Microscopic description of alpha-cluster states in $^{12}$C

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Reactions Involving $^{12}$C: Nucleosynthesis and Stellar Evolution

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Overview

Unitary Correlation Operator Method

Fermionic Molecular Dynamics

Cluster States in $^{12}$C
- FMD and microscopic cluster model
- electron scattering data – form factors
- expansion in HO basis
- investigate intrinsic structure with two-body densities
- include $^8$Be+$\alpha$ continuum
Argonne V18 (T=0)
spins aligned parallel or perpendicular to the relative distance vector

- strong repulsive core: nucleons can not get closer than $\approx 0.5$ fm
  ➤ central correlations

- strong dependence on the orientation of the spins due to the tensor force
  ➤ tensor correlations

The nuclear force will induce **strong short-range correlations** in the nuclear wave function.
Universality of short-range correlations
Two-body densities in $A = 2, 3, 4$ Nuclei — AV8’

**coordinate space**

$S = 1, M_S = 1, T = 0$

- Normalize two-body density in coordinate space at $r = 1.0$ fm
- Normalized two-body densities in coordinate space are identical at short distances for all nuclei
- Use the **same** normalization factor in momentum space – high momentum tails agree for all nuclei

**momentum space**

$S = 1, T = 0$

Correlation Operator

- induce short-range (two-body) central and tensor correlations into the many-body state

\[ \tilde{C} = C_\Omega \tilde{C}_r = \exp[-i \sum_{i<j} g_{\Omega,ij}] \exp[-i \sum_{i<j} g_{r,ij}] , \quad \tilde{C}^\dagger \tilde{C} = 1 \]

- correlation operator should conserve the symmetries of the Hamiltonian and should be of finite-range, correlated interaction **phase shift equivalent** to bare interaction by construction

Correlated Operators

- correlated operators will have contributions in higher cluster orders

\[ \tilde{C}^\dagger Q \tilde{C} = \hat{Q}^{[1]} + \hat{Q}^{[2]} + \hat{Q}^{[3]} + \ldots \]

- two-body approximation: correlation range should be small compared to mean particle distance

Correlated Interaction

\[ \tilde{C}^\dagger (\tilde{T} + \tilde{V}) \tilde{C} = \tilde{T} + V_{\text{UCOM}} + V_{\text{UCOM}}^{[3]} + \ldots \]
Correlations and Energies

Unitary Correlation Operator Method

Central correlator $C_r$ shifts density out of the repulsive core
Tensor correlator $C_\Omega$ aligns density with spin orientation

$\rho^{(2)}_{S,T}(r_1 - r_2) \quad S = 1, M_S = 1, T = 0$

Both central and tensor correlations are essential for binding

Correlated Interaction in Momentum Space

$^3S_1$ bare

bare interaction has strong off-diagonal matrix elements connecting to high momenta

$^3S_1 - ^3D_1$ bare

correlated interaction is more attractive at low momenta

$^3S_1$ correlated

off-diagonal matrix elements connecting low- and high- momentum states are strongly reduced

$^3S_1 - ^3D_1$ correlated

similar to $V_{low-k}$, SRG

states close to one-nucleon, two-nucleon or cluster thresholds can have well developed **halo** or **cluster** structure

these are hard to tackle in the harmonic oscillator basis
Fermionic Molecular Dynamics

Fermionic
Slater determinant

\[ |Q\rangle = \mathcal{A}\left( |q_1\rangle \otimes \cdots \otimes |q_A\rangle \right) \]

- antisymmetrized \( A \)-body state

Molecular
single-particle states

\[ \langle x | q \rangle = \sum_i c_i \exp\left\{ -\frac{(x - b_i)^2}{2a_i} \right\} \otimes |\chi^\dagger_i, \chi^\dagger_i \rangle \otimes |\xi\rangle \]

- Gaussian wave-packets in phase-space (complex parameter \( b_i \) encodes mean position and mean momentum), spin is free, isospin is fixed
- width \( a_i \) is an independent variational parameter for each wave packet
- use one or two wave packets for each single particle state

see also

Antisymmetrized Molecular Dynamics
Horiuchi, Kanada-En’yo, Kimura, ...

Feldmeier, Schnack, Rev. Mod. Phys. 72 (2000) 655
**Interaction Matrix Elements**

**(One-body) Kinetic Energy**

\[
\langle q_k | T | q_l \rangle = \langle a_k b_k | T | a_l b_l \rangle \langle \chi_k | \chi_l \rangle \langle \xi_k | \xi_l \rangle 
\]

\[
\langle a_k b_k | T | a_l b_l \rangle = \frac{1}{2m} \left( \frac{3}{a_k^* + a_l} - \frac{(b_k^* - b_l)^2}{(a_k^* + a_l)^2} \right) R_{kl} 
\]

**Two-body Potential**

- fit radial dependencies by (a sum of) Gaussians

\[
G(x_1 - x_2) = \exp\left\{- \frac{(x_1 - x_2)^2}{2\kappa}\right\}
\]

- Gaussian integrals

\[
\langle a_k b_k, a_l b_l | G | a_m b_m, a_n b_n \rangle = R_{km} R_{ln} \left( \frac{\kappa}{\alpha_{klmn}} + \kappa \right)^{3/2} \exp\left\{- \frac{\rho_{klmn}^2}{2(\alpha_{klmn} + \kappa)} \right\} 
\]

- analytical expressions for matrix elements
\[ \tilde{\mathcal{C}}^\dagger (\tilde{T} + \tilde{V}) \tilde{\mathcal{C}} = \tilde{T} \]

\[ + \sum_{ST} \hat{V}_c^{ST}(r) + \frac{1}{2} (p_r^2 \hat{V}_{p^2}^{ST}(r) + \hat{V}_{p^2}^{ST}(r) p_r^2) + \hat{V}_{l^2}^{ST}(r) l^2 \]

\[ + \sum_{T} \hat{V}_l^{T}(r) l \cdot s + \hat{V}_{l^2s}^{T}(r) l^2 l \cdot s \]

\[ + \sum_{T} \hat{V}_t^{T}(r) S_{12}(r, r) + \hat{V}_{trp}^{T}(r) p_r S_{12}(r, p_\Omega) + \hat{V}_{tll}^{T}(r) S_{12}(l, l) + \hat{V}_{tp_\alpha p_\Omega}^{T}(r) S_{12}(p_\Omega, p_\Omega) + \hat{V}_{l^2tp_\alpha p_\Omega}^{T}(r) l^2 S_{12}(p_\Omega, p_\Omega) \]

one-body kinetic energy

central potentials

spin-orbit potentials

tensor potentials

bulk of tensor force mapped onto central part of correlated interaction
tensor correlations also change the spin-orbit part of the interaction

**FMD**

**PAV, VAP and Multiconfiguration**

**Projection After Variation (PAV)**
- mean-field may break symmetries of Hamiltonian
- restore inversion, translational and rotational symmetry by projection on parity, linear and angular momentum

\[ P^\pi = \frac{1}{2}(1 + \pi\Pi) \]

**Variation After Projection (VAP)**
- effect of projection can be large
- **Variation after Angular Momentum and Parity Projection** (VAP) for light nuclei
- combine VAP with constraints on radius, dipole moment, quadrupole moment, … to generate additional configurations

\[ P'_{MK} = \frac{2J + 1}{8\pi^2} \int d^3\Omega D'_{MK}^*(\Omega) R(\Omega) \]

\[ P^P = \frac{1}{(2\pi)^3} \int d^3X \exp\{-i(P - P) \cdot X\} \]

**Multiconfiguration Calculations**
- **diagonalize** Hamiltonian in a set of projected intrinsic states

\[ \left\{ \left| Q^{(a)} \right\rangle, \quad a = 1, \ldots, N \right\} \]

\[ \sum_{K' b} \langle Q^{(a)} | H P'^{\pi}_{K K' P^P=0} | Q^{(b)} \rangle \cdot c^K_{K' b} = \]

\[ E'^{\pi \alpha} \sum_{K' b} \langle Q^{(a)} | P'^{\pi}_{K K' P^P=0} | Q^{(b)} \rangle \cdot c^K_{K' b} \]
Cluster States in $^{12}$C

Astrophysical Motivation

- Helium burning: triple alpha-reaction

Structure

- Is the Hoyle state a pure $\alpha$-cluster state?
- Other excited $0^+$ and $2^+$ states

- Compare FMD results to microscopic $\alpha$-cluster model
- Intrinsic structure from two-body densities
- Analyze wave functions in harmonic oscillator basis
Cluster States in $^{12}\text{C}$

Microscopic $\alpha$-Cluster Model

**Basis States**
- describe Hoyle State as a system of 3 $^4\text{He}$ nuclei

$$|\psi_{3\alpha}(R_1, R_2, R_3); JMK\pi \rangle = P_{MK}^J P_{\pi A}^\pi \left\{ |\psi_\alpha(R_1) \rangle \otimes |\psi_\alpha(R_2) \rangle \otimes |\psi_\alpha(R_3) \rangle \right\}$$

**Volkov Interaction**
- simple central interaction
- parameters adjusted to give reasonable $\alpha$ binding energy and radius, $\alpha - \alpha$ scattering data, adjusted to reproduce $^{12}\text{C}$ ground state energy

× only reasonable for $^4\text{He}$, $^8\text{Be}$ and $^{12}\text{C}$ nuclei

∀ altogether 165 configurations

**Cluster States in $^{12}$C**

**Basis States**
- 20 FMD states obtained in Variation after Projection on $0^+$ and $2^+$ with constraints on the radius
- 42 FMD states obtained in Variation after Projection on parity with constraints on radius and quadrupole deformation
- 165 $\alpha$-cluster configurations
  ➢ projected on angular momentum and linear momentum

**Interaction**
- UCOM interaction ($I_0=0.30$ fm$^3$ with phenomenological two-body correction term (momentum-dependent central and spin-orbit) fitted to doubly-magic nuclei
- not tuned for $\alpha$-$\alpha$ scattering or $^{12}$C properties
Cluster States in $^{12}$C

$\alpha-\alpha$ Phaseshifts

- Phaseshifts calculated with cluster configurations only (dashed lines)
- Phaseshifts calculated with additional FMD VAP configurations in the interaction region (solid lines)

- only cluster configurations included

$\Rightarrow$ similar quality for description of $\alpha-\alpha$-scattering
Cluster States in $^{12}$C

Comparison

$E - E_{3\alpha}$ [MeV]

$^{12}$C

FMD

Experiment

$\alpha$–cluster
### Cluster States in $^{12}$C

**Comparison**

<table>
<thead>
<tr>
<th></th>
<th>Exp$^1$</th>
<th>Exp$^2$</th>
<th>FMD</th>
<th>$\alpha$-cluster</th>
<th>‘BEC’$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(0^+_1)$</td>
<td>-92.16</td>
<td>-92.64</td>
<td>-89.56</td>
<td>-89.52</td>
<td></td>
</tr>
<tr>
<td>$E^*(2^+_1)$</td>
<td>4.44</td>
<td>5.31</td>
<td>2.56</td>
<td>2.81</td>
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<tr>
<td>$E(3\alpha)$</td>
<td>-84.89</td>
<td>-83.59</td>
<td>-82.05</td>
<td>-82.05</td>
<td></td>
</tr>
<tr>
<td>$E(0^+_2)^-E(3\alpha)$</td>
<td>0.38</td>
<td>0.43</td>
<td>0.38</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>$E(0^+_3)^-E(3\alpha)$</td>
<td>(3.0)</td>
<td>2.7(3)</td>
<td>2.84</td>
<td>2.81</td>
<td></td>
</tr>
<tr>
<td>$E(2^+_2)^-E(3\alpha)$</td>
<td>(3.89)</td>
<td>2.76(11)</td>
<td>2.77</td>
<td>1.70</td>
<td></td>
</tr>
<tr>
<td>$r_{\text{charge}}(0^+_1)$</td>
<td>2.47(2)</td>
<td>2.53</td>
<td>2.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r(0^+_1)$</td>
<td>2.39</td>
<td>2.40</td>
<td>2.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r(0^+_2)$</td>
<td>3.38</td>
<td>3.71</td>
<td>3.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r(0^+_3)$</td>
<td>4.62</td>
<td>4.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r(2^+_1)$</td>
<td>2.50</td>
<td>2.37</td>
<td>2.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r(2^+_2)$</td>
<td>4.43</td>
<td>4.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M(E0, 0^+_1 \rightarrow 0^+_2)$</td>
<td>5.4(2)</td>
<td>6.53</td>
<td>6.52</td>
<td>6.45</td>
<td></td>
</tr>
<tr>
<td>$B(E2, 2^+_1 \rightarrow 0^+_1)$</td>
<td>7.6(4)</td>
<td>8.69</td>
<td>9.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B(E2, 2^+_1 \rightarrow 0^+_2)$</td>
<td>2.6(4)</td>
<td>3.83</td>
<td>0.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B(E2, 2^+_2 \rightarrow 0^+_1)$</td>
<td>0.73(13)</td>
<td>0.46</td>
<td>1.99</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Cluster States in $^{12}\text{C}$

Electron Scattering Data

• compare with precise electron scattering data up to high momenta in Distorted Wave Born Approximation

• use intrinsic density

$$\rho(\mathbf{x}) = \sum_{k=1}^{A} \bra{\psi} \delta(\mathbf{x}_k - \mathbf{X} - \mathbf{x}) \ket{\psi}$$


• elastic cross section described very well by FMD

• transition cross section better described by cluster model

$^{12}\text{C}(e,e)\ ^{12}\text{C}$

$E_x = 7.65\text{ MeV}$

$^{12}\text{C}(e,e')\ ^{12}\text{C}$

$E_x = 0.0\text{ MeV}$

10$^{-5}$ 10$^{-4}$ 10$^{-3}$ 10$^{-2}$ 10$^{-1}$ 10$^{0}$ 10$^{1}$ 10$^{2}$

$(d\sigma/d\Omega) / (d\sigma/d\Omega)_{\text{Mott}}$

q [fm$^{-1}$]

0 0.5 1 1.5 2 2.5 3 3.5
Cluster States in $^{12}\text{C}$

Monopole Matrix Element revisited

- $M(E0)$ determines the pair decay width
- model-independent self-consistent determination of transition form-factor/density in DWBA
- data at high momentum transfer necessary to constrain matrix element $M(E0) = 5.47 \pm 0.09 \text{ e}^2\text{fm}^2$


$^{12}\text{C} (e,e')^{12}\text{C}$

$E_x = 7.654 \text{ MeV}$

$0_1^+ \rightarrow 0_2^+$
• Calculate the overlap with FMD basis states to find the most important contributions to the Hoyle state

\[
\langle \cdot | 0^+_1 \rangle = 0.94 \\
\langle \cdot | 2^+_1 \rangle = 0.93 \\
\langle \cdot | 0^+_2 \rangle = 0.72 \\
\langle \cdot | 0^+_2 \rangle = 0.71 \\
\langle \cdot | 0^+_2 \rangle = 0.61 \\
\langle \cdot | 0^+_2 \rangle = 0.61 \\
\langle \cdot | 3^-_1 \rangle = 0.83 \\
\langle \cdot | 0^+_3 \rangle = 0.50 \\
\langle \cdot | 0^+_3 \rangle = 0.49 \\
\langle \cdot | 0^+_3 \rangle = 0.44 \\
\langle \cdot | 0^+_3 \rangle = 0.41 
\]

FMD basis states are not orthogonal!

0^+_2 and 0^+_3 states have no rigid intrinsic structure.
Cluster States in $^{12}$C

Two-body Densities and Intrinsic Structure

Cluster Model

$$\rho^{(2)}(r) = \langle \psi | \sum_{i<j} \delta(r - r_{ij}) | \psi \rangle$$

- substract contributions from $\alpha$’s to extract “$\alpha-\alpha$” correlations
- (substracted) two-body density peaks at 3.5 fm
- consistent with compact triangular structure

ground state band

![Graph showing two-body densities and intrinsic structure](image-url)
Cluster States in $^{12}\text{C}$

Two-body Densities and Intrinsic Structure

Cluster Model

$$\rho^{(2)}(r) = \langle \psi | \sum_{i<j} \delta(r - r_{ij}) | \psi \rangle$$

- subtract contributions from $\alpha$’s to extract “$\alpha$-$\alpha$ correlations”
- Hoyle state two-body density peaks at 5 fm, extended tail
  ➤ consistent with triangular structure
- tail in $2_2^+$ and $4_2^+$ states more pronounced
  ➤ admixture of open triangle configurations
Cluster States in $^{12}\text{C}$

Two-body Densities and Intrinsic Structure

Cluster Model

$$\rho^{(2)}(r) = \langle \psi \left| \sum_{i<j} \delta(r - \mathbf{r}_{ij}) \right| \psi \rangle$$

- subtract contributions from $\alpha'$s to extract "$\alpha$-$\alpha$" correlations
- two-body density peaks at 4.5 fm and 10 fm
  ➤ consistent with open triangle/chain configuration

third $0^+$ state band

$0_3^+$
$2_3^+$
$4_3^+$

$\rho^{(2)}(r)$

$\rho^{(2)}(r) - 3 \rho^{(2)}_{\alpha}(r)$
Cluster States in $^{12}$C

Two-body Densities and Intrinsic Structure

ground state band

$\rho^{(2)}(r) = \langle \psi | \sum_{i<j} \delta(r - r_{ij}) | \psi \rangle$

- subtract contributions from $\alpha$'s to extract $\alpha-\alpha$ correlations
- (corrected) two-body density peaks at 3.5 fm for $0^+$ and $2^+$
- $4^+$ state strongly mixed with cluster configurations
Cluster States in $^{12}\text{C}$

Two-body Densities and Intrinsic Structure

$\rho^{(2)}(r) = \langle \psi | \sum_{i<j} \delta(r - r_{ij}) | \psi \rangle$

- Subtract contributions from $\alpha$'s to extract $\alpha-\alpha$ correlations
- Hoyle state two-body density peaks at 5 fm, extended tail
  ➤ consistent with extended triangular structure
- $2^+$ and $4^+$ states have different intrinsic structure
  ➤ admixture of open triangle configurations
Cluster States in $^{12}\text{C}$

Two-body Densities and Intrinsic Structure

FMD

\[ \rho^{(2)}(r) = \langle \psi | \sum_{i<j} \delta(r - r_{ij}) | \psi \rangle \]

- subtract contributions from $\alpha$’s to extract $\alpha-\alpha$ correlations
- two-body density peaks at 4.5 fm and 10 fm
- consistent with chain configuration
Calculate the overlap of FMD wave functions with pure $\alpha$-cluster model space

$$N_\alpha = \langle \Psi | P_{3\alpha} | \Psi \rangle$$

Hoyle state has 15% non-alpha admixtures
Cluster States in $^{12}\text{C}$

Harmonic Oscillator $N\hbar\Omega$ Excitations


$$\text{Occ}(N) = \langle \psi | \delta \left( \sum_i (\hat{H}_{i}^\text{HO}/\hbar\Omega - 3/2) - N \right) | \psi \rangle$$

Cluster States in $^{12}$C

Harmonic Oscillator $N\hbar\Omega$ Excitations


\[
\text{Occ}(N) = \langle \psi | \delta \left( \sum_i (\frac{H^i_{HO}}{\hbar\Omega} - 3/2) - N \right) | \psi \rangle
\]

Cluster Model

**0_1^+**

**2_1^+**

**3_1^-**

**0_2^+**

**0_3^+**

**2_2^+**
Preliminary:
Include $^8\text{Be-}\alpha$ continuum

• In principle it should be described as a three-body continuum

• However $^8\text{Be+}\alpha$ states are lower in energy than $3-\alpha$ configurations up to pretty large hyperradii

• Approximation: consider $^8\text{Be}(0^+)$ and $^8\text{Be}(2^+)$ as bound states

• Could be considered as a microscopic CDCC approach
Cluster Model: $^8$Be-$\alpha$ Continuum

$^8$Be-$\alpha$ wave functions

alpha-cluster model calculations with continuum:


$^8$Be wave functions

• $\alpha$-$\alpha$ configurations up to 9 fm distance, project on $0^+$ and $2^+$, $M = 0, 1, 2$

$$|^{8}\text{Be}_{I,K}\rangle = P_{K0}^{I} \sum_{i} \{ |^{4}\text{He}(-R_{i}/2e_{z})\rangle \otimes |^{4}\text{He}(R_{i}/2e_{z})\rangle \} c_{i}^{I}$$

• reproduces ground state energy within 50 keV compared to full calculation

$^{12}$C configurations

• $^8$Be($0^+,2^+$) and $\alpha$ at distance $R$
• $^8$Be($2^+$) can have different orientations with respect to distance vector
• $^8$Be($0^+,2^+$)+$\alpha$ configurations have to be projected on total angular momentum

$$|^{8}\text{Be}_{I,K},^{4}\text{He};R;JM\rangle = P_{MK}^{I} \{ |^{8}\text{Be}_{I,K}(-1/3Re_{z})\rangle \otimes |^{4}\text{He}(2/3Re_{z})\rangle \}$$

Matching to Coulomb asymptotics with $R$-matrix

• at large distances relative wave functions are given by Whittaker/Coulomb functions
• our GCM $^8$Be+$\alpha$ wave functions have to be transformed into RGM wave functions
Cluster Model: $^8$Be-$\alpha$ Continuum

GCM Energy Surfaces

- energy surfaces contain localization energy for relative motion of $^8$Be and $\alpha$
- $2^+$ energy surface depends strongly on orientation of $^8$Be $2^+$ state – $M = 2$ most attractive
Model Space

- Internal region in the cluster model: 3-\(\alpha\) configurations on a grid
- External region: \(^8\text{Be}(0^+, 2^+)-\alpha\) configurations
- Channel radius has to be large: only Coulomb interaction between \(^8\text{Be}\) and \(\alpha\) and Coulomb coupling between different \(^8\text{Be}\) channels should be small
- Check that results are independent from channel radius: used \(\alpha = 16.5\) fm here

Scattering Solutions

- Obtain scattering matrix using multichannel microscopic \(R\)-matrix approach
  Descouvemont, Baye, Phys. Rept. 73, 036301 (2010)
- Diagonal phase shifts and inelastic parameters: \(S_{ii} = \eta_i \exp\{2i\delta_i\}\)
- Eigenphases: \(S = V^{-1}DV, D_{\alpha\alpha} = \exp\{2i\delta_\alpha\}\)
Cluster Model: $^{8}\text{Be-}\alpha$ Continuum

$0^+\text{ Phase shifts}$

**Eigenphaseshifts**

<table>
<thead>
<tr>
<th>$E$ [MeV]</th>
<th>$\Gamma$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0_2^+$</td>
<td>0.29</td>
</tr>
<tr>
<td>$0_3^+$</td>
<td>4.11</td>
</tr>
<tr>
<td>$0_4^+$</td>
<td>4.76</td>
</tr>
</tbody>
</table>

**Phaseshifts**

- non-resonant background
- strong coupling between $^{8}\text{Be}(0^+)$ and $^{8}\text{Be}(2^+)$ channel at 4.1 MeV
- Hoyle state not resolved in phase shifts
- stability of broad resonance with respect to channel radius?

**Inelasticities**

Gamow states

- non-resonant background
- strong coupling between $^{8}\text{Be}(0^+)$ and $^{8}\text{Be}(2^+)$ channel at 4.1 MeV
- Hoyle state not resolved in phase shifts
- stability of broad resonance with respect to channel radius?
Cluster Model: $^8$Be-$\alpha$ Continuum

$2^+$ Phase shifts

### Eigenphaseshifts

### Phaseshifts

### Inelasticities

#### Gamow states

<table>
<thead>
<tr>
<th>Energy [MeV]</th>
<th>Width [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^+_2$</td>
<td>1.51</td>
</tr>
<tr>
<td>$2^+_3$</td>
<td>4.31</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

- non-resonant background
- strong $L = 2$ $^8$Be($0^+$) and $^8$Be($2^+$) resonances
Cluster Model: $^8$Be-$\alpha$ Continuum

$4^+$ Phase shifts

Eigenphaseshifts

Phaseshifts

Inelasticities

Gamow states

<table>
<thead>
<tr>
<th>Energy [MeV]</th>
<th>Width [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4_1^+$</td>
<td>1.17</td>
</tr>
<tr>
<td>$4_2^+$</td>
<td>4.06</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

- $4_1^+$ state very narrow, not resolved in phase shifts
- $4_2^+$ state mostly $^8$Be(0+)
Cluster Model: $^8\text{Be-}\alpha$ Continuum

$3^-$ Phase shifts

- Eigenphaseshifts
- Phaseshifts
- Inelasticities

Gamow states

<table>
<thead>
<tr>
<th>$E$ [MeV]</th>
<th>$\Gamma$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^+_1$</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>$4.46 \cdot 10^{-6}$</td>
</tr>
</tbody>
</table>

$3^+_1$ state very narrow, not resolved in phase shifts
Work in Progress: 
FMD calculation with $^8\text{Be-}\alpha$ Continuum

- Improved UCOM interaction
  - Correlation functions from SRG
  - Modify strength of spin-orbit force to account for omitted three-body forces

$^8\text{Be-}\alpha$ Continuum
  - To get a reasonable description of $^8\text{Be}$ it is essential to include polarized configurations
  - Investigate non-cluster states: non-natural parity states, $T = 1$ states, M1 transitions, $^{12}\text{B}$ and $^{12}\text{N}$ $\beta$-decay into $^{12}\text{C}$, . . .
  - Calculate strength distributions
Summary

Unitary Correlation Operator Method

- Explicit description of short-range central and tensor correlations
- Realistic low-momentum interaction $V_{\text{UCOM}}$

Fermionic Molecular Dynamics

- Gaussian wave-packet basis contains HO shell model and Brink-type cluster states

Cluster States in $^{12}\text{C}$

- Consistent description of ground state band and clustered states including the Hoyle state
- Test Hoyle state structure with electron scattering
- Two-body densities are a model independent tool for understanding structure
- Cluster states need tremendous model spaces in harmonic oscillator basis
- A proper treatment of the continuum above the 3-$\alpha$ threshold is necessary – first results with $^8\text{Be}(0^+,2^+)+\alpha$ continuum in the cluster model

Thanks to my collaborators:
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