RBUU Simulation of Heavy-Ion Collisions for RAON

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Motivation

Heavy-Ion Collision (HIC) produces highly compressed and heated matter (Compact astrophysical objects, early universe …)

Rare isotope accelerator in Korea, RAON

- In operation in 2019
- Maximum beam energy of ~ 200-250 AMeV

Properties of nuclear matter estimated by RBUU simulation
Nuclear transport theory

Heavy-ion collision

Evolution of system away from thermal equilibrium by transferring energy, momentum, particle, ...

Non-equilibrium aspects in the evolution of a collision for a many-body system

The system described by a phase-space distribution function describing the distribution of microscopic states
Consider nucleon & sigma, omega and rho-meson field

\[ \mathcal{L} = \bar{\psi} [i\gamma_{\mu} \partial^{\mu} - (M_N - g_{\sigma}\sigma) - g_{\omega}\gamma_{\mu}\omega^{\mu} - g_{\rho}\gamma_{\mu} \vec{\rho}^{\mu} \cdot \vec{\rho}^{\mu}] \psi + \frac{1}{2} (\partial_{\mu}\sigma \partial^{\mu}\sigma - m_{\sigma}^2\sigma^2) - U(\sigma) + \frac{1}{2} m_{\omega}^2 \omega_\mu \omega^\mu + \frac{1}{2} m_{\rho}^2 \vec{\rho}_\mu \cdot \vec{\rho}^{\mu} - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} - \frac{1}{4} \vec{G}_{\mu\nu} \cdot \vec{G}^{\mu\nu} \]

With the Mean Field approximation:

\[
m_{\sigma}^2\sigma + a\sigma^2 + b\sigma^3 = g_{\sigma}\rho_S
\]
\[
m_{\omega}\omega_0 = g_{\omega}\rho_B
\]
\[
m_{\rho}\rho_0^{(3)} = g_{\rho}\rho_B^{(3)}
\]

\[
\rho_S = \bar{\psi}\psi, \quad \rho_B = \bar{\psi}\gamma_0\psi, \quad \rho_B^{(3)} = \bar{\psi}\gamma_0\tau^{(3)}\psi
\]

\[
U(\sigma) = \frac{1}{3} a\sigma^3 + \frac{1}{4} b\sigma^4
\]
\[
\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu
\]
\[
\vec{G}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu
\]

\[
f_i \equiv \frac{g_i^2}{m_i^2}, \quad A \equiv \frac{a}{g_\sigma^3}, \quad B \equiv \frac{b}{g_\sigma^4} (i = \sigma, \omega, \rho, \delta)
\]

\[
\begin{align*}
  f_\sigma \text{ (fm}^2) & = 10.33 \\
  f_\omega \text{ (fm}^2) & = 5.42 \\
  f_\rho \text{ (fm}^2) & = 0.95 \\
  f_\delta \text{ (fm}^2) & = 0.00 \\
  A \text{ (fm}^{-1}) & = 0.033 \\
  B & = -0.0048
\end{align*}
\]

Relativistic Vlasov equation (MF potential, no collision)

$$\left[ k_\mu^* \partial^\mu_x + (g_\omega k_\nu^* F^{\mu\nu} + M_N^* (\partial^\mu_x M_N^*)) \partial^k_\mu \right] f(x, k^*) = 0$$

Relativistic Boltzmann equation (MF potential, 2-body collision)

$$\left[ k_\mu^* \partial^\mu_x + (g_\omega k_\nu^* F^{\mu\nu} + M_N^* (\partial^\mu_x M_N^*)) \partial^k_\mu \right] f(x, k^*)$$

$$= \frac{1}{2} \int \frac{d^3k_2}{E_2} \frac{d^3k_1'}{E_1'} \frac{d^3k_2'}{E_2'} W(k_1', k_2' \leftarrow k, k_2)(f_1' f_2' - f f_2)$$

Relativistic Boltzmann-Uehling-Uhlenbeck equation

(MF potential, 2-body collisions, Q.M. effect)

$$\left[ k_\mu^* \partial^\mu_x + (g_\omega k_\nu^* F^{\mu\nu} + M_N^* (\partial^\mu_x M_N^*)) \partial^k_\mu \right] f(x, k^*)$$

$$= \frac{1}{2} \int \frac{d^3k_2}{E_2} \frac{d^3k_1'}{E_1'} \frac{d^3k_2'}{E_2'} W(k_1', k_2' \leftarrow k, k_2)$$

$$\times [f_1' f_2' (1 - f)(1 - f_2) - f f_2 (1 - f_1')(1 - f_2')]$$

“Pauli blocking factor”
Numerical calculation

**RBUU code : simulate Heavy-ion collisions**
- (1995) C. Fuchs : first developed in Munich
- (2014) improved stability

**Test particle method > Parallel ensemble method**

single particle phase-space distribution function represented by N covariant Gaussian test particles

\[
g(x, x_i) = \frac{1}{(\pi \sigma^2)^{3/2}} e^{\left(\frac{(x - x_{i\mu})^2 - (u_i^\mu)^2}{\sigma^2}\right)}
\]

\[
g(k^*, k_i^*) = \frac{1}{(\pi \sigma_k^2)^{3/2}} e^{\left(\frac{k^*^2 - (k_i^\mu)^2}{\sigma_k^2}\right)}
\]

\[
f(x, k^*) = \frac{1}{N} \sum_{i=1}^{AN} g(x, x_i) g(k^*, k_i^*)
\]

\[\sigma = 1.4\text{fm} \]
\[\sigma_k = 0.346\text{fm}^{-1} \]
\[N=100\]
Liouville’s theorem

the phase-space density of a certain volume element is constant with time as it follows the trajectory of the system

\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + \sum_i \frac{df}{dx_i} \frac{\partial x_i}{\partial t} + \sum_i \frac{df}{dp_i} \frac{\partial p_i}{\partial t} = 0
\]

Vlasov equation with the test particle method & Liouville’s theorem,

**Relativistic Hamilton’s equation**

\[
\frac{d}{dt} k_{i}^{*\mu} = \frac{1}{u_i^0} \left( \frac{k_{i\nu}^*}{m_i^*} F_i^{\mu\nu} + \partial_i^\mu m_i^* \right)
\]

\[
\frac{d}{dt} x_i^\mu = \frac{1}{u_i^0} u_i^\mu
\]
Baryon density

\[ \rho_B(x) = \frac{1}{N} \sum_{i=1}^{AN} g(x, x_i) u_i \]

Isospin asymmetry

\[ \rho_I(x) = \frac{[\rho_n(x) - \rho_p(x)]}{\rho_B(x)} \]

Local temperature by a fit of the **RBUU momentum distribution** to the **Fermi-Dirac distribution**, assuming **local thermodynamic equilibrium**

\[ n(x, k) = \frac{(2\pi)^3}{4 \pi} \frac{1}{N(\pi \sigma \sigma_k)^3} \sum_{i=1}^{AN} e^{\left[ (x_\mu - x_{i\mu})^2 - (x_\mu - x_{i\mu} u_{i\mu}^\mu)^2 \right] / \sigma_k^2} e^{[k^*2 - (k^* u_{i\mu}^\mu)^2] / \sigma_k^2} \]

\[ n(x, k, T) = \frac{1}{1 + \exp[-(\mu^* - k^* u_{i\mu}^\mu) / T]} \]

\( k_0^* = \sqrt{k^*2 + m^*2} \)

... \( T \) is the only free parameter, \( \mu^* \) is a T-dependent parameter

Determine \( \mu^* \) from the **baryon number conservation**

\[ j_0(x) = 4 \int \frac{d^3 k}{(2\pi)^3} n(x, k, T) \]

Consider the point \( x=0 \)

... practically, a \((5 \times 5 \times 5)\text{fm}^3\) **cubical box** around the center
“Artificial” temperature
diffused momentum distribution function due to the finite \textit{gaussian width} 
\textgreater\textgreater\ \textit{increment in temperature}

\[
n(x, \vec{k}, T) = \frac{1}{1 + \exp[-(\mu^* - k^* u^\mu)/T]}
\]

\[
\tilde{n}(x, \vec{k}, T) = \int dk'^\mathcal{A} \ m^* n(x, \vec{k}', T) \ g(k' - k) \ \delta(k'^2 - m^*2) \ \Theta(k^*')
\]

For a Fermi function in a rest frame

Results ($^{197}$Au + $^{197}$Au @200AMeV)

- **Time evolution of density distribution**
- **Time evolution of baryon density and isospin asymmetry at the center**

$\rho_{\max} \approx 2.0\rho_0$

$\rho_I \approx 0.107$

($\rho_0$ : saturation density, $\sim 0.16\text{fm}^{-3}$)
Take average on $\theta_k, \phi_k$

*1-dim. fct.* of $|k| = \sqrt{k_1^2 + k_2^2 + k_3^2}$

$$\bar{n}(|k|) = \frac{\int n(x, k) |k|^2 d\Omega}{\int d\Omega}$$

$$\chi^2 = 4.85 \times 10^{-7}$$

$$(\chi^2 \equiv \sum_i [\bar{n}(k_i)_{RBUU} - \bar{n}(k_i)_{FD}]^2, \quad i : \text{all grid points})$$

100 mesh points in $|k| = 1 \sim 5$

<table>
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<tr>
<th>time [fm/c]</th>
<th>40</th>
<th>44</th>
<th>48</th>
<th>52</th>
<th>56</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>T [MeV]</td>
<td>26.5</td>
<td>22.1</td>
<td>18.6</td>
<td>15.0</td>
<td>10.6</td>
<td>8.0</td>
</tr>
<tr>
<td>$\mu^*$ [MeV]</td>
<td>753.0</td>
<td>783.1</td>
<td>807.3</td>
<td>832.2</td>
<td>854.8</td>
<td>870.7</td>
</tr>
</tbody>
</table>

$E/A$ [MeV] 250 600

$T_{\text{max}}$ [MeV] 30.3 52.1
Results \( ^{132}\text{Sn} + ^{64}\text{Ni} \) \@ 250 AMeV

\[ \rho_{\text{max}} \approx 2.1 \rho_0 \]

\[ \rho_I \approx 0.100 \]

(\( \rho_0 \): saturation density, \( \sim 0.16 \text{fm}^{-3} \))
The first application of a transport model to HIC at low energy.
Summary

• Simulation of HIC experiments by using the RBUU code for estimation of properties of nuclear matter that is expected to be created in RAON.

• Baryon density, isospin asymmetry, local temperature for $^{197}$Au on $^{197}$Au @200AMeV & $^{132}$Sn on $^{64}$Ni @250AMeV reactions:
  $\rho_{\text{max}} \sim 2\rho_0$, $\rho_I \sim 0.1$, $T_{\text{loc}} \sim 20-40$MeV

• Phase structure of nuclear matter in terms of three thermodynamic variables:
  temperature, baryon chemical potential and isospin chemical potential

• Ongoing development of advanced transport calculation code for better results
Thank you
감사합니다