The Higgs particle in condensed matter

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D. Sherman et. al., Nature Physics (2015)
Outline

- Brief history of the Anderson-Higgs mechanism
- The vacuum is a condensate
- Emergent relativity in condensed matter
- Is the Higgs mode overdamped in d=2?
- Higgs near quantum criticality

Experimental detection:
Charge density waves
Cold atoms in an optical lattice
Quantum Antiferromagnets
Superconducting films
1955: T.D. Lee and C.N. Yang - massless gauge bosons

1960-61 Nambu, Goldstone: massless bosons in spontaneously broken symmetry

Where are the massless particles?

1962

Gauge Invariance and Mass
JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts, and University of California, Los Angeles, California
(Received July 20, 1961)

It is argued that the gauge invariance of a vector field does not necessarily imply zero mass for an associated particle if the current vector coupling is sufficiently strong. This situation may permit a deeper understanding of nucleonic charge conservation as a manifestation of a gauge invariance, without the obvious conflict with experience that a massless particle entails.

1963

Plasmons, Gauge Invariance, and Mass
P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 8 November 1962)

The vacuum is not empty: it is stiff.
like a metal or a charged Bose condensate!
Rewind to 1911

Discovery of Superconductivity
1911

$R = 0$ !

$T_c = 4.2\text{K}$

Lord Kelvin
Mathiessen
mercury

Kamerlingh Onnes
Meissner Effect, 1933

Phil Anderson

Meissner effect ->
1. Wave function rigidity
2. Photons get massive
Symmetry breaking in $O(N)$ theory

$N-$component real scalar field: \[ \phi^t = (\phi_1, \ldots, \phi_N) \]

“Mexican hat” potential: \[ V(\phi) = \frac{m_0^2}{8N} (\phi^2 - N)^2 \]

Spontaneous symmetry breaking

N-1 Goldstone modes (spin waves)

1 Higgs (amplitude) mode

ORDERED GROUND STATE

Dan Arovas, Princeton 1981
Dynamics of bosons

Galilean Gross Pitaevskii bosons (BEC) \( \psi \approx \sqrt{\rho} e^{i\varphi} \)

\[ \mathcal{L} = i\psi^* \dot{\psi} - |\nabla \psi|^2 - g(|\psi|^2 - \bar{n})^2 = \rho \dot{\varphi} - |\nabla \psi|^2 - g(\rho - \bar{n})^2 \rightarrow 1 \text{ massless phase-density phonon} \]

\( \rightarrow \) NO Amplitude-Higgs mode

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Relativistic O(2) theory \( \psi = r(x) e^{i\varphi(x)} \)

\[ \mathcal{L}^{O(2)} = |\dot{\psi}|^2 - |\nabla \psi|^2 - \mu |\psi|^2 + g |\psi|^4 - i\alpha \psi^* \dot{\psi} \]

\( \rightarrow 1 \) massless phase mode

\( \rightarrow 1 \) Amplitude-Higgs mode

Both modes survive weak p-h symmetry breaking
Relativistic Dynamics in Lattice bosons

Bose Hubbard Model
\[ \mathcal{H} = -t \sum_{ij} a_i^\dagger a_j + U \sum n_i^2 - \mu \sum n_i \]

Large \( t/U \): system is a superfluid, (Bose condensate).

Small \( t/U \): system is a Mott insulator, (gap for charge fluctuations).

Mott insulators
incompressible
\[ \langle \psi^\dagger \rangle = 0 \]
\[ \langle n_i^2 \rangle \approx (\langle n_i \rangle)^2 \]

Relativistic Gross Pitaevskii
\[ \mathcal{L} = |\partial_t \psi|^2 - c^2 |\nabla \psi|^2 + r|\psi|^2 - u|\psi|^4 \]
Relativistic Dynamics O(2) model

\[ \mathcal{L} = |\partial_t \psi|^2 - c^2 \nabla |\psi|^2 + r |\psi|^2 - u |\psi|^4 \]

\[ \langle \psi \rangle = 0 \]

\[ \langle \psi \rangle = \Delta e^{i \phi} \]

Collective excitations

Charge excitation gap

Higgs-Amplitude mode
\[ (\partial_t^2 - \nabla^2 - m^2) \delta |\psi| = 0 \]

Phase mode (plasmon)
\[ (\partial_t^2 - \nabla^2) \delta |\phi| = 0 \]

Strong interactions
Boson
Mott insulator

Weak interactions

\[ \frac{t}{U} \]

3D O(2) Quantum critical point

Classical d+1 model at "temperature" U/t

"Relativistic Superfluid"
Higgs - Goldstones coupling

\[ S[\phi] = \frac{1}{g} \int d^d x \int dt \left[ \frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right] \]

Fluctuations in the ordered state:

\[ \phi = (\sqrt{N} + \sigma, \pi) \]

Harmonic theory

\[ \mathcal{L}_0 = \frac{1}{2g} \left[ (\partial_\mu \sigma)^2 - m^2 \sigma^2 + (\partial_\mu \pi)^2 \right] \]

Interactions

\[ \mathcal{L}_1 = \frac{m^2}{2g} \left[ \frac{1}{\sqrt{N}} \sigma \pi^2 + \frac{1}{\sqrt{N}} \sigma^3 + \frac{1}{4N} \sigma^4 + \frac{2}{N} \sigma^2 \pi^2 + \frac{1}{4N} (\pi^2)^2 \right] \]

Higgs coupling to 2 Goldstones

Is the Higgs mode over damped in d=2?
Visibility of the amplitude (Higgs) mode in condensed matter

Daniel Podolsky, 1 Assa Auerbach, 1,2 and Daniel P. Arovas 3

\[
S = \frac{1}{2g} \int_{\Lambda} d^{d+1}x \left[ (\partial_\mu \Phi)^2 + \frac{m_0^2}{4N}(|\Phi|^2 - N)^2 \right]
\]

Quantum Critical point
Landau theory

Sporadically broken symmetry Quantum Critical point Symmetric phase
(quantum disordered)

\[ g < g_c \]
\[ g = g_c \]
\[ g > g_c \]
The Higgs decay

The Higgs mode can decay into a pair of Goldstone bosons:

In neutral systems, and for 2D superconductors, the Goldstones are massless.

\[ d=3 \text{ Higgs decay rate is bounded even at strong coupling} \]

\[ d=2 \text{ self-energy diverges at low frequency, even at weak coupling:} \]

\[ \Sigma_\sigma(k) = \frac{k}{\sigma} \quad \pi \quad \frac{1}{p} \quad \sigma \quad \pi \]

\[ \approx \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2(p+k)^2} = \frac{1}{8|k|} \]

\[ \text{Im} \Sigma(\omega) \propto \frac{1}{|\omega|} \quad \text{infrared divergent!} \]

(Nepomnyashchii)\(^2\) (1978)  
Behavior of different dynamical correlation functions

order parameter susceptibility

\[ \chi_{11}(\omega) = \langle \psi_1(\omega)\psi_1(-\omega) \rangle \sim \omega^{-1} \]

infrared divergent in d=2

scalar susceptibility

\[ \chi_{\rho\rho}(\omega) = \langle |\vec{\psi}|^2(\omega)|\vec{\psi}|^2(-\omega) \rangle \sim \omega^3 \]

infrared regular in d=2

Higgs peak in scalar response is well defined!
vector vs scalar dynamical correlations

Longitudinal versus radial perturbations:

Radial motion is less damped, since it is not effected by azimuthal meandering.

What happens to the spectral function near the quantum critical point?
Numerical simulations

Gazit Podolsky Auerbach PRL (2013), Gazit, Podolsky, AA, Arovas (in preparation)

Conclusion: Higgs peak is visible close to criticality in $d=2$
Apr 26, 2013
Birth of a Higgs boson
Results from ATLAS and CMS now provide enough evidence to identify the new particle of 2012 as ‘a

Narrow Higgs peak —> vacuum is far from criticality

Higgs mass
Experimental detection:

Charge density waves (coupled 1 dimensions)

Quantum Antiferromagnets (3 dimensions)

Cold atoms in an optical lattice (2 dimensions)

Superconducting films (2 dimensions)
CDW systems \((\text{TbTe}_3, \text{DyTe}_3, 2\text{H-TaSe}_2)\):

Magnetic systems in 3 dimensions

Heisenberg antiferromagnet

\[ H = \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j \]

neutron scattering, Headings 2010

Raman scattering, Lyons, 1988

O(3) Relativistic non linear sigma model

Spin waves = Goldstone modes

2 Magnon = Higgs mode

La\(_2\)CuO\(_4\)

TlCuCl\(_3\)

spin waves, Higgs
Cold atoms in _optical lattices_

**Superfluid**

$g > g_c$

**Mott insulator**

$g < g_c$

Sponately broken symmetry

Quantum phase transition

**Relativistic Gross Pitaevskii**

$$\mathcal{L} = |\partial_t \psi|^2 - c^2 |\nabla \psi|^2 + r |\psi|^2 - u |\psi|^4$$

Greiner et. al. Nature 2001
Higgs near criticality: $^{87}\text{Rb}$


- Energy absorption rate of periodically modulated lattice $\propto \omega \chi''_{\rho \rho}(\omega)$
Josephson Junction array (no disorder)

\[ \Delta \]

\[ \Delta \]

Broken pair

\[ E_c, E_J < 2\Delta \text{ Bosonic limit} \]

\[ H = E_c \sum (n_i - \bar{n})^2 - E_J \sum \cos(\phi_i - \phi_i) \]

\[ \mathcal{L} = |\partial_t \psi|^2 - c^2 |\nabla \psi|^2 + r|\psi|^2 - u|\psi|^4 \]

Relativistic Gross Pitaevskii
Superconductor to Insulator transition in thin films

Partial list:
- Haviland et. al. PRL (1989)
- Hebard Palaanen PRL (1990)
- Yazdani & Kapitulnik (PRL 1995) MoGe
- Sambandamurthy et. al. PRL (2004) InO
- Baturina et. al. PRL 99 (2007)
- M. Chand et. Al, PRB (2009)

Theory:
- Finkelstein, Feigelman, Vinokur, Larkin, Ioffe, Trivedi, Randeria, Ghosal, Shimshoni, AA, Meir, Dubi, Michaeli

\(O(2)\) Quantum phase transition at \(T=0\)
Collective modes would prove quantum criticality
We find \( \sigma(\omega) = A \delta(\omega) + \tilde{\sigma}(\omega) \) with \( A = N e^2 g^{-1} + \mathcal{O}(g^0) \).

The finite frequency part is computed from

\[
\hat{K}^p_{\mu\nu}(k) = \frac{1}{(N-1)g^2} \int d^{d+1}x \, e^{ik \cdot (x-x')} \left\langle \left( \sigma \partial_\mu \pi - \pi \partial_\mu \sigma \right)_x \cdot \left( \sigma \partial_\nu \pi - \pi \partial_\nu \sigma \right)_{x'} \right\rangle
\]

We evaluate this perturbatively in the coupling \( g \). Conductivity diagrams:

\[
\mathcal{O}(g^0) : \quad \sigma \quad \pi
\]

\[
\mathcal{O}(g^1) : \quad +
\]

To lowest order,

\[
\tilde{\sigma}_0(\omega) = \frac{\pi S_d e^2}{d\omega^2} \left( \frac{\omega^2 - m^2}{4\pi \omega} \right)^d \Theta(\omega^2 - m^2)
\]

This yields a threshold at the Higgs mass, with

\[
\sigma(\omega) \propto (\omega - m)^d
\]

**Does this change qualitatively at higher orders?**
Sherman et al. THZ spectroscopy

**O(N) model**

Cartesian vs. polar

\[ \phi = \begin{cases} 
(r \sqrt{N} + \sigma, \pi) \\
(r \sqrt{N} (1 + \rho) \hat{n}
\end{cases} \]

**Summary**

- Higgs decays into Goldstone bosons
- \(\phi\): long range ordered \(g_c\), quantum disordered \(g \rightarrow\)
- \(\chi''_{\rho \rho}\) is sharper than longitudinal \(\chi''_{\sigma \sigma}\)

**Conductivity pseudogap**

- \(\chi''_{\sigma \sigma}\)
- \(\chi''_{\rho \rho}\)

Graphs:
- \(d = 2\)
- \(\omega/m\)
- \(16 \pi^{d-2} \sigma_0(\omega)/e_m t^d\)
- \(d = 3\)
- \(d = 2\)
Summary

A Higgs (amplitude) mode in condensed matter appears when

1. The dynamics are relativistic. (granular superconductors, cold atoms on optical lattices, antiferromagnets).

2. Near a quantum critical point, when the Higgs mass is low.

In two dimensional superconductors, the Higgs mode is visible as a threshold for the AC conductivity.

At criticality, the Higgs spectral function scales as a universal peak.