A novel class of equation of state for neutron star interiors with deconfinement

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1. Introduction: The “three-window picture” of dense matter
2. String-flip model density functional for quark matter
4. Application to compact stars: high-mass twin stars!


“New Perspectives on Neutron Star Interiors”, ECT* Trento, 12.10.2017
CEP in the QCD phase diagram: HIC vs. Astrophysics

“Three-window picture” of dense matter
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Hadron-Quark Crossover and Massive Hybrid Stars

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On the basis of the percolation picture from the hadronic phase with hyperons to the quark phase with strangeness, we construct a new equation of state (EOS) with the pressure interpolated as a function of the baryon density. The maximum mass of neutron stars can exceed $2M_\odot$ if the following two conditions are satisfied; (i) the crossover from the hadronic matter to the quark matter takes place at around three times the normal nuclear matter density, and (ii) the quark matter is strongly interacting in the crossover region. This is in contrast to the conventional approach assuming the first order phase transition in which the EOS becomes always soft due to the presence of the quark matter at high density. Although the choice of the hadronic EOS does not affect the above conclusion on the maximum mass, the three-body force among nucleons and hyperons plays an essential role for the onset of the hyperon mixing and the cooling of neutron stars.

Subject Index: Neutron stars, Nuclear matter aspects in nuclear astrophysics, Hadrons and quarks in nuclear matter, Quark matter
2nd attempt: interpolation between energy densities $\varepsilon(\rho)$

Masuda, Hatsuda, Takatsuka, PTP 073D01 (2013); [arxiv:1212.6803v2]
2\textsuperscript{nd} attempt: interpolation between energy densities $\varepsilon(\rho)$

NOTE: After a strong stiffening one observes the “dip” in the speed of sound which is typical for a phase transition and corresponds to the “plateau” in $P(\rho)$
2nd attempt: interpolation between energy densities $\varepsilon(\rho)$

NOTE: This interpolation procedure in $\varepsilon(\rho)$ is not only thermodynamically consistent, but also a true interpolation, as can be seen from $P(\mu)$ or its inversion $\mu(P)$.

Courtesy: Matthias Hempel, using data from arxiv:1212.6803v2

For hybrid star EoS with interpolation in $P(\mu)$, see arxiv:1302.6275; arxiv:1310.3803
2nd attempt: interpolation between energy densities $\varepsilon(\rho)$

Attention:
Results with interpolation between energy densities $\varepsilon(\rho)$ are different from those with interpolation in pressures $P(\rho)$
Which one is correct? ...
Another “three-window picture of dense matter”
Another “three-window picture of dense matter”
Another “three-window picture of dense matter”
Another “three-window picture of dense matter”
Pauli blocking among baryons

a) Low density: Fermi gas of nucleons (baryons)

b) \( \sim \) saturation: Quark exchange interaction and Pauli blocking among nucleons (baryons)

c) high density: Quark cluster matter (string-flip model ...)


Nucleon (baryon) self-energy --> Energy shift

\[
\Delta E_{\nu P}^{\text{Pauli}} = \sum_{123} |\psi_{\nu P}(123)|^2 [E(1) + E(2) + E(3) - E_{\nu P}^0] [f_{\alpha_1}(1) + f_{\alpha_2}(2) + f_{\alpha_3}(3)] \\
+ \sum_{123} \sum_{456} \sum_{\nu \nu'} \psi_{\nu P}(123) \psi_{\nu' P'}(456) [\delta_{\nu \nu} \psi_{\nu P}(123) \psi_{\nu' P'}(456) - \psi_{\nu P}(453) \psi_{\nu' P'}(126)] \\
\times [E(1) + E(2) + E(3) + E(4) + E(5) + E(6) - E_{\nu P}^0 - E_{\nu' P'}^0] \\
= \Delta E_{\nu P}^{\text{Pauli, free}} + \Delta E_{\nu P}^{\text{Pauli, bound}}
\]
Relativistic density functional approach to quark matter - string-flip model (SFM)

Pauli quenching effects in a simple string model of quark/nuclear matter

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and The Niels Bohr Institute, 2100 Copenhagen, Denmark
(Received 16 December 1985)
Relativistic density functional approach* (I)

\[ Z = \int \mathcal{D} \bar{q} \mathcal{D} q \exp \left\{ \int_0^\beta d\tau \int_V d^3x [\mathcal{L}_{\text{eff}} + \bar{q} \gamma_0 \hat{\mu} q] \right\} , \quad q = \begin{pmatrix} q_u \\ q_d \end{pmatrix} , \quad \hat{\mu} = \text{diag}(\mu_u, \mu_d) \]

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{free}} - U(\bar{q}q, \bar{q}\gamma_0 q) , \quad \mathcal{L}_{\text{free}} = \bar{q} \left( -\gamma_0 \frac{\partial}{\partial \tau} + i\vec{\gamma} \cdot \vec{\nabla} - \hat{m} \right) q , \quad \hat{m} = \text{diag}(m_u, m_d) \]

General nonlinear functional of quark density bilinears: scalar, vector, isovector, diquark ...
Expansion around the expectation values:

\[ U(\bar{q}q, \bar{q}\gamma_0 q) = U(n_s, n_v) + (\bar{q}q - n_s)\Sigma_s + (\bar{q}\gamma_0 q - n_v)\Sigma_v + \ldots , \]

\[ \langle \bar{q}q \rangle = n_s = \sum_{f=u,d} n_{s,f} = -\sum_{f=u,d} \frac{T}{V} \frac{\partial}{\partial m_f} \ln Z , \quad \Sigma_s = \frac{\partial U(\bar{q}q, \bar{q}\gamma_0 q)}{\partial (\bar{q}q)} \bigg|_{\bar{q}q=n_s} = \frac{\partial U(n_s, n_v)}{\partial n_s} , \]

\[ \langle \bar{q}\gamma_0 q \rangle = n_v = \sum_{f=u,d} n_{v,f} = \sum_{f=u,d} \frac{T}{V} \frac{\partial}{\partial \mu_f} \ln Z , \quad \Sigma_v = \frac{\partial U(\bar{q}q, \bar{q}\gamma_0 q)}{\partial (\bar{q}\gamma_0 q)} \bigg|_{\bar{q}\gamma_0 q=n_v} = \frac{\partial U(n_s, n_v)}{\partial n_v} \]

\[ Z = \int \mathcal{D} \bar{q} \mathcal{D} q \exp \{ S_{\text{quasi}}[\bar{q}, q] - \beta V \Theta[n_s, n_v] \} , \quad \Theta[n_s, n_v] = U(n_s, n_v) - \Sigma_s n_s - \Sigma_v n_v \]

\[ S_{\text{quasi}}[\bar{q}, q] = \beta \sum_n \sum_{\vec{p}} \bar{q} \left( \gamma_0 \omega_n - \hat{\mu}^* \right) q , \quad G^{-1}(\omega_n, \vec{p}) = \gamma_0 (-i\omega_n + \hat{\mu}^*) - \vec{\gamma} \cdot \vec{p} - \hat{m}^* \]

* This approach can be formulated using chirally invariant combinations of quark bilinears
Relativistic density functional approach (II)

\[ Z = \int D\bar{q} Dq \exp \{ S_{\text{quasi}}[\bar{q}, q] - \beta V \Theta[n_s, n_v] \}, \quad \Theta[n_s, n_v] = U(n_s, n_v) - \Sigma_s n_s - \Sigma_v n_v \]

\[ Z_{\text{quasi}} = \int D\bar{q} Dq \exp \{ S_{\text{quasi}}[\bar{q}, q] \} = \det[\beta G^{-1}], \quad \ln \det A = \text{Tr} \ln A \]

\[ P_{\text{quasi}} = \frac{T}{V} \ln Z_{\text{quasi}} = \frac{T}{V} \text{Tr} \ln[\beta G^{-1}] \]

\[ = 2N_c \sum_{f=u,d} \int \frac{d^3 p}{(2\pi)^3} \left\{ T \ln \left[ 1 + e^{-\beta(E_f^* - \mu_f^*)} \right] + T \ln \left[ 1 + e^{-\beta(E_f^* + \mu_f^*)} \right] \right\} \]

\[ P_{\text{quasi}} = \sum_{f=u,d} \int \frac{dp}{\pi^2 E_f^*} \frac{p^4}{E_f^*} \left[ f(E_f^* - \mu_f^*) + f(E_f^* + \mu_f^*) \right] \]

\[ E_f^* = \sqrt{p^2 + m_f^*}, \quad f(E) = 1/[1 + \exp(\beta E)] \]

\[ P = \sum_{f=u,d} \int_0^{p_{F,f}} \frac{dp}{\pi^2 E_f^*} \left[ f(E_f^* - \mu_f^*) + f(E_f^* + \mu_f^*) \right] - \Theta[n_s, n_v], \quad p_{F,f} = \sqrt{\mu_f^* - m_f^*} \]

Selfconsistent densities

\[ n_s = - \sum_{f=u,d} \frac{\partial P}{\partial m_f} = \frac{3}{\pi^2} \sum_{f=u,d} \int_0^{p_{F,f}} dpp^2 \frac{m_f^*}{E_f^*}, \quad n_v = \sum_{f=u,d} \frac{\partial P}{\partial \mu_f} = \frac{3}{\pi^2} \sum_{f=u,d} \int_0^{p_{F,f}} dpp^2 = \frac{p_{F,u}^3 + p_{F,d}^3}{\pi^2}. \]
Relativistic density functional approach (III)

Density functional for the SFM

\[ U(n_s, n_v) = D(n_v)n_s^{2/3} + an_v^2 + \frac{bn_v^4}{1 + cn_v^2}, \]

Quark selfenergies

\[ \Sigma_s = \frac{2}{3}D(n_v)n_s^{-1/3}, \quad \text{Quark “confinement”} \]

\[ \Sigma_v = 2an_v + \frac{4bn_v^3}{1 + cn_v^2} - \frac{2bcn_v^5}{(1 + cn_v^2)^2} + \frac{\partial D(n_v)}{\partial n_v}n_s^{2/3}, \]

String tension & confinement due to dual Meissner effect (dual superconductor model)

\[ D(n_v) = D_0 \Phi(n_v) \]

Effective screening of the string tension in dense matter by a reduction of the available volume \( \alpha = \nu|\nu|/2 \)

\[ \Phi(n_B) = \begin{cases} 1, & \text{if } n_B < n_0 \\ e^{-\alpha(n_B - n_0)^2}, & \text{if } n_B > n_0 \end{cases} \]
Phase transition to SFM quark matter

Hadronic matter: DD2 with excluded volume

\[
\Phi_n = \Phi_p = \begin{cases} 
1, & \text{if } n_B < n_0 \\
\frac{v}{2} |v| (n_B - n_0)^2, & \text{if } n_B > n_0
\end{cases}
\]

Varying the hadronic excluded volume parameter, p00 → v=0, … , p80 → v=8 fm^3

[S. Typel, EPJA 52 (3) (2016)]
Quark Pauli Blocking in Nuclear Matter
(mimicked by an excluded volume)
Pauli blocking among baryons

a) Low density: Fermi gas of nucleons (baryons)

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+ \sum_{123} \sum_{456} \sum_{\nu P'} \psi_{\nu P}(123) \psi_{\nu P'}(456) f_3(E_{\nu P}^0) \left[ \delta_{36} \psi_{\nu P}(123) \psi_{\nu P'}^*(456) - \psi_{\nu P}(453) \psi_{\nu P'}^*(126) \right] \\
\times \left[ E(1) + E(2) + E(3) + E(4) + E(5) + E(6) - E_{\nu P}^0 - E_{\nu P'}^0 \right] \\
= \Delta E_{\nu P}^{Pauli, \text{free}} + \Delta E_{\nu P}^{Pauli, \text{bound}} .
\]
Pauli quenching effects in a simple string model of quark/nuclear matter

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Pauli blocking among baryons - details

\[ \Sigma_{\nu}(p, p_{F\nu}, p_{F'}) = \sum_{\nu'=\{n,p\}} \sum_{\alpha=1,2} C_{\nu\nu'}^{(\alpha)} W_{\alpha}(p_{F\nu'}, p) \]

\[ W_{\alpha}(p_{F\nu'}, p) = \frac{\Omega}{2\pi^2} \int_{0}^{p_{F\nu'}} p'^2 V^{(\alpha)}(p, p') dp'; \]

\[ V^{(\alpha)}(\vec{p}, \vec{p}') = \frac{1}{2} \int_{-1}^{1} V^{(\alpha)}(\vec{p}, \vec{p}') dz; \]

\[ V^{(\alpha)}(\vec{p}, \vec{p}') = \frac{V_0 b}{\Omega m} \left( \frac{15}{2} - \lambda_{\alpha}^2 (\vec{p} - \vec{p}')^2 \right) \exp(-\lambda_{\alpha}^2 (\vec{p} - \vec{p}')^2). \]

\[ b^{-2} = \sqrt{3} m \omega \]

\[ \omega = 178.425 \text{ MeV} \]

\[ m = 350 \text{ MeV} \quad b = 0.6 \text{ fm} \]

\[ V_0 = \frac{9\sqrt{3} \pi^{3/2}}{2} \text{ and } \lambda_{\alpha} = \frac{b}{\sqrt{3} \alpha}. \]

\[ W_{\alpha}(p_{F\nu'}, p) = \frac{V_0 b}{32\pi^2 \lambda_{\alpha}^4 m} \left[ 12 \lambda_{\alpha} \sqrt{\pi} \left( \text{erf} (\lambda_{\alpha} (p_{F\nu'} - p)) + \text{erf} (\lambda_{\alpha} (p_{F\nu'} + p)) \right) + \frac{1}{p} \left( 11 - 2 \lambda_{\alpha}^2 p_{F\nu'} (p_{F\nu'} + p) \right) e^{-\lambda_{\alpha}^2 (p_{F\nu'} + p)^2} + (11 - 2 \lambda_{\alpha}^2 p_{F\nu'} (p_{F\nu'} - p)) e^{-\lambda_{\alpha}^2 (p_{F\nu'} - p)^2} \right] \]

\[ \Delta_{n, p}^{\text{Pauli}} = \frac{1}{24\sqrt{3\pi}} \frac{b}{m} \sum_{\nu'} \left[ 15 \alpha_{\nu, \nu'} P_F (\nu')^3 + \frac{17}{12} b_{\nu, \nu'} b^2 (P^2 + P_F (\nu')^2) P_F (\nu')^3 \right] \]

![Diagram](image1.png)

Nucleons (baryons) in medium

One-quark exchange

Two-quark exchange
Pauli blocking in NM – details

New aspect: chiral restoration --> dropping quark mass

Increased baryon swelling at supersaturation densities: --> dramatic enhancement of the Pauli repulsion !!

Pauli blocking among baryons – results

New EoS: Joining RMF (Linear Walecka) for pointlike baryons with chiral Pauli blocking

\[ p = \frac{1}{4\pi^2} \sum_\nu \left[ -E^*_\nu m^*_\nu p_{F\nu} + \frac{2}{3} E^*_\nu p_{F\nu}^3 + m^*_\nu \log \left( \frac{E^*_\nu + p_{F\nu}}{m^*_\nu} \right) \right] \]

\[ + \frac{1}{2} \left( \frac{g_\omega}{m_\omega} \right)^2 n^2 - \frac{1}{2} \left( \frac{g_\sigma}{m_\sigma} \right)^2 n_s^2 + p_{ex}; \]

\[ \epsilon = \frac{1}{4\pi^2} \sum_\nu \left[ 2 E^*_\nu p_{F\nu} - E^*_\nu m^*_\nu p_{F\nu} - m^*_\nu \log \left( \frac{E^*_\nu + p_{F\nu}}{m^*_\nu} \right) \right] \]

\[ + \frac{1}{2} \left( \frac{g_\omega}{m_\omega} \right)^2 n^2 + \frac{1}{2} \left( \frac{g_\sigma}{m_\sigma} \right)^2 n_s^2 + \epsilon_{ex}, \]

\[ \mu_{ex,\nu} = \Delta_\nu(n, x) = \sum_\nu (P_{F\nu}; P_{Fn}, P_{Fp}), \]

\[ \epsilon_{ex} = \sum_\nu \int_0^n dx \Delta_p(n', x) + (1 - x) \Delta_n(n', x), \]

\[ p_{ex} = \sum_\nu \mu_{ex,\nu} n_\nu - \epsilon_{ex}, \]

\[ n_{s,\nu} = \frac{m^*_\nu}{\pi^2} \left[ E^*_\nu p_{F\nu} - m^*_\nu \log \left( \frac{E^*_\nu + p_{F\nu}}{m^*_\nu} \right) \right], \]

\[ E^*_\nu = \sqrt{m^*_\nu^2 + p_{F\nu}^2}, \]

\[ n_\nu = \frac{p_{F\nu}^3}{3\pi^2}, \]

\[ m^*_\nu = m_\nu - \left( \frac{g_\sigma}{m_\sigma} \right)^2 n_{s,\nu}, \]

\[ \mu_\nu = E^*_\nu + \left( \frac{g_\omega}{m_\omega} \right)^2 n_\nu + \mu_{ex,\nu}. \]
Pauli blocking among baryons – results

Parametrization: from saturation properties

Prediction: symmetry energy
Pauli blocking in NM – results neutron stars
Pauli blocking in NM – nucleon excluded volume

New aspect: chiral restoration --> dropping quark mass

Increased baryon swelling at supersaturation densities: --> dramatic enhancement of the Pauli repulsion !!

Pauli blocking in NM – Summary

Pauli blocking selfenergy (cluster meanfield) calculable in potential models for baryon structure

Partial replacement of other short-range repulsion mechanisms (vector meson exchange)

Modern aspects:
- onset of chiral symmetry restoration enhances nucleon swelling and Pauli blocking at high n
- quark exchange among baryons -> six-quark wavefunction -> “bag melting” -> deconfinement

Chiral stiffening of nuclear matter --> reduces onset density for deconfinement

Hybrid EoS:
Convenient generalization of RMF models,
Take care: eventually aspects of quark exchange already in density dependent vertices!

Other baryons:
- hyperons
- deltas
Again calculable, partially done in nonrelativistic quark exchange models, chiral effects not yet!

Relativistic generalization:
Box diagrams of quark-diquark model ...

M-R relationships for hybrid stars – High-mass “twin” stars
Two high-mass pulsars with $M \sim 2M_\text{sun}$

- PSR J0348+0432: $M=2.01 \pm 0.04$ $M_\odot$
- PSR J1614-2230: $M=1.928 \pm 0.017$ $M_\odot$

What if they were high-mass twin stars? → radius measurement required! → NICER (2017)
Two high-mass pulsars with $M \sim 2M_{\odot}$

**Neutron Star**
- Hadronic matter
- $M_{\text{star}} = 2.0~M_\odot$
- $R_{\text{star}} = 13.9~\text{km}$

**Hybrid Star**
- Hadronic and Quark matter
- $M_{\text{star}} = 2.0~M_\odot$
- $R_{\text{star}} = 11.1~\text{km}$
- $R_{\text{quark-core}} = 7.36~\text{km}$
**Motivation – Neutron stars (Twins?)**

- Star configurations with same masses, but different radii

- New class of EOS, that features high mass twins

- NASA NICER mission: radii measurements $\sim 0.5$ km

- Existence of twins implies 1$^{st}$ order phase-transition and hence a critical point

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Support a CEP in QCD phase diagram with Astrophysics?

Crossover at finite T (Lattice QCD) + First order at zero T (Astrophysics) = Critical endpoint exists!
First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the “latent heat” (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → “third family of CS”.

Measuring two disconnected populations of compact stars in the M-R diagram would be the detection of a first order phase transition in compact star matter and thus the indirect proof for the existence of a critical endpoint (CEP) in the QCD phase diagram!
Density functional approach to quark matter


\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} = \bar{q} \left( i \gamma^0 \frac{d}{dt} - m_0 \right) q + \mu_0 \bar{q} \gamma^0 q + U(\bar{q}q, \bar{q} \gamma^0 q) \]

\[ U(\bar{q}q, \bar{q} \gamma^0 q) = U(\langle \bar{q}q \rangle, \langle \bar{q} \gamma^0 q \rangle) + \left. \frac{\partial U(\bar{q}q, \bar{q} \gamma^0 q)}{\partial \bar{q}q} \right|_{\bar{q}q = \langle \bar{q}q \rangle, \bar{q} \gamma^0 q = \langle \bar{q} \gamma^0 q \rangle} (\bar{q}q - \langle \bar{q}q \rangle) \]

\[ + \left. \frac{\partial U(\bar{q}q, \bar{q} \gamma^0 q)}{\partial \bar{q} \gamma^0 q} \right|_{\bar{q}q = \langle \bar{q}q \rangle, \bar{q} \gamma^0 q = \langle \bar{q} \gamma^0 q \rangle} (\bar{q} \gamma^0 q - \langle \bar{q} \gamma^0 q \rangle) + \ldots \]

\[ \mathcal{L}_{\text{eff}} \approx \mathcal{L}_{\text{free}} + U + U_s \bar{q}q - U_s \langle \bar{q}q \rangle + U_v \bar{q} \gamma^0 q - U_v \langle \bar{q} \gamma^0 q \rangle \]

\[ \bar{n}_s = \langle \bar{q}q \rangle \quad , \quad \bar{n}_v = \langle \bar{q} \gamma^0 q \rangle \quad , \quad \left. \frac{\partial U(\bar{q}q, \bar{q} \gamma^0 q)}{\partial \bar{q}q, \bar{q} \gamma^0 q} \right|_{\bar{q}q = \langle \bar{q}q \rangle, \bar{q} \gamma^0 q = \langle \bar{q} \gamma^0 q \rangle} = U_{s,v} \]

\[ P = - \left. \frac{\partial \Omega}{\partial V} \right|_{\mu, T} = g \int \frac{d^3 p}{(2\pi)^3} \left[ T \ln(1 + e^{-\beta(\text{E} - \mu^*)}) + T \ln(1 + e^{-\beta(\text{E} + \mu^*)}) \right] + \Theta \quad E = \sqrt{p^2 + m^2}. \]

\[ m_i^* = m_{0,i} - \Sigma_{s,i} \bar{n}_s = m_{0,i} + \left[ D(\bar{n}_s) \bar{n}_s^{-\frac{1}{3}} \right] \text{confinement} \]

\[ D(\bar{n}_s, \bar{n}_v) = D_0 \Phi(\bar{n}_s, \bar{n}_v) \]

\[ \Phi(\bar{n}_s, \bar{n}_v) = e^{-\alpha(\bar{n}_v - n_0)^2} \]

Available volume fraction:

Thermodynamic consistency --

Rearrangement selfenergies
Density functional approach to quark matter

Hadronic matter: DD2 with excluded volume

\[ \Phi_n = \Phi_p = \begin{cases} 
1, & \text{if } n_B < n_0 \\
\exp\left( -\frac{v}{2} (n_B-n_0)^2 \right), & \text{if } n_B > n_0 
\end{cases} \]

Varying the hadronic excluded volume parameter, \( p00 \rightarrow v=0, \ldots, p80 \rightarrow v=8 \text{ fm}^3 \)

[S. Typel, EPJA 52 (3) (2016)]
Hybrid EOS - parameters $\alpha, a, b$

Kaltenborn, Bastian, Blaschke, arXiv:1701.04400

Hybrid EOS - parameters

\( \alpha, a, b \)

Kaltenborn, Bastian, Blaschke, arXiv:1701.04400
Density functional approach to quark matter

Varying the 4-quark coupling parameter $a$

Robustness of HMTs against mixed phase

KVOR_cut02 RMF EoS vs. SFM-RDF_alpha02

Towards “measuring” the EoS in the T – μ plane (QCD phase diagram)

Speed-of-sound diagram from the INT program in Seattle, Summer 2016

Interpolation between lattice QCD and Compact star physics (2 M$_{\odot}$)

Towards “measuring” the EoS in the T – mu plane (QCD phase diagram)

Speed-of-sound diagram from the INT program in Seattle, Summer 2016

Interpolation between lattice QCD and Compact star physics (2 M_{sun})
**Conclusion:**

High-mass twins (HMTs) with quark matter cores can be obtained within different hybrid star EoS models, e.g.,
- constant speed of sound
- higher order NJL
- piecewise polytrope
- density functional

HMTs require stiff hadronic and quark matter EoS with a strong phase transition (PT)

Existence of HMTs can be verified, e.g., by precise compact star mass and radius observations (and a bit of good luck) → Indicator for strong PT !!

Extremely interesting scenarios possible for dynamical evolution of isolated (spin-down and accretion) and binary (NS-NS merger) compact stars

**Critical endpoint search in the QCD phase diagram with Heavy-Ion Collisions goes well together with Compact Star Astrophysics**
New compstar!

29 member countries!! (MP1304)

Kick-off: Brussels, November 25, 2013
21 member countries!
(CA15213)

"Theory of HOT Matter in Relativistic Heavy-Ion Collisions"

New: THOR!
Network: CA16214

Newest: PHAROS

http://www.cost.eu/COST_Actions/ca/CA16214

Kick-off: Brussels, late 2017
Topical Issue on Exploring Strongly Interacting Matter at High Densities - NICA White Paper
edited by David Blaschke, Jörg Aichelin, Elena Bratkovskaya, Volker Fries, Marek Gazdzicki, Jörgen Randrup, Oleg Rogachevsky, Oleg Teryaev, Viacheslav Toneev

From: Three stages of the NICA accelerator complex by V. D. Kekeidze et al.

NICA

Società Italiana di Fisica

Springer

EPJA Topical Issues can be found at http://epja.epj.org/component/list/?task=topic
Key fact: Mass “twins” ↔ 1\textsuperscript{st} order PT

Systematic Classification [Alford, Han, Prakash: PRD88, 083013 (2013)]

EoS $P(\varepsilon) \leftrightarrow$ Compact star phenomenology $M(R)$

Most interesting and clear-cut cases: (D)isconnected and (B)oth – high-mass twins!
Twins prove existence of disconnected populations (third family) in the M-R diagram.

Consequence of a first order phase transition

**Question:** Do twins prove the 1st order phase transition?
High mass twins: more examples!

MESSAGE:
- excluded volume (quark Pauli blocking) important
- high-density quark matter slightly stiffer $\eta_v=0.25$
- the scaled energy density jump (0.65) fulfills the twin condition of the schematic model by Alford et al. (2013)

→ Astronomers: Find disconnected star branches!!

DB, Alvarez, Benic, arxiv:1310.3803
Proceedings of CPOD 2013
2. Piecewise polytrope EoS – high mass twins?


\[ P_i(n) = \kappa_i n^{\Gamma_i} \]

- \( i = 1 \) : \( n_1 \leq n \leq n_{12} \)
- \( i = 2 \) : \( n_{12} \leq n \leq n_{23} \)
- \( i = 3 \) : \( n \geq n_{23} \)
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Here, 1\textsuperscript{st} order PT in region 2:

\[ \Gamma_2 = 0 \text{ and } P_2 = \kappa_2 = P_{\text{crit}} \]

\[ P(n) = n^2 \frac{d(\varepsilon(n)/n)}{dn}, \]

\[ \varepsilon(n)/n = \int dn \frac{P(n)}{n^2} = \int dn \kappa n^{\Gamma-2} = \frac{\kappa n^{\Gamma-1}}{\Gamma-1} + C, \]

\[ \mu(n) = \frac{P(n) + \varepsilon(n)}{n} = \frac{\kappa \Gamma}{\Gamma-1} n^{\Gamma-1} + m_0, \]

Seidov criterion for instability:

\[ \frac{\Delta \varepsilon}{\varepsilon_{\text{crit}}} \geq \frac{1}{2} + \frac{3}{3} \frac{P_{\text{crit}}}{\varepsilon_{\text{crit}}} \]

\[ n(\mu) = \left[ (\mu - m_0) \frac{\Gamma-1}{\kappa \Gamma} \right]^{1/(\Gamma-1)} \]

\[ P(\mu) = \kappa \left[ (\mu - m_0) \frac{\Gamma-1}{\kappa \Gamma} \right]^{\Gamma/(\Gamma-1)} \]

Maxwell construction:

\[ P_1(\mu_{\text{crit}}) = P_3(\mu_{\text{crit}}) = P_{\text{crit}} \]
\[ \mu_{\text{crit}} = \mu_1(n_{12}) = \mu_3(n_{23}) \]
2. Piecewise polytrope EoS – high mass twins?

Set with same onset of Phase transition:

\[ P_{\text{crit}} = 68.18 \text{ MeV/fm}^3 \]

\[ \epsilon_{\text{crit}} = 318.26 \text{ MeV/fm}^3 \]

\[ \Delta \epsilon = 253.89 \text{ MeV/fm}^3 \]

\[ n_{12} = 0.32 \text{ fm}^3 ; \quad n_{23} = 0.53 \text{ fm}^3 \]

Third family solutions in the 2M_\text{sun} mass range (HMT) exist!!

[arxiv:1703.02681]
Walecka \((\sigma - \omega)\) model of asymmetric nuclear matter
(Functional integral approach)
Walecka model for dense nuclear matter (I)

Meson exchange model

eample: scalar (σ) meson

\[
(-\Delta + m_\sigma^2)\sigma(\vec{r}) = -g_\sigma \delta(\vec{r})
\]

\[
\Rightarrow \sigma(r) = -\frac{g_\sigma e^{-m_\sigma r}}{4\pi} \frac{1}{r}
\]

\[
V_{NN}^{(\sigma)}(r) = \sigma(r) = -\frac{g_\sigma^2 e^{-m_\sigma r}}{4\pi} \frac{1}{r}
\]

<table>
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<tr>
<th>Meson</th>
<th>$I^\pi$</th>
<th>$T$</th>
<th>$S$</th>
<th>$M$ [MeV]</th>
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<td>0</td>
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<td>0</td>
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<td>$1/2$</td>
<td>±1</td>
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<td>0</td>
<td>0</td>
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<tr>
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<tr>
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<td>0</td>
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<tr>
<td>$\delta$</td>
<td>$0^+$</td>
<td>1</td>
<td>0</td>
<td>900</td>
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</tbody>
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Walecka model for dense nuclear matter (II)

Field theoretical formulation: Lagrangian and Path Integral for Partition Function

\[
Z_gk(T, V, \{\mu_i\}) = \int [d\bar{\Psi}][d\Psi] \exp \left\{ \int_0^{\beta=1/T} d\tau \int d^3\vec{x} \left( \mathcal{L}_0 + \mathcal{L}_I + \mu_p \Psi^+_p \Psi_p + \mu_n \Psi^+_n \Psi_n \right) \right\}
\]

\[
\mathcal{L}_0(\tau, \vec{x}) = \bar{\Psi}(\tau, \vec{x}) (i\gamma_\mu \partial_\mu - m_N) \Psi(\tau, \vec{x}) , \quad \mathcal{L}_I(\tau, \vec{x}) = j_{\omega_\mu}(\tau, \vec{x}) \frac{G_\omega}{2} j_{\omega_\mu}(\tau, \vec{x}) - j_\sigma(\tau, \vec{x}) \frac{G_\sigma}{2} j_\sigma(\tau, \vec{x})
\]

\[
j_\sigma(\tau, \vec{x}) = \bar{\Psi}(\tau, \vec{x}) \Psi(\tau, \vec{x}) \quad \Psi = \begin{pmatrix} \psi_n \\ \psi_p \end{pmatrix} ; \quad \psi_n = \begin{pmatrix} u_n, \uparrow \\ u_n, \downarrow \\ v_n, \uparrow \\ v_n, \downarrow \end{pmatrix} \quad \text{Neutron} \quad \text{Antineutron}
\]

\[
\mu_n = \mu_p \quad \rightarrow \text{symmetric nuclear matter}
\]

\[
\mu_n \neq 0; \mu_p = 0 \quad \rightarrow \text{pure neutron matter}
\]

\[
\mu_n = \mu_p + \mu_e^- \quad \rightarrow \text{neutron star matter (}\beta\text{-equilibrium)}
\]
Walecka model for dense nuclear matter (III)

Evaluation of the Path Integral: Hubbard-Stratonovich trick

\[
\exp \left( - (\bar{\Psi} \Psi) \frac{G_\sigma}{2} (\bar{\Psi} \Psi) \right) = \left( \det G_{\sigma}^{-1} \right)^{\frac{1}{2}} \int [d\sigma] \exp \left( \frac{\sigma^2}{2G_\sigma} + \sigma \bar{\Psi} \Psi \right)
\]

Effective action quadratic \implies\ Gaussian Path Integral

\[
S \equiv \int_0^\beta d\tau \int d^3 \vec{x} \left\{ \bar{\Psi}(\vec{x}, \tau) \left( -\gamma_0 \frac{\partial}{\partial \tau} + i \vec{\gamma} \vec{\nabla} - m_N + \gamma_0 \mu + \sigma - \gamma_\mu \omega_\mu \right) \Psi(\vec{x}, \tau) + \frac{\sigma^2}{2G_\sigma} - \frac{\omega_\mu^2}{2G_{\omega_\mu}} \right\}
\]

Fourier representation: \( \Psi(\vec{x}, \tau) = \sqrt{\frac{T}{V}} \sum_n \sum_{\vec{p}} e^{i(\vec{p} \cdot \vec{x} + \omega_n \tau)} \psi_n(\vec{p}) \), with \( \omega_n \equiv \pi T (2n + 1) \)

\[
\int_0^\beta d\tau \int d^3 \vec{x} \bar{\Psi}(\vec{x}, \tau) \left( -\gamma_0 \frac{\partial}{\partial \tau} + i \vec{\gamma} \vec{\nabla} - m_N + \gamma_0 \mu + \sigma - \gamma_0 \omega_0 \right) \Psi(\vec{x}, \tau)
\]

\[
= \frac{1}{\beta V} \int_0^\beta d\tau \int d^3 \vec{x} \sum_n \sum_{\vec{p}, \vec{p}'} \bar{\Psi}_{n'}(\vec{p}') (-i\gamma_0 \omega_n - \vec{\gamma} \vec{p} - m_N^* + \gamma_0 \mu^*) \psi_n(\vec{p}) e^{i \left\{ (\vec{p} - \vec{p}') \cdot \vec{x} + (\omega_n - \omega_{n'}) \tau \right\}}
\]

\[
= \beta \sum_n \sum_{\vec{p}} \bar{\Psi}_n(\vec{p}) (-\gamma_\mu p_\mu - m_N^*) \psi_n(\vec{p}) = \sum_n \sum_{\vec{p}} \bar{\Psi}_n(\vec{p}) G^{-1}[\sigma, \omega_0] \psi_n(\vec{p})
\]

Effective mass \( m_N^* = m_N - \sigma \), chemical potential \( \mu^* = \mu - \omega_0 \) and quasiparticle propagator

\[
G^{-1}[\sigma, \omega] = -\beta (\gamma_\mu p_\mu + m_N^*) , \quad p_0 = i\omega_n - \mu^*
\]
Walecka model for dense nuclear matter (IV)

Evaluate fermionic Path Integral and mean field approximation:

\[
Z_{gk}(T, V, \{\mu_i\}) = \mathcal{N} \prod_{n, \bar{\rho}} \int [d\bar{\Psi}_n(\bar{\rho})] [d\Psi_n(\bar{\rho})] [d\sigma] [d\omega_0] e^{\left\{ \frac{\sigma^2 - \omega_0^2}{2G\omega_0} + \sum_{n, \bar{\rho}} \bar{\Psi}_n(\bar{\rho}) G^{-1}[\sigma, \omega_0] \Psi_n(\bar{\rho}) \right\}}
\]

\[
= \int [d\sigma] [d\omega_0] \exp \left\{ Tr \ln G^{-1}[\sigma, \omega_0] + \frac{\sigma^2}{2G\sigma} - \frac{\omega_0^2}{2G\omega_0} \right\}
\]

\[
= \exp \left\{ Tr \ln G^{-1}[\bar{\sigma}, \bar{\omega}_0] + \frac{\bar{\sigma}^2}{2G\sigma} - \frac{\bar{\omega}_0^2}{2G\omega_0} \right\}
\]

Stationarity condition: \( \partial \ln Z_{gk} / \partial \bar{\sigma} = \partial \ln Z_{gk} / \partial \bar{\omega}_0 = 0 \) corresponds to "gap equations":

\[
\bar{\sigma} = -G_\sigma \ Tr \ G[\bar{\sigma}, \bar{\omega}_0] = G_\sigma n_s , \quad \bar{\omega}_0 = -G_\omega \ Tr \gamma_0 G[\bar{\sigma}, \bar{\omega}_0] = G_\omega n .
\]

Thermodynamics: \( \Omega(T, V, \mu) = -T \ln Z_{gk} = -pV \)

\[
p(\mu, T) = \frac{1}{2} G_\omega n^2 - \frac{1}{2} G_\sigma n_s^2 + 4T \int \frac{d^3 \bar{\rho}}{(2\pi)^3} \left[ \ln \left( 1 + e^{-\beta(E^* - \mu^*)} \right) + \ln \left( 1 + e^{-\beta(E^* + \mu^*)} \right) \right]
\]

\[
n = 4 \int \frac{d^3 \bar{\rho}}{(2\pi)^3} \left[ f_-(E^*) - f_+(E^*) \right] , \quad n_s = 4 \int \frac{d^3 \bar{\rho}}{(2\pi)^3} \frac{m^*_N}{E^*} \left[ f_-(E^*) - f_+(E^*) \right] , \quad f_\pm(E^*) = \frac{1}{e^{\beta(E^* \pm \mu^*)} + 1}
\]

Quasiparticle properties \( E^* = \sqrt{\bar{\rho}^2 + m^*_N^2} \), \( m^*_N = m_n - G_\sigma n_s \), \( \mu^* = \mu - G_\omega n \).
Walecka model for dense nuclear matter (V)

Evaluate Traces: $Tr \ln G^{-1} = 2tr_p tr_D \ln G^{-1} = 2tr_p \ln \det_D G^{-1} = 2 \sum n \sum \bar{p} \ln \det_D G^{-1}$

Scalar mean field

$\bar{\sigma} = -G_\sigma Tr G[\bar{\sigma}, \bar{\omega}_0]$

$= -2G_\sigma T \sum_n \int \frac{d^3 \bar{p}}{(2\pi)^3} tr_D \left[ \gamma_\mu \rho_\mu - (m - \bar{\sigma}) + i\gamma_0 (\mu - \bar{\omega}) \right]^{-1}$

$= 2G_\sigma T \sum_n \int \frac{d^3 \bar{p}}{(2\pi)^3} \left( \frac{4m^*}{\bar{p}^2 + m^* (\omega_n + i\mu^*)^2} \right)$

$= 4G_\sigma \int \frac{d^3 \bar{p}}{(2\pi)^3} \frac{m^*}{E^*} \left( \frac{1}{e^{\beta(E^* - \mu^*)} + 1} - \frac{1}{e^{\beta(E^* + \mu^*)} + 1} \right)$

$\equiv G_\sigma n_s$

Vector mean field

$\bar{\omega}_0 = -G_{\bar{\omega}_0} Tr \gamma_0 G[\bar{\sigma}, \bar{\omega}_0]$

$= 4G_\omega \int \frac{d^3 \bar{p}}{(2\pi)^3} \left( \frac{1}{e^{\beta(E^* - \mu^*)} + 1} - \frac{1}{e^{\beta(E^* + \mu^*)} + 1} \right)$

$\equiv G_\omega n$

Matsubara sums $\rightarrow$ Exercise!!
Walecka model for dense nuclear matter - results

Effective mass

Energy per nucleon

Pressure

Symmetric nuclear matter \((n_p/n_B = 0.5)\) saturates with a binding energy per nucleon of 16 MeV at \(n_B = n_p + n_n = 0.16 \text{ fm}^{-3}\). Increasing the asymmetry towards pure neutron matter \((n_p = 0)\) makes the system unbound.

See, e.g., Kapusta's book "Finite temperature field theory" for the nuclear liquid-gas phase transition.