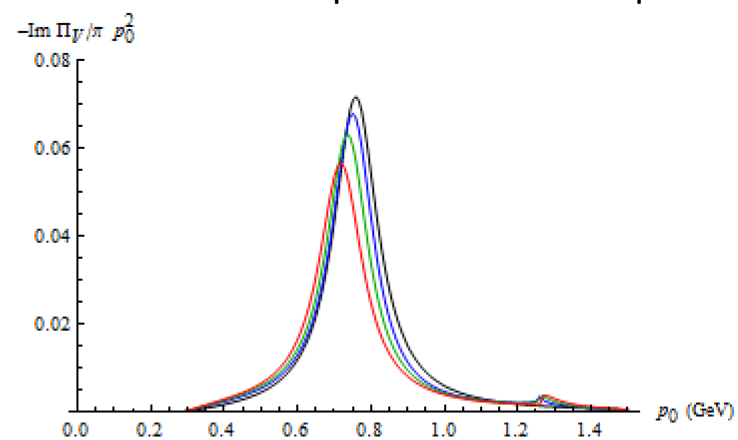
T=0

T=120 MeV

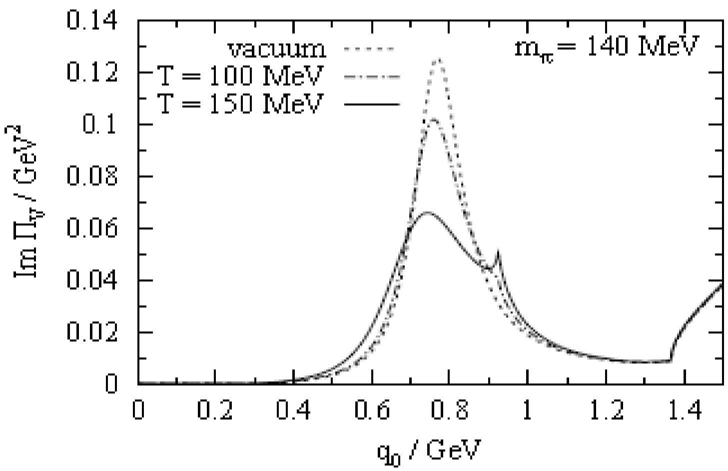
T=150 MeV

T=180 MeV

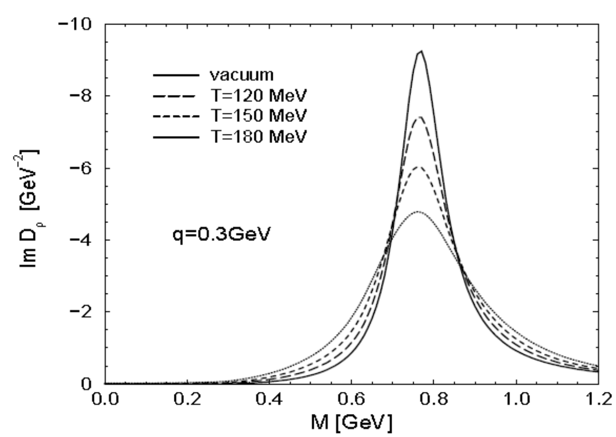
Plus tadpole and a1-π loop



Urban et al 2001



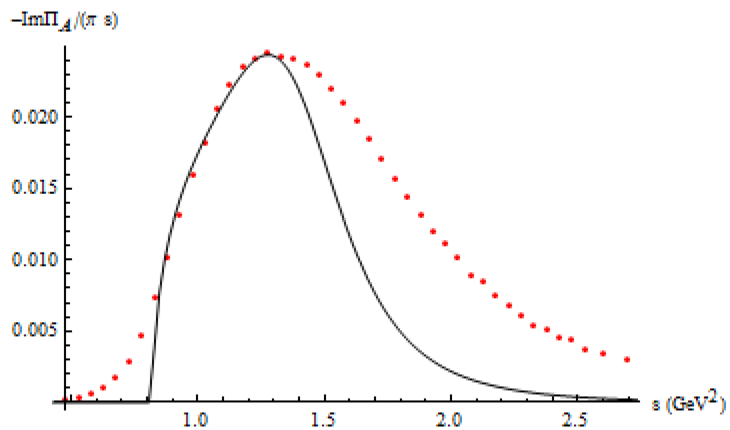
Gale and Rapp 1999



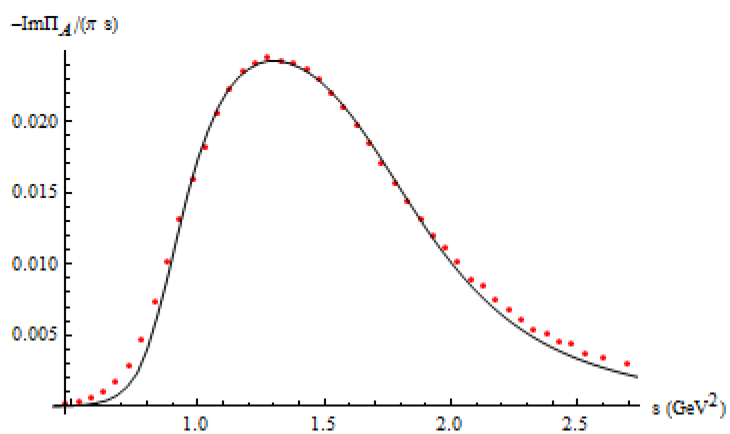
(Includes higher resonances)

V. Axial Vector Channel
VA. Vacuum

- Sharp rho



- Broad rho



Fit parameters: 2 from Lagrangian and 5 counter terms.

•Strong decay at higher energies – not broad enough peak.

•Understood from energy dependence of width.

•PCAC can be verified, but with a renormalized f_π

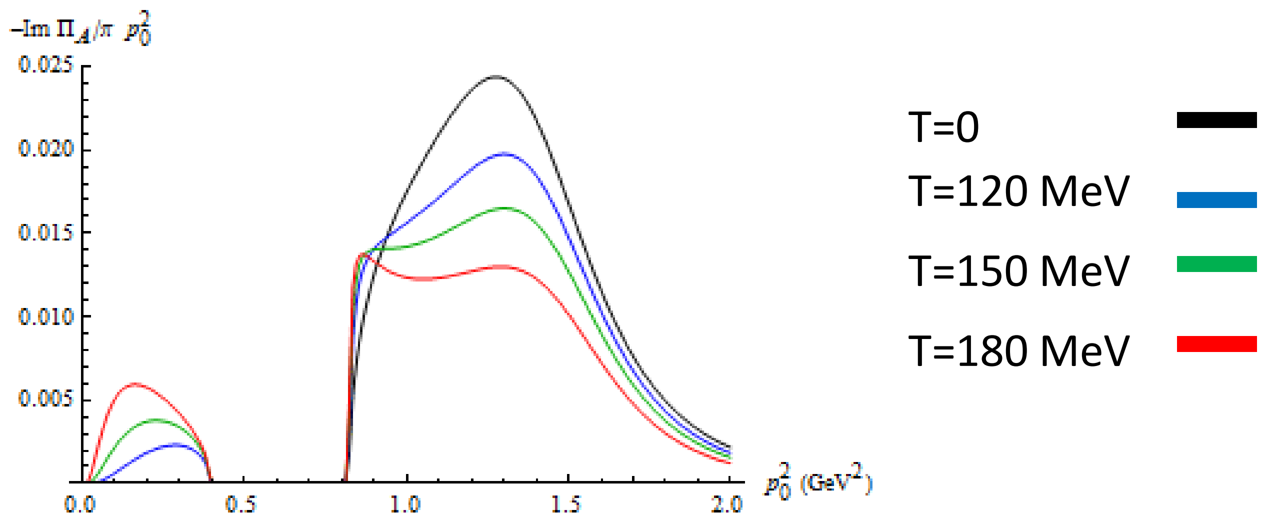
	Calc.	Expt.
$\Gamma(a_1 \to \gamma\pi)$	884 keV	640±246 keV
D/S	-0.108	-0.09±.03

Reasonable agreement with data.

Full chiral properties not yet completely understood.

VB. Finite temperature: Preliminary

Sharp rho scenario in medium



Reduction of a1 peak, formation of low energy peak (a1 + π → ρ)

Need to use a broad rho at finite temperature.

Examine Π_A^L to extract chiral parameters

$$\int ds \Pi_A^L(s) s^{-1} = f_\pi^2$$

$$\int ds \Pi_A^L(s) = f_\pi^2 m_\pi^2$$

VI. Summary

- Explored systems which relate the vector and axial vector meson properties so that dilepton data of ρ can be used to infer chiral symmetry restoration.

- Sum Rules
  - Deduced an excited axial vector state in vacuum.
  - Low T limit of SRs break down T~ m_{π}.
  - Using "realistic" in medium rho SFs, finite T axial vector spectral functions can satisfy SRs, compatible with chiral restoration.

- Massive Yang-Mills
  - Vector is consistent with previous works both in vacuum and finite T
  - Axial vector vacuum fits need a broad rho for best agreement though chiral properties still need to be explored.
  - Finite T axial-vector spectral functions show marked medium effects.
  - Trend towards chiral restoration? -- much more work is needed.