Introduction to deeply virtual Compton Scattering

Dieter Müller
Ruhr-University Bochum

- Generalized parton distributions
- Status of Theory and Phenomenology
- Conclusions

some recent and upcoming work in collaboration with:
K. Kumerički (KK), E. Aschenauer, S. Firzo, M. Murray
K. Passek-Kumerički (KP-K), T. Lautenschlager, A. Schäfer; M. Meskauskas
A. Belitsky, Y. Ji; V. Braun, A. Manashov, B. Pirnay
D.S. Hwang
**GPDs embed non-perturbative physics**

GPDs appear in various hard exclusive processes, e.g., hard electroproduction of photons (DVCS)

\[ Q^2 > 1 \text{GeV}^2 \]

\[
\mathcal{F}(\xi, Q^2, t) = \int_{-1}^{1} dx \ C(x, \xi, \alpha_s(\mu), Q/\mu) F(x, \xi, t, \mu) + O\left(\frac{1}{Q^2}\right)
\]

**Compton form factor observable**

**hard scattering part** (perturbation theory)

**GPD universal (conventional)**

**higher twist** depends on approximation
GPD related hard exclusive processes

• Deeply virtual Compton scattering (clean probe)
  \[ ep \rightarrow e' p' \gamma \]
  \[ ep \rightarrow e' p' \mu^+ \mu^- \]
  \[ \gamma p \rightarrow p' e^- e^+ \]

• Deeply virtual meson production (flavor filter)
  \[ ep \rightarrow e' p' \pi \]
  \[ ep \rightarrow e' p' \rho \]
  \[ ep \rightarrow e' n \pi^+ \]
  \[ ep \rightarrow e' n \rho^+ \]

• etc.

scanned area of the surface as a functions of lepton energy

\[ \eta \]

\[ x \]

twist-two observables:
  longitudinal cross sections
  transverse target spin asymmetries

factorization proof for longitudinal cross sections

[Collins, Frankfurt, Strikman (96)]
Field theoretical GPD definition

GPDs are defined as matrix elements of renormalized light-ray operators:

\[ F(x, \eta, \Delta^2, \mu^2) = \int_{-\infty}^{\infty} d\kappa \, e^{i\kappa x \cdot n \cdot P} \langle P_2 | RT : \phi(-\kappa n)[(-\kappa n), (\kappa n)] \phi(\kappa n) : | P_1 \rangle, \, n^2 = 0 \]

momentum fraction \( x \), skewness \( \eta = \frac{n \cdot \Delta}{n \cdot P} \) \( \Delta = P_2 - P_1 \) \( P = P_1 + P_2 \) \( \Delta^2 \equiv t \)

For a nucleon target we have four chiral even twist-two GPDs:

\[
\begin{align*}
\bar{\psi}_i \gamma^+ \psi_i & \Rightarrow \quad i q^V = \bar{U}(P_2, S_2) \gamma^+ U(P_1, S_1) H_i + \bar{U}(P_2, S_2) \frac{i\sigma^+ \gamma^\nu}{2M} U(P_1, S_1) E_i \\
\bar{\psi}_i \gamma^+ \gamma_5 \psi_i & \Rightarrow \quad i q^A = \bar{U}(P_2, S_2) \gamma^+ \gamma_5 U(P_1, S_1) \tilde{H}_i + \bar{U}(P_2, S_2) \frac{\gamma^5 \Delta^+}{2M} U(P_1, S_1) \tilde{E}_i
\end{align*}
\]

shorthands:

\[
\begin{align*}
\text{chiral even GPDs:} & \quad F = \{H, E, \tilde{H}, \tilde{E}\} \quad & \text{& CFFs:} & \quad \mathcal{F} = \{\mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \tilde{\mathcal{E}}\} \\
\text{chiral odd GPDs:} & \quad F_T = \{H_T, E_T, \tilde{H}_T, \tilde{E}_T\} \quad & \quad \mathcal{F}_T = \{\mathcal{H}_T, \mathcal{E}_T, \tilde{\mathcal{H}}_T, \tilde{\mathcal{E}}_T\}
\end{align*}
\]
Partonic interpretation of GPDs

\[ \frac{x + \eta}{2} \quad \frac{x - \eta}{2} \]

\[ \Delta_\perp = p'_\perp - p_\perp \]

GPDs simultaneously carry information on longitudinal and transverse distribution of partons in a proton

for \( \eta = 0 \) they have a probabilistic interpretation (infinite momentum frame) \([B]urkhardt (00)\]

\[ b_\perp = \sqrt{4 \frac{d}{dt} \ln H(x, 0, t)} \bigg|_{t=0} \]

GPDs contain also information on partonic angular momentum \([X. \text{ Ji (96)}\]

\[ \frac{1}{2} = \sum_{a=q,G} J^z_a \]

\[ J^z_a = \lim_{\Delta \to 0} \frac{1}{2} \int_{-1}^{1} dx \, x \left( H_a + E_a \right)(x, \eta, \Delta^2) \]

\[ q(x, \Delta^2) \]
A partonic duality interpretation

quark GPD (anti-quark $x \rightarrow -x$):

$$F(x, \eta, t) = \theta(-\eta \leq x \leq 1) \omega(x, \eta, t) + \theta(\eta \leq x \leq 1) \omega(x, -\eta, t)$$

$$\omega(x, \eta, t) = \frac{1}{\eta} \int_{0}^{\frac{x+\eta}{1+\eta}} dy \left(a - bx \right)^p f(y, (x - y)/\eta, t)$$

dual interpretation on partonic level:

central region - $\eta < x < \eta$

mesonic exchange in $t$-channel

support extension is unique [DM et al. 92]

ambiguous ($D$-term) [DM, A. Schäfer (05) KMP-K (07)]

outer region $\eta < x$

partonic exchange in $s$-channel
Can one `measure’ GPDs?

- **CFF** given as **GPD** convolution:

\[ F(\xi, t, Q^2) \overset{\text{LO}}{=} \int_{-1}^{1} dx \left( \frac{1}{\xi - x - i\epsilon} + \frac{1}{\xi + x - i\epsilon} \right) F(x, \eta = \xi, t, Q^2) \]

\[ \overset{\text{LO}}{=} i\pi F^\pm (x = \xi, \eta = \xi, t, Q^2) + \text{PV} \int_{0}^{1} dx \frac{2x}{\xi^2 - x^2} F^\pm (x, \eta = \xi, t, Q^2) \]

- **F(x, x, t, Q^2)** viewed as "spectral function" (s-channel cut):

\[ F^\pm (x, x, t, Q^2) \equiv F(x, x, t, Q^2) \mp F(-x, x, t, Q^2) \overset{\text{LO}}{=} \frac{1}{\pi} \text{Im} F(\xi = x, t, Q^2) \]

- **CFFs** satisfy `dispersion relations’ (not the physical ones, threshold \( \xi_0 \) set to 1)

\[ \text{Re} F(\xi, t, Q^2) = \frac{1}{\pi} \text{PV} \int_{0}^{1} d\xi' \left( \frac{1}{\xi - \xi'} \mp \frac{1}{\xi + \xi'} \right) \text{Im} F(\xi', t, Q^2) + C(t, Q^2) \]

[Frankfurt et al (97)
Chen (97)
Terayev (05)
KMP-K (07)
Diehl, Ivanov (07)]

**access** to the **GPD** on the cross-over line \( \eta = x \) (at LO)

**access** to the subtraction constant (for \( H, E \) related to `\( D \)-term’)

[Terayev (05)]
Modeling & Evolution

outer region governs the evolution at the cross-over trajectory

\[ \mu^2 \frac{d}{d\mu^2} F(x, x, t, \mu^2) = \int_x^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu)) F(y, x, \mu^2) \]

GPD at \( \eta = x \) is `measurable’ (LO)

central region follows (polynomiality of moments)

outer region governs evolution

net contribution of outer + central region is governed by a sum rule:

\[ \text{PV} \int_0^1 dx \frac{2x}{\eta^2 - x^2} F^+(x, \eta, t) \]

\[ = \text{PV} \int_0^1 dx \frac{2x}{\eta^2 - x^2} F^+(x, x, t) + C(t) \]
Photon leptoproduction $e^\pm N \rightarrow e^\pm N\gamma$

measured by H1, ZEUS, HERMES, CLAS, HALL A collaborations

planned at COMPASS, JLAB@12GeV, perhaps at ? EIC, ?? LHeC

$$\frac{d\sigma}{dx_B j dyd|\Delta^2|d\phi d\varphi} = \frac{\alpha^3 x_B j y}{16 \pi^2 Q^2} \left(1 + \frac{4M^2 x_B j^2}{Q^2}\right)^{-1/2} \left|\frac{T}{e^3}\right|^2,$$

\[ x_B j = \frac{Q^2}{2P_1 \cdot q_1} \approx \frac{2\xi}{1 + \xi}, \]

\[ y = \frac{P_1 \cdot q_1}{P_1 \cdot k}, \]

\[ \Delta^2 = t \text{ (fixed, small)}, \]

\[ Q^2 = -q_1^2 (> 1\text{GeV}^2). \]
interference of **DVCS** and **Bethe-Heitler** processes

12 Compton form factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}, \cdots$ elastic form factors $F_1, F_2$

(helicity amplitudes)

\[
|\mathcal{T}_{\text{BH}}|^2 = \frac{e^6 (1 + \epsilon^2)^{-2}}{x_B^2 y^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_{\text{BH}}^0 + \sum_{n=1}^{2} c_{\text{BH}}^n \cos(n\phi) \right\}, \quad \text{exactly known (LO, QED)}
\]

\[
|\mathcal{T}_{\text{DVCS}}|^2 = \frac{e^6}{y^2 Q^2} \left\{ c_{\text{DVCS}}^0 + \sum_{n=1}^{2} \left[ c_{\text{DVCS}}^n \cos(n\phi) + s_{\text{DVCS}}^n \sin(n\phi) \right] \right\}, \quad \text{harmonics 1:1 helicity ampl.}
\]

\[
\mathcal{I} = \frac{\pm e^6}{x_B y^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^\mathcal{I} + \sum_{n=1}^{3} \left[ c_n^\mathcal{I} \cos(n\phi) + s_n^\mathcal{I} \sin(n\phi) \right] \right\}. \quad \text{harmonics 1:1 helicity ampl.}
\]
\[ \mathcal{T}^\text{VCS}(\phi) = (-1)^{a-1} \varepsilon_2^{\mu*}(b) T_{\mu\nu} \varepsilon_1^\nu(a) \]

\[ \mathcal{T}^\text{VCS} = \mathcal{V}(\mathcal{F}_{ab}) - b A(\mathcal{F}_{ab}) \quad \text{for} \quad a \in \{0, +, -\}, \quad b \in \{+, -\} \]

parameterization of (DV)CS helicity amplitudes

\[ \mathcal{V}(\mathcal{F}_{ab}) = \bar{u}_2 \left( \eta \mathcal{H}_{ab} + i \sigma_{\alpha\beta} \frac{m^\alpha \Delta^\beta}{2M} \mathcal{E}_{ab} \right) u_1 \]

\[ A(\mathcal{F}_{ab}) = \bar{u}_2 \left( \eta \gamma_5 \mathcal{H}_{ab} + \gamma_5 \frac{m \cdot \Delta}{2M} \mathcal{E}_{ab} \right) u_1 , \quad m^\mu = \frac{q^\mu_1 + q^\mu_2}{(p_1 + p_2) \cdot (q_1 + q_2)} \]

(one) parameterization of (DV)CS tensor equivalent to Tarrach’s one

relations of CFFs to helicity dependent CFFs are easily calculated:

\[ \mathcal{F}_{+b} = \left[ 1 + b \sqrt{1 + \frac{2}{e^2}} \right] \mathcal{F}_T + \frac{(1 - x_B) x_B^2 (4M^2 - t) (1 + \frac{i}{Q \sqrt{1 + e^2}})}{Q^2 \sqrt{1 + e^2} (2 - x_B + \frac{x_B t}{Q^2})^2} \mathcal{F}_T + \frac{2 x_B K^2}{2 \sqrt{1 + e^2} M^2 (2 - x_B + \frac{x_B t}{Q^2})^2} \mathcal{F}_{TT} + \frac{2 x_B K^2}{2 \sqrt{1 + e^2} M^2 (2 - x_B + \frac{x_B t}{Q^2})^2} \mathcal{F}_{LT} \]

\[ \mathcal{F}_{0+} = \frac{\sqrt{2} K}{\sqrt{1 + e^2} Q (2 - x_B + \frac{x_B t}{Q^2})} \left\{ \left[ 1 + \frac{2 x_B (4M^2 - t)}{Q^2 (2 - x_B + \frac{x_B t}{Q^2})} \right] \mathcal{F}_L + x_B \left[ 2 - \frac{4M^2 - t}{M^2 (2 - x_B + \frac{x_B t}{Q^2})} \right] \mathcal{F}_{TT} \right\} \]

usable for DVCS - RCS, extendable to timelike (D)VCS, double(D)VCS or DIS
relations among harmonics and GPDs are not more based on $1/Q$ expansion:

(all harmonics are expressed by twist-2 and -3 GPDs) [Diehl et. al (97)
Belitsky, DM, Kirchner (01)
Belitsky, DM, Ji (12)]

$$\begin{align*}
\left\{ c_1 \right\}_{s_1}^T & \propto \frac{\Delta}{Q} \text{tw-2(GPDs)} + O(1/Q^3), \\
\left\{ c_2 \right\}_{s_2}^T & \propto \frac{\Delta^2}{Q^2} \text{tw-3(GPDs)} + O(1/Q^4), \\
\left\{ c_3 \right\}_{s_3}^T & \propto \frac{\Delta s}{Q} (\text{tw-2})^T + O(1/Q^3), \\
c_0^{CS} & \propto (\text{tw-2})^2, \\
\left\{ c_1 \right\}_{s_1}^{CS} & \propto \frac{\Delta}{Q} (\text{tw-2}) (\text{tw-3}), \\
\left\{ c_2 \right\}_{s_2}^{CS} & \propto \alpha_s (\text{tw-2})(\text{tw-2})^{GT}
\end{align*}$$

(setting up the perturbative framework):

✓ twist-two coefficient functions at next-to-leading order
✓ anomalous dimensions and evolution kernels at next-to-leading order
✓ next-to-next-to-leading order in a specific conformal subtraction scheme
✓ twist-three including quark-gluon-quark correlation at LO
✓ partially, twist-three sector at next-to-leading order

? `target mass corrections’ (not understood) [Belitsky DM (01)]
✓ kinematical twist-four corrections [Braun, Manashov, (11)]
DVCS world data set

Current DVCS data at colliders:
- ZEUS- total xsec
- H1- total xsec
- ZEUS- dσ/dt
- H1- dσ/dt
- H1- A_{CU}

Current DVCS data at fixed targets:
- HERMES- A_{LT}
- HERMES- A_{CU}
- HERMES- A_{LL}, A_{UL}, A_{LU}
- HERMES- A_{UT}
- Hall A- CFFs
- CLAS- A_{LU}
- CLAS- A_{UL}

Planned DVCS at fixed targets:
- COMPASS- dσ/dt, A_{CSU}, A_{CST}
- JLAB12- dσ/dt, A_{LU}, A_{UL}, A_{LL}

EIC v/s = 140 GeV, 0.01 ≤ y ≤ 0.95
EIC v/s = 45 GeV, 0.01 ≤ y ≤ 0.95
Q^2 = 100 GeV^2
Q^2 = 50 GeV^2
y ≤ 0.6
y ≤ 0.6
Strategies to analyze DVCS data

(ad hoc) modeling:

VGG code [Goeke et. al (01) based on Radyushkin’s DDA]
BMK model [Belitsky, DM, Kirchner (01) based on RDDA]
`aligned jet’ model [Freund, McDermott, Strikman (02)]
Goloskokov/Kroll (05) based on RDDA (pinned down by DVMP)
`dual’ model [Polyakov, Shuvaev 02; Guzey, Teckentrup 06; Polyakov 07]
  " -- " [KMP-K (07) in MBs-representation]
polynomials [Belitsky et al. (98), Liuti et. al (07), Moutarde (09)]

dynamical models:  
not applied [Radyushkin et.al (02); Tiburzi et.al (04); Hwang DM (07)]...
(respecting Lorentz symmetry)

flexible models:
any representation by including unconstrained degrees of freedom
(for fits)  
KMP-K (07/08) for H1/ZEUS in MBs-integral-representation

CFFs (real and imaginary parts) and GPD fits/predictions

i. CFF extraction with formulae (local) [BMK (01), HALL-A (06)] and [KK,DM, Murray]
   least square fits (local) [Guidal, Moutarde (08...)]
   neural networks – a start up [KMS (11)]

ii. `dispersion integral’ fits [KMP-K (08), KM (08...)]

iii. flexible GPD modeling [KM (08...)]

vi. model comparisons & predictions
   VGG code, however also BMK01 (up to 2005)
   Goloskokov/Kroll (07) model based on RDDA
Asking for CFFs (physics case)

- CFFs are defined for the whole kinematical region
- contain (generalized) polarizabilities
- their access requires a complete measurement

**toy example DVCS off a scalar target**

- for the first step we use s-channel helicity conservation hypothesis (neglecting twist-three and transversity associated CFFs)

  • linearized set of equations (approximately valid)

    \[ A_{\sin(1\phi)}^{\sin(1\phi)} \approx N c_{jm}^{-1} H^{jm} \text{ and } A_{\cos(1\phi)}^{\cos(1\phi)} \approx N c_{re}^{-1} H^{re} \]

  • normalization \( N \) is bilinear in CFFs

    \[
    0 \lesssim N(A) \approx \frac{1}{1 + \frac{k}{4}|H|^2} \lesssim \frac{\int_{-\pi}^{\pi} d\phi \mathcal{P}_1(\phi)\mathcal{P}_2(\phi) d\sigma_{BH}(\phi)}{\int_{-\pi}^{\pi} d\phi \mathcal{P}_1(\phi)\mathcal{P}_2(\phi) [d\sigma_{BH}(\phi) + d\sigma_{DVCS}(\phi)]} \lesssim 1
    \]

  • cubic equation for \( N \) with two non-trivial solutions

    \[
    N(A) \approx \frac{1}{2} \left( 1 \pm \sqrt{1 - k c_{jm}^2 \left( A_{\sin(1\phi)}^{\sin(1\phi)} \right)^2 - k c_{re}^2 \left( A_{\cos(1\phi)}^{\cos(1\phi)} \right)^2} \right)
    \]

  + BH regime
  - DVCS regime

- standard error propagation

**NOTE:** there is no need to linearize, one can do mapping numerically
a complete measurement allows in principle to pin down all CFFs
missing information in incomplete measurements can be filled with noise

larger statistics:
some $E$ CFF constraint might have been obtained by HERMES
Neural Networks

- Kinematical values are represented by the input layer.
- Propagated through the network, where weights are set randomly.
- Random values for $\text{Im} \mathcal{H}$ and $\text{Re} \mathcal{H}$.
- Calculation of $\chi^2$.
- Backwards propagation (PyBrain).
- Adjusting weights so that error decreases.
- Repeat procedure.
- Taking next kinematical point.

Monte Carlo procedure to propagate errors, i.e., generating a replica data set avoiding over fitting (fitting to noise), dividing data set, taking a control example if error increases after decreasing – one stops.
A first use of neural network fits

(ideal) tool for error propagation and quantifying model uncertainties used to access real and imaginary part of $\mathcal{H}$ CFF from HERMES results are compatible to model, CFF fits, and mapping
Model prediction versus unbiased error propagation

- model fits and neural networks are complimentary
- meaning of error bands should be properly understood
- error propagation is practically an art (full information is not given)
GPD ansatz from t-channel view

- at short distance a quark/anti-quark state is produced, labeled by \textit{conformal spin} \( j+2 \)
- they form an intermediate mesonic state with total angular momentum \( J \)
- strength of \textit{coupling} is
- mesons propagate with
- decaying into nucleon anti-nucleon pair with given angular momentum \( J \), described by an \textit{impact form factor}

\[ f^J_J, J \leq j (+1) \]
\[ \frac{1}{m^2(J) - t} \propto \frac{1}{J - \alpha(t)} \]
\[ \frac{1}{(1 - \frac{t}{M^2(J)})^p} \]

- (conformal) GPD moments expanded in Wigner's rotation matrices

\[ F_j(t, \eta) = \sum^j_{J} \frac{f^J_J}{J - \alpha(t)} \left( 1 - \frac{t}{M^2(J)} \right)^p \eta^{j (+1) - J} \hat{d}^F_J(\eta), \quad \hat{d}^F_J(\eta = 0) = 1 \]

- labeling by \textit{t-channel quantum numbers} \( J^{PC} \)
- so-called D-term arises from \( 0^{++}, (f^0 \text{ or } \sigma) \) \( 2^{++}, 4^{++}, \ldots \)
  has even \( J = j+1 \) (or \( j = -1 \) in DR) pole (\( J (=0) \) has multiple meanings [KMP-K(07&08)])
- usable for large \( x \) (employing effective rotation matrices)
DIS+DVCS+DVMP phenomenology at small-$x_B$ (H1,ZEUS) works somehow without DIS at LO [T. Lautenschlager, DM, A. Schäfer (soon)] works at NLO ($Q^2 > 4 \text{ GeV}^2$), done with Bayes theorem (probability distribution function)
fixed:
meson DA flavor content

errors might be perhaps larger

entirely model dependency for $x > 10^{-2}$

• going from LO to NLO increases the skewness ratios (known since `ever’, [KMP-K(07)])
• gluons are more centralized as sea quarks (expected from DVCS & $J/\psi$ interpretation)
• cross-talk of skewness and $t$-dependency has been addressed by pdf
• NLO GPDs look rather compatible to Goloskokov/Kroll and Martin et. al finding
• there is also DVCS beam charge and perhaps beam spin data are coming up
A simple valence quarks GPD model

• model of GPD $H(x,x,t)$ within DD motivated ansatz at $Q^2=2\text{ GeV}^2$

**fixed:**

- PDF normalization
- eff. Reage pole
- large $t$-counting rules

$$H(x,x,t) = \frac{n r 2^\alpha}{1 + x} \left( \frac{2x}{1 + x} \right)^{-\alpha(t)} \left( \frac{1 - x}{1 + x} \right)^b \left( 1 - \frac{1-x}{1+x} \frac{t}{M^2} \right)^p .$$

**free parameters:** $r$-ratio at small $x$

- large $x$-behavior
- $p$-pole mass

• unpolarized valence quarks: asking for $r$, $b$, $M$ parameters

  $n = 1.0$, $\alpha(t) = 0.43 + 0.85t/\text{GeV}^2$, $p = 1$

• flexible parameterization of subtraction constant (so-called D-term convoluted with hard amplitude)

• analogous ansatz for polarized quark GPD + pion-pole contribution

• no $E(x,x,t)$ nor $\hat{E}(x,x,t)$ is set up

• KM...> 2010 hybrid models GPD evolution for sea /gluon + DR for valence
Fixed target DVCS data

- HERMES(02-12) $12 \times 34$ asymmetries (+ few bins) $0.05 \leq \langle x_B \rangle \leq 0.2$, $\langle |t| \rangle \leq 0.6 \text{ GeV}^2$
  $[\sin(\phi), \ldots, \cos(3 \phi),$
  two kinds of electrons, all polarization options]

- HERMES(12) $A_{LU}$ with recoil detector
  (compatible with old data, differences in GPD interpretation)

- CLAS(07) $12 \times 12$ $[A_{LU}(\phi)]$
  $0.14 \leq \langle x_B \rangle \leq 0.35$, $\langle |t| \rangle \leq 0.3 \text{ GeV}^2$
  $<Q^2> \approx 1.8 \text{ GeV}^2$
  $40 \times 12$ $[A_{LU}(\phi)]$ (large $|t|$ or bad sta.)

- HALL A(06) $12 \times 24$ $[\Delta \sigma(\phi)]$
  $A_{UL}$ and $A_{LU}$
  $\langle x_B \rangle = 0.36$, $\langle |t| \rangle \leq 0.33 \text{ GeV}^2$
  $\langle Q^2 \rangle \approx 1.8 \text{ GeV}^2$

- HALL A(06) $3 \times 24$ $[\sigma(\phi)]$
  $\langle Q^2 \rangle \approx 2.5 \text{ GeV}^2$
KM10 fits to DVCS off unpolarized proton

- a hybrid model: three effective SO(3) PWs for sea quarks/gluons
  dispersion relations for valence
  still $E$ GPD is neglected (only D-term)
  still $\hat{E}$ GPD only flexible pion pole contribution

- asking for GPD $H$ and `D-term' ($\hat{H}$ is considered as effective d.o.f.)

leading order, including evolution for sea quarks/gluons
quark twist-two dominance hypothesis within CFF convention [BM10]

- data selection (taking moments of azimuthal angle harmonics)

KM10a: neglecting HALL-A data
KM10b: forming ratios of moments
KM10: original HALL-A data
neglecting large $-t$ BSA CLAS data

15 parameter fit, e.g., including all HALL-A data

175 data points
$\chi^2$/d.o.f. = 132/165

- results are given as xs.exe on http://calculon.phy.hr/gpd/
recoil detector data are compatible with missing mass technique ones
fits produce curves were data are scattered around
recoil data: RDDA is not so much disfavored as it was before the case
HALL A $\phi$-dependence

- $\phi$-dependence is described (if we fit to it)
higher twist nor NLO corrections can explain data with standard models e.g., Guidal Polyakov Radyushkin Vanderhaeghen 04 model

wrong understanding on CFF hierarchy?

exclusivity issue in all other fixed target data?

Is (QED) correction procedure understood?

naive understanding of `power corrections’ [VGG (99)] is entirely misleading
**KM... versus CFF fits & large-χ² “model” fit**

**GUIDAL**
- Twist-two dominance hypothesis
- 7 parameter fit to all harmonics of unpolarized cross section
- Propagated errors + “theoretical” error estimate

**Moutarde**
- H dominance hypothesis within a smeared polynomial expansion
- Propagated errors + “theoretical” error estimate

**NN**
- Neural network within H dominance hypothesis
- Green (blue) [red] curves (KM10...) without (with) HALL A data (ratios)

**GK08**
- Black curve GPDs (based on RDDA) obtained from handbag approach to DVMP

- Reasonable agreement for HERMES and CLAS kinematics
- Large x-region and real part remains unsettled
• KMM12 (KM10 type model) includes polarized target DVCS data (global fit to most of data, $\chi^2/d.o.f \approx 1.6$ - best what is there at present e.g., transverse polarized HERMES asymmetries looks as)
The Future

✓ Compass
✓ JLAB@12 GeV
? ENC@GSI
? LHeC@CERN
? EIC@BNL or EIC@JLAB

Aschenauer, Firzo KK, DM (13)

from stage II
20×250 GeV² simulations
quantifying the partonic content

- elastic processes
- hard excl. processes
- exclusive processes @ large t
- inclusive processes
- semi-inclusive processes
- GPDs
- PDFs
- uPDFs
- TMDs
- dynamical models
- partonic phase space functions
- effective LCWFs
- spin cont. imaging
- looks doable [Hwang, DM (07,11,12,??)]
Summary
GPDs are intricate and (thus) a promising tool

- to reveal the transverse distribution of partons (to some extent done at small $x_B$)
- to address the spin content of the nucleon (not possible at present)
- providing a bridge to LCWFs & non-perturbative methods (e.g., lattice)
- modeling in terms of effective LCWFs is doable (require efforts)

*first decade of hard exclusive leptoproduction measurements*

- CFFs have their own interest, bridging low and high virtuality regimes
- should be straightforward to improve global (flexible) model fits to DVCS
- DVCS and DVMP data are describable in global fits at small $x$
- moving on: to NLO, kinematical twist, full GPD models, DVCS+DVMP+...
- covering the kinematical region between HERA (COMPASS) experiments within a high luminosity machine and dedicated detectors is needed to quantify exclusive and inclusive QCD phenomena: handle on GPD $E$ & 3D

**need:**

tools/technology for global NLO QCD fits (inclusive + exclusive)

theory development (desired but not urgent needed for phenomenology)
back ups
GPD phenomenology lessons: first decade

- qualitatively GPD formalism works in DVCS (from the start up)
- first look: no serious problems in DVMP (apart from ? about very large $x_B$ data) also supported by hand-bag model description of Goloskokov/Kroll
- description of present DVCS data is reached/feasible with flexible models for unpolarized target— but GPD understanding induces tension among data large unidentified contribution called $\hat{H}$ is disfavored by polarized target data
- many uncertainties: exclusivity, correction procedure, assumptions
- HERMES gave proof of principle that on can go for a complete measurement

partonic interpretation:

- RDDA (GVP01,BMK01, VGG code in its many versions, GK07, ...) a bit disfavored at LO can not reach a $\chi^2/dof \sim 1...1.6$ (its like $\chi^2/nop \sim 5...10$) should work at NLO \cite{Freund, McDermott (02)}
- GPD $H$ is dominant (? 15% accuracy), tomography at small-$x_B$
- GPD $\hat{H}$ is constrained
- no access to GPD $E$ from present data, pion pole model for $\hat{E}$ is disfavored
- D-term related subtraction constant comes out negative (& sizable)

Goke et. al model prediction (perhaps fit result might be not stable)
The Future

✓ Compass
✓ JLAB@12 GeV
?
ENC@GSI
?
LHeC@CERN
?
EIC@BNL or EIC@JLAB

Aschenauer, Firzo
KK, DM (13)

from stage II
20x250 GeV^2 simulations
two simulations from S. Fazio for DVCS cross section \( \sim 650 \) data points

\[ -t < \sim 0.8 \text{ GeV}^2 \] for \( \sim 10/\text{fb} \)

\[ 1 \text{ GeV}^2 < -t < 2 \text{ GeV}^2 \] for \( \sim 100/\text{fb} \) (cut: \(-t < 1.5 \text{ GeV}^2, 4 \text{ GeV}^2 < Q^2 \) to ensure \(-t < Q^2 \))

pseudo data are re-generated with GeParD

statistical errors rescaled

5% systematical errors added in quadrature, 3% Bethe-Heitler uncertainty
Imaging (probabilistic interpretation)

\[ q(x, \vec{b}, \mu^2) = \frac{1}{4\pi} \int_0^\infty dt |t| J_0(|\vec{b}| \sqrt{|t|}) H(x, \eta = 0, t, \mu^2) \]

- skewness effect vanishes \((s_2, s_4 \rightarrow 0)\)
- reduce fit uncertainties
- increase model uncertainties

extrapolation errors for \(-t \rightarrow 0\)

(large b uncertainties – small effect)

extrapolation errors into \(-t > 1.5 \text{ GeV}^2\)

(small b uncertainties)
Single transverse target spin asymmetry

20x250 2x5/fb mock data
(~1200 data points with statistical errors + 5% systematics at cross section level)

flexible GPD model for $E_{\text{sea}}$ and $E_G$

normalization (and $t$-dependency) of $E_{\text{sea}}$
is reasonable constraint

$E_G$ is essentially unconstraint