Emergent BCS regime of 2D fermionic Hubbard model: ground-state phase diagram

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\[ H = -t \sum_{\langle ij \rangle} a_{\sigma_i}^+ a_{\sigma_j} + U \sum_i n_{\uparrow i} n_{\downarrow i}, \quad n_{\sigma_i} = a_{\sigma_i}^+ a_{\sigma_i} \]
Cooper instability via linear response

Modify the Hamiltonian:

\[ H \rightarrow H + \left( \eta_{12}^* \psi_1 \psi_2^* + \text{H.c.} \right) \]

Study linear response

\[ \langle \psi_1 \psi_2 \rangle \]

infinitesimally small

Diagrammatically:

\[ \langle \psi_1 \psi_2 \rangle = G_{13} G_{24} \eta_{34} + \]

irreducible
(in the Cooper channel)
four-pole vertex
Singular response: eigenvector-eigenvalue problem

Response diverges when the following eigenvalue becomes equal to 1.

Corresponding eigenvector defines the pairing channel.
The four-pole vertex $T$ is small and temperature independent.

Green’s function has a Fermi-liquid form (close to the Fermi surface):

$$G(k, \xi) \approx \frac{z(\hat{k})}{i\xi - v_F(\hat{k}) \cdot [k - k_F(\hat{k})]}$$

Temperature dependence of the eigenvalue is due to the Green’s function factor:

$$\lambda(T) = g \ln \left( \# E_F / T \right) \Rightarrow \quad T_c = \# E_F e^{-1/g}$$

$g \ll 1$
Ladder summation trick

\[ \Gamma_{12} = -U \delta(\tau_1 - \tau_2) \]

\[ = \Gamma + \Pi_{13} \Gamma_{32} \]

\[ \Gamma(\tau, k) = -U \delta(\tau) + \tilde{\Gamma}(\tau, k) \]

The two terms substantially compensate each other, but only in the integral sense.

Introduce new object:

\[ \int_0^\beta \tilde{\Gamma}_U(\tau) d\tau = -U \]

Now we can combine the two:

\[ A_{1234} = \tilde{\Gamma}_U(\tau_1 - \tau_2) G_{\uparrow}(\tau_1 - \tau_3) G_{\downarrow}(\tau_1 - \tau_4) + \tilde{\Gamma}(\tau_1 - \tau_2) G_{\uparrow}(\tau_2 - \tau_3) G_{\downarrow}(\tau_2 - \tau_4) \]
The eigenvector-eigenvalue problem in 2D

Momenta live on the Fermi surface.

Frequencies approach zero.

Fermi surface is parameterized in terms of polar angle $\theta$.

$$\int_0^{2\pi} \tilde{T}_{\theta,\theta'} \phi_{\theta'} \frac{d\theta'}{2\pi} = g\phi_{\theta}$$

$$\tilde{T}_{\theta,\theta'} = Q_{\theta}^2 T_{\theta,\theta'} Q_{\theta'}^2$$

$$Q_{\theta} = \frac{k_F(\theta) z^2(\theta)}{2\pi \hat{\theta} \cdot v_F(\theta)}$$
$D_{4h}$ nomenclature for the square lattice

\[ f_s(\theta) = \sum_{m=0}^{\infty} A_m \cos(4m\theta) \]

\[ f_g(\theta) = \sum_{m=1}^{\infty} B_m \sin(4m\theta) \]

\[
\begin{align*}
\left\{ \begin{array}{c}
p_y \\ p_x \\
\end{array} \right\} (\theta) &= \sum_{m=0}^{\infty} C_m \left\{ \begin{array}{c}
\cos[(2m+1)\theta] \\
\sin[(2m+1)\theta] \\
\end{array} \right\}
\end{align*}
\]

\[
\begin{align*}
\left\{ \begin{array}{c}
d_{x^2-y^2} \\ d_{xy} \\
\end{array} \right\} (\theta) &= \sum_{m=0}^{\infty} \left\{ \begin{array}{c}
D_m \cos[(4m+2)\theta] \\
E_m \sin[(4m+2)\theta] \\
\end{array} \right\}
\end{align*}
\]
Weakness of BCS coupling

\[ T_c = \# E_F e^{-1/g} \]
Nonexistence of the Luttinger-Ward Functional and Misleading Convergence of Skeleton Diagrammatic Series for Hubbard-Like Models

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Shifted-action expansion: controlled dressed diagrammatic schemes

R. Rossi, F. Werner, N. Prokof'ev, and BS, 2015
Partial dressing

original action

\[ S[\psi] = \langle \psi \mid G_0^{-1} \mid \psi \rangle + S_{\text{int}}[\psi] \]

auxiliary action

\[ S_{\xi}^{(N)}[\psi] = \langle \psi \mid \tilde{G}_N^{-1} + \xi \Lambda_1 + \ldots + \xi^N \Lambda_N \mid \psi \rangle + \xi S_{\text{int}}[\psi] \]

\[ \tilde{G}_N^{-1} + \Lambda_1 + \Lambda_2 + \ldots + \Lambda_N = G_0^{-1} \quad \Rightarrow \quad S_{\xi=1}^{(N)} = S \]

self-energies of corresponding orders, playing the role of counter-terms

equivalence to the original action
Full dressing:
sufficient condition for converging to correct answer

(i) The sequence  $\tilde{G}_N$ converges and is uniformly bounded.

(ii) The sequence

$$\xi \Lambda_1[\tilde{G}_N] + \xi^2 \Lambda_2[\tilde{G}_N] + \ldots + \xi^N \Lambda_N[\tilde{G}_N]$$

converges and is uniformly bounded within a circle $|\xi| < \xi_0$, where $\xi_0 > 1$. 