Predicting the Axion Mass with large String Tension

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Motivation

Axion Cosmology

Axionic String Networks

Results
Why are we investigating the Axion

- The **Axion** can solve the strong CP problem
- **Dark Matter** is still a mystery

We know so far that it is

- **matter**: makes up 25% of the density of the Universe
- **dark**: interaction is feeble (except gravitationally)
- **cold**: almost pressureless

The **Axion** is also a possible **Dark Matter** candidate
Axion Cosmology

Lagrangian

\[ \mathcal{L}_a = \partial^\mu \phi^* \partial_\mu \phi + \frac{\lambda}{8} (2\phi^* \phi - f_a^2)^2 + \chi(T) \text{Re}\phi \]

- \( f_a \) is unknown
- \( \chi(T) \) strong function of \( T \)
  - \( \chi(T) \approx (76 \text{MeV})^4 \) if \( T \ll T_C \)
  - \( \chi(T) \propto T^{-8} \) if \( T \gg T_C \), with much larger error

Cortona et al, arXiv:1511.02867

Axion Cosmology

Evolution

Inflationary epoch

After inflation

Before inflation

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Axion Cosmology

Evolution

Inflationary epoch

Before inflation

After inflation

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Axion Cosmology

Evolution

Inflationary epoch

Before inflation

$T_{PQ}$ $T_{QCD}$ present

distance

time

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Axion Cosmology
Solving on Lattice

- Put $\mathcal{L}_a$ as classical field theory on lattice
- Simulation starts after Inflation
- $\phi$ is a complex field
- $\theta$ is chosen randomly
- Topological defects will arise
- Simulate with Hubble drag
- Count axions at the end
Axion Cosmology

String

String arises when the spontaneous symmetry gets broken

- $\phi$ is a complex number
- $\theta$ can take values between $[-\pi, \pi]$.
- $\theta$ is undefined in the center of the string.
Axion Cosmology

String

- 2D slice of an Axion field
- Field generically has **vortices (strings)**
Axion Cosmology

String

- Circling around the string, $\theta$ varies by $\pm 2\pi$
- In the center of the string $\theta$ is undefined $\rightarrow \phi = 0$
Domain walls arise when the potential tilts

- Each string is connected to a domain wall
- Domain walls can appear without strings
- Change of $\theta$ happens in domain wall
- Domain walls pull strings
Axion Cosmology

Domain Wall

Evolution of the Axion field, when the potential tilts
Axion Cosmology
String Network

Evolution of a string Network
Axion Cosmology
String Energy

\[ E_{\text{str}} = \int \int \int r \, dr \, dz \, d\phi \left( \nabla \phi^* \nabla \phi \simeq \frac{f_a^2}{2r^2} \right) \simeq \pi f_a^2 \int_{f_a^{-1}}^{H^{-1}} \frac{r \, dr}{r^2} \]

- **Log-large** string tension \( T_{\text{str}} = \pi f_a^2 \ln(f_a/H) \)
- \( f_a/H \simeq 10^{30} \)
- Equal energy in each x2 scale
- Scale range is \( 10^{30} \rightarrow \kappa = \ln(10^{30}) \simeq 70 \), **achievable is** \( \kappa = 6 \)
Should we care that $\kappa = 6$ and not $\kappa = 70$

Things which scale as $\kappa$

- String tension $T_{str} \sim \pi \kappa f_a^2$
- Energy in network $E \sim L_{\text{network}} \pi \kappa f_a^2$

Things which do not scale with $\kappa$

- String-string interaction $dF/dl \sim f_a^2/r_{\text{sep}}$
- Power radiated from strings $dE/dldt \sim f_a^2 R_{\text{curv}}^{-1}$

Small $\kappa$ leads to **wrong physics**
What must an effective model contain

- **Long-range** (light) degree of freedom to be the axion
- **Thin cores** with high tension $T \sim 70\pi f_a^2$
- Correct **string-field interactions**

Any modified **high-mass physics** which does this is OK
Axionic String Networks

Abelian Higgs

\[ \mathcal{L}(\phi, A_\mu) = \frac{1}{4} (\partial_\mu u A_\nu - \partial_\nu A_\mu)^2 + (D_\mu \phi)^\ast (D^\mu \phi) + \frac{\lambda}{8} (2\phi^\ast \phi - f_a^2)^2 \]

with the covariant derivative \( D_\mu = \partial_\mu - ieA_\mu \)

- Magnetic flux centered on string
- Outside the core \( D\phi \rightarrow 0 \)

Finite tension \( T \sim \pi f_a^2 \), no long-range interactions
Axionic String Networks
Hybrid Theory

Using two complex scalars and $A_\mu$

$$
\mathcal{L}(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2
+ \frac{\lambda}{8} \left[ (2\phi_1^* \phi_1 - f^2) + (2\phi_2^* \phi_2 - f^2) \right]
+ |(\partial_\mu - iq_1 eA_\mu)\phi_1|^2 + |(\partial_\mu - iq_2 eA_\mu)\phi_2|^2
$$

- Pick $q_1 \neq q_2$
- Two rotation symmetries $\phi_1 \rightarrow e^{i\theta_1} \phi_1$ and $\phi_2 \rightarrow e^{i\theta_2} \phi_2$
- $q_1\theta_1 + q_2\theta_2$ gauged, $q_2\theta_1 - q_1\theta_2$ global
Axionic String Networks
Global String, Local Core

- B-Field almost compensates gradients outside string
  \[ f_a^2 = f^2 / (q_1^2 + q_2^2) \]

- Tension \( T \simeq 2\pi f^2 \)
- string-string interaction \( \frac{dF}{dl} = \frac{f^2}{(q_1^2 + q_2^2)r} \)
- \( \kappa_{\text{eff}} = 2(q_1^2 + q_2^2) \)
- \( \kappa_{\text{eff}} \) has no log-scale
Axionic String Networks
Different String Tensions
Results
Axion Production Rate

Axion production changes **only a little**

Axion Production vs String Tension

- Efficiency $n_{ax}/n_{misalign}$
- $\kappa$ (string tension)
- $(q_1, q_2)$:
  - $(2,1)$
  - $(3,2)$
  - $(4,3)$
- Physical range
- $m_t = 300$ ma = 1.0
- $k_{start} = 24$ $m_{t_{start}} = 80$
- $1536^3$ box: $L_t = 5.12$
Results
Calculating the mass

Using

- \( n_{ax}(T = T_*) = KH(T_*) f_a^2 \)
- \( K \) is a constant and determines the produced density
  - \( K = 13.0 \pm 2.0 \) lattice result
  - \( K = 16.61 \) for Misalignment

\[
\frac{\rho_{dm}}{s} = \frac{n_{ax}(T=T_*) m_a(T=0)}{s}
\]


We predict

- \( f_a = (2.21 \pm 0.29) \times 10^{11} \) GeV
- \( m_a = 26.2 \pm 3.4 \mu\text{eV} \)
Results

- 10x string tension leads to 3x denser networks
- Only 40% more axions than with axion-only simulation
- Energy in "walls" is part of misalignment energy
- Walls get "eaten" by strings → energy lost for axions
- Strings **bad at making axions**
Axion can explain the **CP-problem** and the **dark matter**

In early Universe the Axion dynamics are **string defects**

With the **two-field-model** we get the string defect physics right

We find **Axion mass** $m_a = 26.2 \pm 3.4\mu eV$
  - Assuming Axions make all DM
  - $\theta$ is chosen randomly
Results
String Density

\[ \xi = \frac{\nu L_{str}}{V} \]

- Value of different string tensions

Abelian
\((q_1, q_2) = (4, 3)\)
\((q_1, q_2) = (3, 2)\)
\((q_1, q_2) = (2, 1)\)

Pure Global

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Results

Starting Density

\[
\begin{align*}
(q_1, q_2) &= 4, 3  \quad m_t = 400 \\
L_t &= 5.12  \quad n = 7 \\
\xi_{\text{init}} &= \xi(m_t = 128) \\
\xi_{\text{late}} &= \xi(m_t = 1024)
\end{align*}
\]
Results

Volume Dependence

Volume dependence of axion production

\[(q_1, q_2) = 4, 3 \quad m_t = 200 \quad m_a = 1.0\]

\[k_{\text{start}} = 24, \quad m_{t_{\text{start}}} = 100\]