EXTRAGALACTIC PHOTON-ALP OSCILLATIONS UP TO 1000 TeV

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Morphology and strength of $B_{\text{ext}}$ totally unknown. The only information is $10^{-7} \, \text{nG} < B_{\text{ext}} < 1.7 \, \text{nG}$ from blazar observations and from Faraday rotation measures. Slightly less stringent upper bounds but still $\sim \, \text{nG}$ from the CMB.

I assume that $B_{\text{ext}}$ is produced by galactic outflows: ionized galactic matter is ejected into extragalactic space with the magnetic field frozen in, which gets amplified by turbulence and magnetizes the surrounding space.

MOTIVATION
First proposed by Rees and Setti (1968) and Hoyle (1969) to explain properties of radio sources.

Further, according Kronberg et al. (1996) galactic superwinds of primeval galaxies ($z > 6$) give rise to $B_{\text{ext}} \sim \, \text{nG}$ on Mpc scale, in agreement with Lyman-alpha forest clouds.
Similar proposal by Furlanetto and Loeb (2001) for quasars outflows. All this in agreement with numerical simulations of S. Bertone, C. Vogt and T. Ensslin (2006). Also normal galaxies (ellipticals and S0) possess galactic outflows owing to the AGN feedback and supernova explosions. Uncontroversial evidence for galactic outflows in regular clusters, due to iron present in the ICM.

MORAL: in all these models the seeds of $B_{\text{ext}}$ are galaxies, which imply three main features of $B_{\text{ext}}$: its domain-like morphology, why it has nearly the same strength in all domains – which means around each galaxy – and why its direction changes randomly from the neighborhood of a galaxy to that of another one.
ALTERNATIVE STRATEGY
Magnetohydrodynamic simulations: an initial condition for $B_{\text{ext}}$ is chosen *ad hoc* in the dark age and its evolution as driven by structure formation is investigated. The link with the real world is to reproduce cluster magnetic field. As a by-product: prediction of $B_{\text{fil}}$ today. What is the seed?

Vazza et al. (2014): ‘the maximal amplification for large filaments is of the order of $\sim 100$ for the magnetic energy, corresponding to a typical field of a few $\sim nG$ starting from a primordial weak field of $10^{-10} G$ (comoving)’

Vazza et al. (2016): seed field $B_{\text{ext}}(z = 49, 33.5, 30.7) = 0.1 nG$ comoving, conclusion for filaments: ‘even in the largest objects the average magnetic field only show a growth of order $\sim 10$ compared to the comoving seed field’. Whence even in the largest filaments $\langle B_{\text{fil}} \rangle \sim 1 nG$.

UNCERTAINTY: Donnert et al. (2009) show that the whole $B_{\text{cl}}$ can be produced by galactic outflows. So, it is unclear how much cosmological $B$ contribution to $B_{\text{cl}}$ is needed!
So, I will work with the standard domain-like network of $B_{\text{ext}}$.

So far, the *domain-like sharp-edges* (DLSHE) model has been employed, wherein all magnetic domains have the same size $L_{\text{dom}}$ equal to the $B_{\text{ext}}$ coherence length, and in each domain $B_{\text{ext}}$ is assumed to be uniform and to have the same strength, but that its direction changes *randomly* and *discontinuously* passing from one domain to the next.

Of course, it is not a physical model but only a highly idealized model.

Consider now

$$P_{\gamma\rightarrow a}^{\text{single}} \propto \sin^2 \left( \frac{\pi L_{\text{dom}}}{l_{\text{osc}}} \right), \quad (1)$$

As long as $l_{\text{osc}} \gg \pi L_{\text{dom}}$ we can expand the sinus and so $P_{\gamma\rightarrow a}^{\text{single}}$ is insensitive to the domain’s shape and to the sharp edges.
But when $l_{\text{osc}} \leq L_{\text{dom}}$ a whole oscillation – or even several oscillations – are contained inside a single domain, thereby implying that $P_{\text{single}}$ depends not only on $L_{\text{dom}}$ but also on the domain’s *shape*. It goes without saying that in such a situation the use of the DLSHE model gives rise to unphysical results because of its sharp edges.

Yet, the condition $l_{\text{osc}} < L_{\text{dom}}$ actually occurs because of photon dispersion on the CMB, which becomes important for $\mathcal{E} > 5$ TeV. Because CTA, HAWC, GAMMA 400, LHAASO and TAIGA-HiSCORE will explore the whole VHE range $100$ GeV $< \mathcal{E} < 100$ TeV and even beyond, the DLSHE model is doomed to failure!
In order to save the situation – namely retaining the advantages of the DLSHE model while at the same time getting physically meaningful results – I consider the \textit{domain-like smoothed-out} (DLSME) model where the edges get smoothed out in such a way that that the components of $B_{\text{ext}}$ change \textit{continuously} across adjacent domains.

As always, there is a price to pay. While in the DLSHE model the propagation equation in a single domain is easy to solve because $B_{\text{ext}} = \text{constant}$, in the DLSME model the above equation is more difficult to solve because $B_{\text{ext}} \neq \text{constant}$. Still, by a clever use of the Laplace transform we have solved it exactly in an analytic fashion.
Reduced Schrödinger-like equation

\[
\left(i \frac{d}{dy} + M^{(n)}(y)\right) \psi(y) = 0 ,
\] (2)

with

\[
\psi(y) \equiv \begin{pmatrix} \gamma_1(y) \\ \gamma_2(y) \\ a(y) \end{pmatrix} ,
\] (3)

Mixing matrix

\[
M^{(n)}(y) \equiv \begin{pmatrix} \Delta^{(n)}_{\text{CMB}} + \Delta^{(n)}_{\text{abs}} + \Delta^{(n)}_{\text{pl}} & 0 & \Delta^{(n)}_{a\gamma} \sin \phi_n(y) \\ 0 & \Delta^{(n)}_{\text{CMB}} + \Delta^{(n)}_{\text{abs}} + \Delta^{(n)}_{\text{pl}} & \Delta^{(n)}_{a\gamma} \cos \phi_n(y) \\ \Delta^{(n)}_{a\gamma} \sin_n \phi(y) & \Delta^{(n)}_{a\gamma} \cos \phi_n(y) & \Delta^{(n)}_{aa} \end{pmatrix} .
\] (4)
with

\[ \Delta^{(n)}_{\text{CMB}} = 0.522 \cdot 10^{-42} \mathcal{E}_n, \quad \Delta^{(n)}_{\text{abs}} = i/(2\lambda^{(n)}_\gamma), \quad \Delta^{(n)}_{\text{pl}} = -\omega^2_{\text{pl}}/(2\mathcal{E}_n), \]
\[ \Delta^{(n)}_{a\gamma} = g_{a\gamma\gamma} B_{T,n}/2 \]

Transfer matrix \( U(y, y_0; \phi_n(y)) \)

Von Neumann-like equation

\[ i \frac{d\rho(y)}{dy} = \rho(y) \mathcal{M}^{(n)\dagger}(y) - \mathcal{M}^{(n)}(y) \rho(y), \quad (5) \]

Solution

\[ \rho(y) = U(y, y_0; \phi_n(y)) \rho_0 U^\dagger(y, y_0; \phi_n(y)). \quad (6) \]
Probability that the beam in the initial state polarization state $\rho_0$ at $y_0$ will be found in the final polarization state $\rho$ at $y$ reads

$$P_{\gamma \rightarrow \gamma}^{\text{ALP}}(\rho, y; \rho_0 y_0; \phi(y)) =$$

$$\text{Tr} \left[ \rho U(y, y_0; \phi(y)) \rho_0 U^\dagger(y, y_0; \phi(y)) \right]$$

with $\text{Tr} \rho_0 = \text{Tr} \rho = 1$. 

In the cosmological context it is better to use a uniform mesh in redshift space with a single domain having $\Delta z$. Accordingly

$$\mathcal{E}_n = \mathcal{E}_0 \left[ 1 + (n - 1)\Delta z \right], \quad (8)$$

$$B_{T,n} = B_{T,0} \left[ 1 + (n - 1)\Delta z \right]^2, \quad (9)$$

$$\lambda^{(n)} = \frac{L^{(n)}_{\text{dom}}}{\tau_\gamma(\mathcal{E}_0, n \Delta z) - \tau_\gamma(\mathcal{E}_0, (n - 1)\Delta z)}, \quad (10)$$

$$L(z_a, z_b) \simeq 4.29 \cdot 10^3 \int_{z_a}^{z_b} \frac{dz}{(1 + z)[0.7 + 0.3(1 + z)^3]^{1/2}} \text{Mpc} \simeq 2.96 \cdot 10^3 \ln \left( \frac{1 + 1.45 z_b}{1 + 1.45 z_a} \right) \text{Mpc}. \quad (11)$$
I denote by \( \{ \phi_n \}_{1 \leq n \leq N} \) the set of angles that \( B_{T,n} \) forms with the above fiducial fixed \( x \)-direction in the plane orthogonal to the beam. I also define the two quantities \( y_{0,n} \) and \( y_{1,n} \) as

\[
y_{0,n} \equiv y_{D,n} - \frac{\sigma}{2} (y_{D,n} - y_{D,n-1}) , \quad (1 \leq n \leq N - 1) ; \quad (12)
\]

\[
y_{1,n} \equiv y_{D,n} + \frac{\sigma}{2} (y_{D,n+1} - y_{D,n}) , \quad (1 \leq n \leq N - 1) ; \quad (13)
\]

where \( \sigma \in [0, 1] \) is the smoothing parameter. In a generic interval \( [y_{1,n-1}, y_{1,n}] \) \((1 \leq n \leq N - 1)\) I set

\[
\phi(y) = \begin{cases} 
\phi_{0,n} = \text{constant} , & y \in [y_{1,n-1}, y_{0,n}] ; \\
\phi_{0,n} + \frac{\phi_{1,n} - \phi_{0,n}}{y_{1,n} - y_{0,n}} (y - y_{0,n}) , & y \in [y_{0,n}, y_{1,n}] .
\end{cases} \quad (14)
\]
Schematically, the DLSME model is defined as follows.

Figura: Behavior of the angle $\phi$ between $B_T$ and the fixed $x$-axis as respect to the propagation of the beam in the $y$-direction: the solid black line is the new smooth version, while the broken gray line represents its usual jump from one domain to the next. The horizontal solid and broken lines partially overlap.
Once I have solved the propagation equation in a single \( n \)-th domain I have the corresponding transfer matrix

\[
U(\mathcal{E}; y_n, y_{n-1}; \phi_n(y)) = U_{\text{var}}(\mathcal{E}; y_n, y_{n-1}; \phi_n(y)) U_{\text{const}}(\mathcal{E}; y_n, y_{n-1}; \phi_n),
\]

so that the total transfer matrix pertaining to a single realization of the beam propagation from the blazar to us reads

\[
U(\mathcal{E}; y_N, y_0; \{\phi_n\}_{1 \leq n \leq N}) = U_{\text{const}}(\mathcal{E}; y_N, y_{N-1}; \phi_N) \cdot \prod_{n=1}^{N-1} U_{\text{var}}(y_n, y_{n-1}; \phi_n(y)) U_{\text{const}}(\mathcal{E}; y_n, y_{n-1}; \phi_n).
\]
Because all $\phi_n$ are unknown, they have to be regarded as $N$ random variables, and so I plug $\mathcal{U}(\mathcal{E}; y_N, y_0; \{\phi_n\}_{1 \leq n \leq N})$ into

$$P_{\gamma \rightarrow \gamma}^\text{ALP}(y_N, y_0; \{\phi_n\}_{1 \leq n \leq N}) =$$

$$\sum_{i=x, z} \text{Tr}\left[\rho_i \mathcal{U}(y_N, y_0; \{\phi_n\}_{1 \leq n \leq N}) \rho_{\text{unp}} \mathcal{U}^\dagger(y_N, y_0; \{\phi_n\}_{1 \leq n \leq N})\right]$$

$$\rho_x \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \rho_z \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \rho_{\text{unp}} \equiv \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (19)$$

and finally I average over all angles $\phi_n$. 


I define

\[ \xi \equiv (g_{a\gamma} 10^{11} \text{GeV}) \left( \frac{B}{\text{nG}} \right), \tag{20} \]

and \( B_{\text{ext}} < 1.7 \text{ nG} \) and \( g_{a\gamma\gamma} < 0.66 \cdot 10^{-10} \text{ GeV}^{-1} \) imply

\[ \xi < 11.22. \tag{21} \]

Then:

solid yellow line corresponds to \( \xi = 5.0 \),
dotted-dashed blue line corresponds to \( \xi = 1.0 \),
dashed red line corresponds to \( \xi = 0.5 \),
dotted green line corresponds to \( \xi = 0.1 \),
solid black line corresponds to conventional physics.
RESULTS

$z = 0.031$
$L = 4 \text{ Mpc}$

$z = 0.188$
$L = 10 \text{ Mpc}$
$z = 0.444$
$L = 10^{10}$ Mpc

$z = 0.444$
$L = 10$ Mpc

$z = 0.536$
Survival probability

$E_0$ [TeV]

$z = 1$
$L = 10^{10}$ Mpc
$L = 10^{\text{Mpc}}$

Survival probability

$z = 2$

$E_0 [\text{TeV}]$