Development of a high precision X-ray spectrometer for diffused sources with HAPG crystals in the range 2-20 keV: the VOXES experiment

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ASTRA: Advances and open problems in low-energy nuclear and hadronic STRAangeness physics

ECT*, Trento, 26/10/2017
The discrepancies between different theoretical models and approaches could be eliminated with ~ 1eV precision measurement.

Best measurements (SIDDHARTA):

<table>
<thead>
<tr>
<th>Target</th>
<th>$\varepsilon$</th>
<th>$\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4$He</td>
<td>$+5 \pm 3 \text{ (stat.)} \pm 4 \text{ (syst.)}$</td>
<td>$14 \pm 8 \text{ (stat.)} \pm 5 \text{ (syst.)}$</td>
</tr>
<tr>
<td>$^3$He</td>
<td>$-2 \pm 2 \text{ (stat.)} \pm 4 \text{ (syst.)}$</td>
<td>$6 \pm 6 \text{ (stat.)} \pm 7 \text{ (syst.)}$</td>
</tr>
</tbody>
</table>

New measurements are needed.
A possible application: the K⁻ mass puzzle

**K⁻ mass puzzle**

**K⁻ mass is a fundamental quantity in physics**

Call for new measurement!!

**Requirements:**
1. **high-resolution** detector
2. K-atom with low-Z **gas** target to reduce the electron screening effect

Needs precision below 0.1 eV!
Project’s goal

- High resolution (few eV) measurements of the X rays (2-20 keV) emitted in various processes is strongly demanded in: particle and nuclear physics, fundamental science, astrophysics, biology, medical and industrial applications.

- Additionally, for some applications (like exotic atoms measurements) X-ray detector systems have to be operated in high background environment.

- These X-rays not always are produced by a point-like source; it is mandatory to develop detectors working with ‘extended’ (diffused) sources.

**VOXES’s goal**: to develop, test and qualify the first prototype of ultra-high resolution and high efficiency X-ray spectrometer in the range of energies 2 - 20 keV using HAPG bent crystals able to work with ‘extended’ sources.
The solid state detectors have intrinsic resolution (FWHM ~ 120 eV at 6 keV) given by the electronic noise and the Fano Factor.

\[ \sigma = \frac{FWHM}{2.35} = \omega \sqrt{\frac{W^2}{N} + \frac{F \times E}{\omega}} \]

Presently, to achieve ~ eV resolution, two options are available:

- Transition Edge Sensors (TES)
- Crystals and position detectors (Bragg spectrometers)
Transition Edge Sensors (TES).

Excellent energy resolution (few eV at 6 keV)

LIMITATIONS:
• not optimised for E < 5 keV
• very small active area
• prohibitively high costs
• rather laborious use (complex cryogenic system needed)

\[ T_C \sim 50 \text{ mK} !!! \]
High resolution can be achieved depending on the quality of the crystal and the dimensions of the detectors.

Geometry of the detector determines also the energy range of the spectrometer.

But....

Crystals response may not be uniform (shape, impurities, ecc.)

In accelerator environments particles may hit the detector

Typical $d$ (Si) $\approx$ 5.5 Å

$\lambda = 2d \sin \theta_B$

Limitation in efficiency

Lineshapes are difficult to be measured within few eV precision (surface scan)

Background reduction capability is mandatory

$\theta_B < 10^\circ$ for $E > 6$ keV (forward & difficult)
Mosaic crystal consist in a large number of nearly perfect small crystallites.

**Mosaicity** makes it possible that even for a fixed incidence angle on the crystal surface, an energetic distribution of photons can be reflected.

Increase of efficiency (focusing) ~ 50

Loss in resolution

Pyrolitic Graphite mosaic crystals (d = 3.354 Å):

Highly Oriented Pyrolitic Graphite (HOPG, Δθ≈1°)

Highly Annealed Pyrolitic Graphite (HAPG, Δθ≈0.07°)

Flexible HAPG has twice higher spectral resolution, while flexible HOPG – approximately twice higher reflectivity.

H. Legall, H. Stiel, I. Grigorieva, A. Antonov et al., FEL Proc. 2006
HAPG: High Annealed Pyrolitic Graphite

- Bending does not influence resolution and intensity
- Mosaic spread down to 0.05 degree
- Integral reflectivity \( \sim 10^2 \) higher than for other crystals
- Variable thickness (efficiency)
- Excellent thermal and radiation stability

Characterization of HAPG mosaic crystals using synchrotron radiation

Martin Gerlach, Lars Anklam, Alexander Antonov, Inna Grigorieva, Ina Holfelder, Birgit Kanningßer, Herbert Legall, Wolfgang Malzer, Christopher Schlesiger and Burkhard Beckhoff


\( \rightarrow \) The integral reflectivity can be more than 50 times higher compared to Si(111) reflection.
\( \rightarrow \) The use of the von Hamos geometry can increase the overall efficiency even more.
Von Hamos configuration

**PRO:**
- Focusing
- Energy range given by the crystal
- Distance from the source (background...)
- Perfect (linear) Bragg spectrum

**CON:**
- Absorption in air
- ‘Point-like’ source needed (low geom. eff.)

\[
\rho = 206.7 \text{ mm} \\
\text{Cu (K}_\alpha_1\text{) = 8048 eV } \rightarrow \theta_B = 13.28^\circ \\
\gamma \text{ path } \approx 180 \text{ cm}
\]
**Johann configuration**

\[ \rho = 206.7 \text{ mm} \]
\[ \text{Cu (K}\alpha_1) = 8048 \text{ eV} \rightarrow \theta_B = 13.28^\circ \]
\[ \gamma \text{ path} \approx 10 \text{ cm} \]

**PRO:**
- Higher efficiency (geometrical, distance from source…)
- No ‘point-like’ source needed
- Less absorption in air

**CON:**
- Non linear Bragg spectrum (unless using curved detectors…..)
- Near to source (background)
- No vertical focusing (partially restore with spherical crystals)
- Energy range fixed by the target
Von Hamos configuration

<table>
<thead>
<tr>
<th>Line (eV)</th>
<th>θ</th>
<th>L1 (mm)</th>
<th>L2 (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu (Kα1)</td>
<td>8047.78</td>
<td>13.28</td>
<td>900.54</td>
</tr>
<tr>
<td>Cu (Kα2)</td>
<td>8027.83</td>
<td>13.31</td>
<td>898.31</td>
</tr>
</tbody>
</table>

ρ = 206.7 mm

Δθ_{min} = 0.03°
## Von Hamos configuration

- **$S_0$** = X spread @ Source
- **$S_{1,2}$** = X spread @ Slits
- **$S_M$** = X spread @ Mythen
- **$\Delta S_M$** = background X spread @ Mythen

- **$d_F$** = ‘point-like’ source Z distance
- **$d_{0,1}$** = Slits Z distance
- **$d_2$** = Mythen Z distance

### Angular Spreads

- $\Delta \theta = $ signal opening angle
- $\Delta \theta_{bkg} = $ background opening angle

### Dimensions

<table>
<thead>
<tr>
<th>$d_0$ (mm)</th>
<th>$d_1$ (mm)</th>
<th>$d_2$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>715</td>
<td>900.54</td>
</tr>
</tbody>
</table>

### Angular Spreads Table

<table>
<thead>
<tr>
<th>$\Delta \theta$</th>
<th>$\tan(\Delta \theta)$</th>
<th>$S_0$ (mm)</th>
<th>$d_F$ (mm)</th>
<th>$S_1$ (mm)</th>
<th>$S_2$ (mm)</th>
<th>$S_M$ (mm)</th>
<th>$\tan(\Delta \theta_{bkg})$</th>
<th>$\Delta S_M$ (mm)</th>
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<tbody>
<tr>
<td>0.1</td>
<td>0.00174533</td>
<td>0.113</td>
<td>1.248</td>
<td>1.572</td>
<td>0.002</td>
<td></td>
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<td></td>
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<tr>
<td>0.2</td>
<td>0.00349067</td>
<td>0.227</td>
<td>2.496</td>
<td>3.143</td>
<td>0.003</td>
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</tr>
<tr>
<td>0.3</td>
<td>0.00523604</td>
<td>0.340</td>
<td>3.744</td>
<td>4.715</td>
<td>0.006</td>
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<td></td>
<td></td>
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<tr>
<td>0.1</td>
<td>0.00174533 0.1</td>
<td>57.30</td>
<td>0.213</td>
<td>1.348</td>
<td>1.672</td>
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<tr>
<td>0.2</td>
<td>0.00349067 0.1</td>
<td>26.65</td>
<td>0.327</td>
<td>2.596</td>
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<td>0.3</td>
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<tr>
<td>0.1</td>
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<td>286.48</td>
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<tr>
<td>0.2</td>
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<td>143.24</td>
<td>0.727</td>
<td>2.996</td>
<td>3.643</td>
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<tr>
<td>0.3</td>
<td>0.00523604 0.5</td>
<td>95.49</td>
<td>0.840</td>
<td>4.244</td>
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<tr>
<td>0.4</td>
<td>0.00698143 0.5</td>
<td>71.62</td>
<td>0.954</td>
<td>5.492</td>
<td>6.767</td>
<td>0.008</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Starting VOXES: test Setup

DECTRIS Mythen2 strip detector
- 450 µm thickness
- 32 x 8 mm² area
- 640 strips (50 µm width)

1.25 µm step positioners (XZ)
(STANDA 8MT167S-25LS)

0.01 µm step positioners (Y)
(STANDA 8MVT40-13)

0.00125° step rotator (θθ)
(STANDA 8MR191-28)

3D-printed ABS Plus box with 45° target prism
OXFORD Instruments XTF 5011 X-ray tube
1 mm Cu solid target

Dectris Ltd
MYTHEN2 detector:
- 32 x 8 mm surface
- 640 channels
- 50 mm resolution
- 4-40 keV range

Working @ room temperature

HAPG:
- Mosaicity 0.07° ↔ 0.1°
- 20, 40, 100 µm thickness
- 3x3, 2 cm² surface

8MUP21-2 - Motorized Optical Mount (STANDA)

< 1 arcsec resolution

- ρ = 206.7 mm
- ρ = 206.7 mm
- ρ = 206.7 mm

ρ = 20 µm
ρ = 40 µm
ρ = 100 µm

INFIN
Istituo Nazionale di Fisica Nucleare
Laboratori Nazionali di Frascati
Location for VOXES development

‘ssemi’ Von Hamos configuration
Increasing dynamic range

Out Of Focusing configuration:

Exact focusing is not important within the detector strip length

\[ S(\theta) = \frac{S(0^\circ)}{\cos \theta} \]
VH vs ‘semi’-VH configuration

counts / 2 chan

0 100 200 300 400 500 600

channels

0 100 200 300 400 500 600

X-ray energy (eV)

“semi-VH”

VH
Best spectrum with Cu Kα lines

XZ opening angle: $\Delta \theta = 0.1^\circ$
Point-like source ($S_0 = 0$)
$\theta_B = 13.28^\circ$ (Kα₁ line = 8047.78 eV)

Cu Kα₁

Cu Kα₂

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Entries</td>
<td>847968</td>
</tr>
<tr>
<td>$\chi^2 / \text{ndf}$</td>
<td>1276.48 / 62</td>
</tr>
<tr>
<td>bkg0</td>
<td>$3267.32 \pm 1300.50$</td>
</tr>
<tr>
<td>bkg1</td>
<td>$-0.13 \pm 0.16$</td>
</tr>
<tr>
<td>A(kα₁)</td>
<td>$26071.28 \pm 98.37$</td>
</tr>
<tr>
<td>E(kα₁)</td>
<td>$8047.78 \pm 0.01$</td>
</tr>
<tr>
<td>$\sigma(kα₁)$</td>
<td>$4.19 \pm 0.01$</td>
</tr>
<tr>
<td>A(kα₂)</td>
<td>$12711.50 \pm 71.34$</td>
</tr>
<tr>
<td>E(kα₂)</td>
<td>$8027.83 \pm 0.02$</td>
</tr>
<tr>
<td>$\sigma(kα₂)$</td>
<td>$4.34 \pm 0.02$</td>
</tr>
</tbody>
</table>
Beam angular spread dependence

Source width = 500 µm
Source width dependence

$\Delta \tan \theta = 0.1^\circ$
Relative (reflection) efficiency estimate

\[ \varepsilon = \frac{N^b_R}{N^d_R} \]

b=brag, d=direct, R=rescaled
Reflection efficiency estimate

\[ \frac{A_{MYTH}}{A_{HAPG}} = \frac{S^M \times 8\text{ mm}}{S^M \times Y^M_{\text{spread}}} = \frac{8\text{ mm}}{Y^M_{\text{spread}}} \]

10.5 mm\(^2 < A_{HAPG} < 35\text{ mm}\(^2\)

\[ \varepsilon = \frac{N^{bragg}}{N^{\text{direct}} \times \text{abs}(Air) \times Y^M_{\text{spread}}} \times \frac{t^d_{\text{daq}} \times 8\text{ mm}}{t^b_{\text{daq}} \times R \left( \frac{K_\alpha}{K_\beta} \right)} \]
Refl. efficiency: Beam angular spread dependence

Source width = 500 μm
Refl. efficiency: Source width dependence

$\Delta \text{tg} \theta = 0.1^\circ$

![Graphs showing efficiency dependence on source width and $x$ spread for different source widths and $x$ spreads.](image-url)
Alloy: **Cu (58%) + Zn(40%) + Pb(2%)**

\[ \Delta \theta = 0.5 \text{ deg}, \text{ target size} = 0 \]

![Graph showing X-ray energy versus counts](image)

<table>
<thead>
<tr>
<th>Entry</th>
<th>Count / 2.371463 eV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25000</td>
</tr>
<tr>
<td></td>
<td>20000</td>
</tr>
<tr>
<td></td>
<td>15000</td>
</tr>
<tr>
<td></td>
<td>10000</td>
</tr>
<tr>
<td></td>
<td>5000</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

**Cu**
- Line (eV): 8905.29
- Line (Å): 1.39
- \( \sin \theta \): 0.21
- \( \theta \): 11.98°
- \( L_1 \) (mm): 996.49
- \( L_2 \) (mm): 974.65

**Zn**
- Line (eV): 8638.86
- Line (Å): 1.44
- \( \sin \theta \): 0.21
- \( \theta \): 12.35°
- \( L_1 \) (mm): 966.68
- \( L_2 \) (mm): 944.16

\[ \Delta \theta_{\text{min}} = 0.37° \]
Quality check measurement @ PSI

πM1 Line
Low momentum $\pi, \mu$ ($\approx 100$ MeV/c)

Prog. Theor. Exp. Phys. 2016, 091D01

$E(4f \rightarrow 3d) = 6428.39 \pm 0.13\text{(stat.)} \pm 0.09\text{(syst.)} \text{ eV},$

$E(4d \rightarrow 3p) = 6435.76 \pm 0.30\text{(stat.)}^{+0.11}_{-0.07}\text{(syst.)} \text{ eV},$
New setup and targets for measurement @ PSI

<table>
<thead>
<tr>
<th>Line (eV)</th>
<th>θ</th>
<th>eV/mm</th>
<th>Range (eV)</th>
<th>eV/mm</th>
<th>Range (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>πC (4-3)</td>
<td>6428,39</td>
<td>16,71</td>
<td>29,79</td>
<td>953,23</td>
<td>39,74</td>
</tr>
<tr>
<td>πC (4-3)</td>
<td>6435,76</td>
<td>16,69</td>
<td>29,83</td>
<td>954,42</td>
<td>39,79</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Element</th>
<th>$K\alpha_1$</th>
<th>$K\alpha_2$</th>
<th>$K\beta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mn</td>
<td>5,898.75</td>
<td>5,887.65</td>
<td>6,490.45</td>
</tr>
<tr>
<td>Fe</td>
<td>6,403.84</td>
<td>6,390.84</td>
<td>7,057.98</td>
</tr>
<tr>
<td>Co</td>
<td>6,930.32</td>
<td>6,915.30</td>
<td>7,649.43</td>
</tr>
</tbody>
</table>

Thanks to Doris Pristauz @ SMI

Possible calibration lines
Impact
(scientific, technological, socioeconomic)

HAPG technology development

Medical Applications (Mammography)

Particle and Nuclear Physics

X-ray spectroscopy (DAΦNE-Luce)

Industry, art and Safety: Elemental Mapping

Foundations: Quantum Mechanics

FAIR (exotic atoms)

JPARC (K-atoms)

PSI (π-atoms)

DAΦNE (K-atoms)

LNGS (PEP)
Next steps…

- Measurement of the pionic atoms transitions at PSI
- Fast & triggerable position detectors → SDD ? Linearly Graded SiPM ? (FBK)
- Parallel measurement of energy & position to improve background reduction

& conclusions

- The VOXES project aims to investigate the possibility to use Bragg spectrometers with diffused sources and in high background environments
- HAPG crystals are ideal candidates for this purpose and can be used in different geometrical configurations (Von Hamos, Johann, ecc…)
- Promising and improvable results have been already obtained, measuring Cu Kα lines with ≈ 10 eV resolution (FWHM) and 0.01 eV precision
- Such a spectrometer may have a strong impact in several fields like nuclear and fundamental physics, medical, elemental mapping, astrophysics

Thank you for the attention
Spare
The main disagreement is between the two most recent and precise results,
\[ m_{K^\pm} = 493.696 \pm 0.007 \text{ MeV} \quad \text{DENISOV 91} \]
\[ m_{K^0} = 493.636 \pm 0.011 \text{ MeV} \quad (S = 1.5) \quad \text{GALL 88} \]
Average \[ m_{K^\pm} = 493.679 \pm 0.006 \text{ MeV} \]
\[ \chi^2 = 21.2 \quad \text{for 1 D.F., Prob.} = 0.0004\% \quad \text{(3)} \]
both of which are measurements of x-ray energies from kaonic atoms. Comparing the average in Eq. (3) with the overall

(6→5) kaonic nitrogen transition: \( 7560 \pm 32 \text{ eV} \),
(7→6) kaonic nitrogen transition: \( 4589 \pm 37 \text{ eV} \).

Exploratory test with DEAR @ DAΦNE

Not yet performed

Calculated efficiency ~ 400 times less than @ DAFNE
Un-efficient background reduction (statistics loss)
Angular spread and attenuation

$$S(\theta) = \frac{S(0^\circ)}{\cos \theta}$$

\[\chi^2 / \text{ndf} \quad 2.70 / 8\]

\[p_0 \quad 1.48 \pm 0.03\]
Angular spread and attenuation
Angular spread and attenuation

\[ S(\theta=78^\circ) = \frac{S(0^\circ)}{\cos(78^\circ)} \]

\[ \chi^2 / \text{ndf} \quad 0.25 / 7 \]

\[ p_0 \quad 4.72 \pm 0.04 \]
Angular spread and attenuation

Protective layer
12 µm Mylar

Incident (VH) angle $\theta \approx 10^\circ$

450 µm thickness (optimized for 6-8 keV)
Angular spread and attenuation

Incident (VH) angle $\theta \approx 10^\circ$
\[ \varepsilon = \frac{N^b_R}{N^d_R} \]

b = brag, d = direct, R = rescaled

\[ N^b_R = N^{bragg} \times \frac{1}{\text{abs}(\text{Air})} \times \frac{1}{\text{abs}(\text{Mylar})} \times \frac{1}{Q E_{MYTH}} \times \frac{1}{t^b_{daq}} \]

\[ N^d_R = N^{direct} \times \frac{A_{HAPG}}{A_{MYTH}} \times \frac{1}{\text{abs}(\text{Mylar})} \times \frac{1}{Q E_{MYTH}} \times \frac{1}{t^d_{daq}} \times R \left( \frac{K_\alpha}{K_\beta} \right) \]

\[ t^d_{daq} = \text{tempo acquisizione diretta} \]

\[ t^b_{daq} = \text{tempo acquisizione brag} \]
\[ N^b_R = N^{bragg} \times \frac{1}{\text{abs}(\text{Air})} \times \frac{1}{\text{abs}(\text{Mylar})} \times \frac{1}{Q E_{\text{MYTH}}} \times -t^\nu_{\text{daq}} \]

\[ N^{bragg} = \int K_{\alpha 1} + \int K_{\alpha 2} \]

\[ \sigma N^{bragg} = \text{IntegralError}(\text{ROOT}) \]

\[ \text{abs}(\text{Air}) = 0.362 \pm 0.004 \pm 1 \text{ cm path} \]
\[
\begin{align*}
\delta \tan \phi (S_2, S_2^z) &= \tan \phi \sqrt{\left( \frac{\delta S_2}{S_2 - \Delta Y} \right)^2 + \left( \frac{\delta S_2^z}{S_2^z - 50 \text{ mm}} \right)^2} \\
\tan \phi &= \frac{S_2 - \Delta Y}{S_2^z - 50 \text{ mm}} \\
Y_{\text{spread}}^i &= (S_i^z - S_2^z) \tan \phi + S_2 \\
S_M &= \frac{S_i}{S_i^z} \times S_M^z
\end{align*}
\]
\[ N_R^d = N_{\text{direct}} \times \frac{A_{\text{HAPG}}}{A_{\text{MYTH}}} \times \frac{1}{\text{abs(Mylar)}} \times \frac{1}{QE_{\text{MYTH}}} \times \frac{1}{t_{\text{daq}}^d} \times R\left(\frac{K_{\alpha}}{K_{\beta}}\right) \]

\[ N_{\text{direct}} = \text{Numero eventi a } \theta = 0 \quad R\left(\frac{K_{\alpha}}{K_{\beta}}\right) = 0.8988 \quad (K_{\alpha2} = 0.51K_{\alpha1} \& K_{\beta} = 0.17K_{\alpha1}) \]

\[ \sigma N_{\text{direct}} = \text{Propagazione errori (} \sqrt{N} \text{)} \]

\[ \frac{A_{\text{MYTH}}}{A_{\text{HAPG}}} = \frac{S^M \times 8 \text{ mm}}{S^M \times Y^M_{\text{spread}}} = \frac{8 \text{ mm}}{Y^M_{\text{spread}}} \]

\[ \varepsilon = \frac{N^{\text{bragg}}}{N_{\text{direct}} \times \text{abs(Air)} \times Y^M_{\text{spread}}} \times \frac{t_{\text{daq}}^d \times 8 \text{ mm}}{t_{\text{daq}}^b} \times R\left(\frac{K_{\alpha}}{K_{\beta}}\right) \]

10.5 mm\(^2\) < \( A_{\text{HAPG}} \) < 35 mm\(^2\)
Direct measurement ($t_{\text{daq}} = 30 \text{ m}$)
Bragg measurement (t = variable)

\[ \Delta t \theta = 0.1 \]

100 \( \mu \)m HAPG
40 \( \mu \)m HAPG
20 \( \mu \)m HAPG
Bragg measurement ($t = \text{variable}$)

$\Delta t g \theta = 0.2$

100 µm HAPG
40 µm HAPG
20 µm HAPG
Bragg measurement \((t_{\text{daq}} = \text{variable})\)

\[ \Delta t \theta = 0.3 \]

100 \(\mu\text{m}\) HAPG

40 \(\mu\text{m}\) HAPG

20 \(\mu\text{m}\) HAPG
Efficiency evaluation

\[ \Delta \text{tg} \theta = 0.1 \]

100 µm HAPG
40 µm HAPG
20 µm HAPG

10.5 mm² < A_{HAPG} < 35 mm²
Efficiency evaluation

$\Delta t g \theta = 0.2$

100 μm HAPG
40 μm HAPG
20 μm HAPG
Efficiency evaluation

\[ \Delta \tan \theta = 0.3 \]

Graphs showing the efficiency for different materials:
- 100 µm HAPG
- 40 µm HAPG
- 20 µm HAPG