Structure of light neutron-rich nucleus, $^5$H

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Theoretical important issue:

- Can we describe observed 4n system using realistic NN interaction and $T=3/2$ three-body force?

Motivated by experimental data, we started to study tetra neutron system.
Possibility of generating a 4-neutron resonance with a $T = 3/2$ isospin 3-neutron force

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We consider the theoretical possibility of generating a narrow resonance in the 4-neutron system as suggested by a recent experimental result. To that end, a phenomenological $T = 3/2$ 3-neutron force is introduced, in addition to a realistic $NN$ interaction. We inquire what the strength should be of the 3$n$ force to generate such a resonance. The reliability of the 3-neutron force in the $T = 3/2$ channel is examined, by analyzing its consistency with the low-lying $T = 1$ states of $^4$He, $^3$He, and $^3$Li and the $^4$H $+n$ scattering. The \textit{ab initio} solution of the 4$n$ Schrödinger equation is obtained using the complex scaling method with boundary conditions appropriate to the four-body resonances. We find that to generate narrow 4$n$ resonant states a remarkably attractive 3$N$ force in the $T = 3/2$ channel is required.
For the study of tetraneutron system

We should consider interaction and method:

NN interaction: realistic NN interaction
Method: They reported the energy of tetraneutron was bound energy region to resonant energy region. Especially, for the resonant energy region, we should use Complex scaling method.

For this purpose, we use AV8 NN interaction (central, LS, Tensor). The NN potential is applicable for complex scaling method. Also, we need $T=\frac{3}{2}$ three-nucleon force.
As for 3 nucleon forces, we have Illinois potentials, for example. However, this potential is too complicated to use in order to get resonant state with CSM. At present, we use a simple potential. For this purpose, we use the following shape.

\[ V_{ijk}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^{2} W_n(T) e^{-\left(r_{ij}^2 + r_{jk}^2 + r_{ki}^2\right)/b_n^2} \mathcal{P}_{ijk}(T) \]

\[
W_1(T = 1/2) = -2.04 \text{ MeV} \quad b_1 = 4.0 \text{ fm} \\
W_2(T = 1/2) = +35.0 \text{ MeV} \quad b_2 = 0.75 \text{ fm}
\]

Two range Gaussian potentials
Four parameters are fixed so as to reproduce the low-energy properties of $^3\text{H}$, $^3\text{He}$ and $^4\text{He}$($T=0$).
In order to solve few-body problem accurately,

**Gaussian Expansion Method (GEM), since 1987**
- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

**Developed by Kyushu Univ. Group, Kamimura and his collaborators.**
Review article:
E. Hiyama, M. Kamimura and Y. Kino,
Prog. Part. Nucl. Phys. 51 (2003), 223.

**High-precision calculations of various 3- and 4-body systems:**

- Exotic atoms / molecules,
- 3- and 4-nucleon systems,
- Multi-cluster structure of light nuclei,
- Light hypernuclei,
- 3-quark systems,
- \( ^4 \)He-atom tetramer
\[(H - E)\Psi_{JM} = 0\]

\[H = T + V_1(r_1) + V_2(r_2) + V_3(r_3)\]

\[T = \frac{\hbar^2}{2m} \nabla^2_r + \frac{\hbar^2}{2M} \nabla^2_R\]

\[\Psi_{JM} = \Phi^{(1)}_{JM}(r_1, R_1) + \Phi^{(2)}_{JM}(r_2, R_2) + \Phi^{(3)}_{JM}(r_3, R_3)\]
This Schrödinger equation is solved with Rayleigh-Ritz variational method and we obtain eigen value $E$ and eigen function $\Psi$.

\[
(H - E)\Psi = 0
\]

Here, we expand the total wavefunction in terms of a set of $L^2$-integrable basis function $\{\Phi_n : n = 1, \ldots, N \}$

\[
\Psi = \sum_{n=1}^{N} C_n \Phi_n
\]

The Rayleigh-Ritz variational principle leads to a generalized matrix eigenvalue problem.

\[
\langle \Phi_i | H - E | \sum_{n=1}^{N} C_n \Phi_n \rangle = 0 , \quad (i = 1, \ldots, N)
\]
Where the energy and overlap matrix elements are given by

\[ H_{in} = \langle \Phi_i | H | \Phi_n \rangle \]

\[ N_{in} = \langle \Phi_i | 1 | \Phi_n \rangle \]

Next, by solving eigenstate problem, we get eigenenergy \( E \) and unknown coefficients \( C_n \).

\[
\begin{pmatrix}
( H_{in} ) - E ( N_{in} )
\end{pmatrix}
\begin{pmatrix}
C_n
\end{pmatrix} = 0
\]
An important issue of the variational method is how to select a good set of basis functions.

\[ \Phi_{lmn}(r) = r^\ell e^{-\nu_n r^2} Y_{\ell m}(\mathbf{r}) \]

\[ \nu_n = (1/r_n)^2 \]

\[ r_n = r_1 a^{n-1} \quad (n=1-n_{\text{max}}) \]

The Gaussian basis function is suitable not only for the calculation of the matrix elements but also for describing short-ranged correlations, long-ranged tail behaviour.
Using our method and AV8 NN potential, we start to study the structure of tetraneutron system.
To answer these issues, we employ AV8 NN potential + a phenomenological three-body force.

\[ V_{ij}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^{2} W_n(T) e^{-(r_{i,j}^2 + r_{j,k}^2 + r_{k,i}^2) / b_n^2} \mathcal{P}_{ijk}(T) \]

\[
W_1(T = 1/2) = -2.04 \text{ MeV} \quad b_1 = 4.0 \text{ fm} \\
W_2(T = 1/2) = +35.0 \text{ MeV} \quad b_2 = 0.75 \text{ fm}
\]

These parameters \((W_1, W_2, b_1, b_2)\) are determined so as to reproduce the binding energies of the ground states of \(^3\text{H}\), \(^3\text{He}\) and \(^4\text{He}\).

For 4n system, we need \(T=3/2\) three-body force. We use the same potential with \(T=1/2\), but, different parameter of \(W_1\).

\(W_1(T=3/2)\) = free \(b_1=4.0\text{fm} \Rightarrow W_1\) should be adjusted so as to reproduce the observed 4n system

\(W_2(T=3/2) = +35 \text{ MeV} \quad b_2 = 0.75\)
The observed 4n system was reported from the bound region to resonant region. In order to obtain energy position ($E_r$) and decay width ($\Gamma$), we use complex scaling method.

\[
[H(\theta) - E(\theta)]\Psi_{JM,TT_z}(\theta) = 0
\]

\[
\Psi_{JM,TT_z}(\theta) = \sum_\alpha C_{\alpha}^{(K)}(\theta)\Phi^{(K)}_{\alpha} + \sum_\alpha C_{\alpha}^{(H)}(\theta)\Phi^{(H)}_{\alpha}
\]

\[
r_c \rightarrow r_c e^{i\theta}, \quad R_c \rightarrow R_c e^{i\theta}, \quad \rho_c \rightarrow \rho_c e^{i\theta} \quad (c = K, H)
\]

The energy pole is stable with respect to $\theta$. Re($E$) corresponds to energy With respect to 4n breakup threshold. Im($E$) corresponds to $\Gamma/2$. 

4n breakup threshold
energy trajectory of $J=0^+$ state changing $W_1$
In order to reproduce the data of 4n system, we need $W_1(T=3/2) = -36$ MeV $\sim -30$ MeV.

It should be noted that $W_1(T=1/2) = -2.04$ MeV to reproduce the observed binding energy of $^4\text{He}$, $^3\text{He}$ and $^3\text{H}$.

Attraction is 15 times Stronger.

To check the validity of three-body force, we calculate the energies of $^4\text{H}$, $^4\text{He}(T=1)$, $^4\text{Li}$.

$$V_{i\bar{j}k}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^{2} W_n(T) e^{-\left(\frac{r_{i\bar{j}}^2 + r_{\bar{j}k}^2 + r_{k\bar{i}}^2}{b_n^2}\right)} P_{i\bar{j}k}(T)$$

$W_1(T=3/2) = \text{free} \quad b_1 = 4.0 \text{fm}$
$W_2(T=3/2) = +35$ MeV $b_2 = 0.75 \text{ fm}$

Question: $W_1$ value for $T=3/2$ is reasonable?
Table 4.1: Energy levels of $^3$H defined for channel radius $a_0 = 4.9$ fm. All energies and widths are in the c.m. system.

<table>
<thead>
<tr>
<th>$E_0$ (MeV)</th>
<th>$J^P$</th>
<th>$T$</th>
<th>$\Gamma$ (MeV)</th>
<th>Decay</th>
<th>Reactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>g.s. a</td>
<td>2</td>
<td>1</td>
<td>3.42</td>
<td>a,$^2$H</td>
<td>1.11</td>
</tr>
<tr>
<td>0.31</td>
<td>1</td>
<td>1</td>
<td>6.73 b</td>
<td>a,$^2$H</td>
<td>11.12</td>
</tr>
<tr>
<td>2.08</td>
<td>0</td>
<td>1</td>
<td>8.92</td>
<td>a,$^2$H</td>
<td>11.12</td>
</tr>
<tr>
<td>2.83</td>
<td>1</td>
<td>1</td>
<td>12.99</td>
<td>a,$^2$H</td>
<td>11.12</td>
</tr>
</tbody>
</table>

$^a$ 3.19 MeV above the n+$^3$He mass  
$^b$ Primarily $^3$He

Table 4.24: Energy levels of $^4$Li defined for channel radius $a_0 = 4.9$ fm. All energies and widths are in the c.m. system.

<table>
<thead>
<tr>
<th>$E_0$ (MeV)</th>
<th>$J^P$</th>
<th>$T$</th>
<th>$\Gamma$ (MeV)</th>
<th>Decay</th>
<th>Reactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>g.s. a</td>
<td>2</td>
<td>1</td>
<td>6.03</td>
<td>p,$^4$He</td>
<td>3</td>
</tr>
<tr>
<td>0.32</td>
<td>1</td>
<td>1</td>
<td>7.35 b</td>
<td>p,$^4$He</td>
<td>3</td>
</tr>
<tr>
<td>2.08</td>
<td>0</td>
<td>1</td>
<td>9.35</td>
<td>p,$^4$He</td>
<td>3</td>
</tr>
<tr>
<td>2.85</td>
<td>1</td>
<td>1</td>
<td>13.51 c</td>
<td>p,$^4$He</td>
<td>3</td>
</tr>
</tbody>
</table>

$^a$ 4.07 MeV above the p+$^4$He mass  
$^b$ Primarily $^3$He
$^c$ Primarily $^1$P.
If we use $W_1 = -36 \text{ MeV} \sim -30 \text{ MeV}$ to reproduce the observed data of $4n$, we have strong binding energies of $^4\text{H}$, $^4\text{He}$ ($T=1$) and $^4\text{Li}$.

This result is inconsistent with the data of $A=4$ nuclei. The $J=2^-$ state of $A=4$ nuclei should be resonant states.

On the contrary, when $W_1 \sim -18 \text{ MeV}$, we have unbound states for $A=4$ nuclei. How about the tetraneutron system?
We calculated 3n system. The lowest state should be $J=3/2^-$. We need $W_1 = -40 \text{ MeV}$ to have a resonant state for 3n system. $W_1 = -40 \text{ MeV}$ is much more attractive than the case of 4n system. Then, there might not exist a 3n system as a resonant state.

$$V_{ijk}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^{2} W_n(T) e^{-\left(\frac{r_{ij}^2 + r_{jk}^2 + r_{ki}^2}{b_n^2}\right)} P_{ijk}(T)$$

$W_1(T=3/2) = \text{free} \quad b_1 = 4.0 \text{ fm}$

$W_2(T=3/2) = +35 \text{ MeV} \quad b_2 = 0.75 \text{ fm}$
How do we consider this inconsistency?

• The T=3/2 force is just a phenomenological.

\[ V_{ij\;k}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^{2} W_n(T) e^{-(r_{ij}^2+r_{jk}^2+r_{ki}^2)/b_n^2} \mathcal{P}_{ijk}(T) \]

Should we consider spin-dependent term in three-body force?
Tensor force, spin-orbit force???

In this way, at present, in our calculation, it would be difficult to describe resonant state for a tetraneutron system.
Further investigation of $T=3/2$ force and existence of tetra neutron system:

structure of $^5\text{H}$ is one of good candidate.

$^1\text{H}+n+n = 1.7 \pm 0.3 \text{ MeV}$

$\Gamma = 1.9 \pm 0.4 \text{ MeV}$

transfer reaction $p(^6\text{He}, ^2\text{He})^5\text{H}$


However, I found that $T=3/2$ and $T=1/2$ three-body forces contribute to the energies of $^5\text{H}$. 
We cited this experiment. However, you have many different decay widths.

To confirm the energy and width of $^5\text{H}$ is important for the study of hypernuclear physics.

I shall explain why.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$E_R$ (MeV)</th>
<th>$\Gamma_R$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^5\text{H}$ (full)</td>
<td>(1.57, 1.53)</td>
<td></td>
</tr>
<tr>
<td>$^5\text{H}$ ($d = 0$)</td>
<td>(1.55, 1.35)</td>
<td></td>
</tr>
<tr>
<td>Theor. [16]</td>
<td>(2.26, 2.93)</td>
<td></td>
</tr>
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<td>Theor. [12]</td>
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<td>Exp. [3]</td>
<td>(1.7 ± 0.3, 1.9 ± 0.4)</td>
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<td>(1.8 ± 0.1, &lt; 0.5)</td>
<td></td>
</tr>
<tr>
<td>Exp. [4]</td>
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<td></td>
</tr>
<tr>
<td>Exp. [5]</td>
<td>(2, 2.5)</td>
<td></td>
</tr>
<tr>
<td>Exp. [6]</td>
<td>(3, 6)</td>
<td></td>
</tr>
<tr>
<td>Exp. [9]</td>
<td>(5.5 ± 0.2, 5.4 ± 0.6)</td>
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</tr>
</tbody>
</table>

Ground-state properties of $^5\text{H}$ from the $^6\text{He}(d,^3\text{He})^5\text{H}$ reaction


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$^5\text{H}$ is important from side of hypernuclear physics.
Evidence for Heavy Hyperhydrogen $^6_\Lambda \text{H}$

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(FINUDA Collaboration)

A. Gal

Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel
(Received 2 November 2011; published 24 January 2012)
No Pauli principle between $N$ and $\Lambda$

Due to the attraction of $\Lambda N$ interaction, the resultant hypernucleus will become more stable against the neutron decay. $\Lambda$ particle can reach deep inside, and attract the surrounding nucleons towards the interior of the nucleus.

We call this phenomena ‘gluelike’ role of $\Lambda$ particle.
Another interesting issue is to study the excited states of $^7\Lambda$He.

One of the excited state was observed at JLab.

Observed at J-Lab experiment


In this way, due to attendance of $\Lambda$ particle, we have more stable hypernuclear states.
$t + n + n + \Lambda$

$1/2^+$
$1.7 \pm 0.3 \text{ MeV}$

FINUDA experiment

$\Gamma = 1.9 \pm 0.4 \text{ MeV}$

$4H + n + n$

$0.3 \text{ MeV}$

$5H: \text{super heavy hydrogen}$

$6H$

$\Lambda$
Before experiment, the following authors calculated the binding energies by shell model picture and G-matrix theory.


Motivated by the experimental data, I calculated the binding energy of $^6_\Lambda^1\text{H}$ and I shall show you my result.
Four-body calculation of $^6\Lambda H$

Before doing full 4-body calculation, it is important and necessary to reproduce the observed binding energies of all the sets of subsystems in $^6\Lambda H$.

Namely, all the potential parameters are needed to adjust in the 2- and 3-body subsystems.

Among the subsystems, it is extremely important to adjust the energy of $^5H$ core nucleus.
To calculate the binding energy of $^6_\Lambda H$, it is very important to reproduce the binding energy of the core nucleus $^5H$.

Transfer reaction $p(^6He, ^2He)^5H$


To reproduce the data, for example, R. De Diego et al, Nucl. Phys. A786 (2007), 71. calculated the energy and width of $^5H$ with $t+n+n$ three-body model using complex scaling method. The calculated binding energy for the ground state of $^5H$ is 1.6 MeV with respect to $t+n+n$ threshold and width has 1.5 MeV.
Even if the potential parameters were tuned so as to reproduce the lowest value of the Exp., \( E=1.4 \text{ MeV}, \Gamma=1.5 \text{ MeV}, \)
we do not obtain any bound state of \( ^6_\Lambda \text{H}. \)

On the contrary, if we tune the potentials to have a bound state in \( ^6_\Lambda \text{H}, \) then what is the energy and width of \( ^5_\text{H}? \)

$\Gamma = 1.9 \pm 0.4 \text{ MeV}$

FINUDA experiment

But, FINUDA group provided the bound state of $^6\Lambda H$.  

$^5H$: super heavy hydrogen
How should I understand the inconsistency between our results and the observed data?

We need more precise data of $^5\text{H}$.


To get bound state of $^6\text{H}$, the energy should be lower than the present data.
We cited this experiment. However, you have many different decay widths. width is strongly related to the size of wavefunction.

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</table>

I hope that the confirmation experiment for $^6\Lambda$H is important. Also, the confirmation experiment, especially to determine decay width of $^5$H is important to conclude whether or not to have a bound state in $^6\Lambda$H.
Therefore, we started to calculate $^5$H as five-body problem.

K-channel (I used 60 kinds of channel.)

3+2 channel (30 kinds)

H-channel (30 kinds)

Totally, 120 Jacobian coordinates
Hamiltonian:

To discuss the validity of $T=3/2$ 3-body force and NN two-body force, I should use AV8' and $T=3/2$ force used our 4n paper. And I should use CSM for $5H$ to get resonant energy and decay width.

But, first, as the first step, to calculate $5H$, it is much easier to use just central force +5-body force. For this purpose, I use MT13 potential (central force) and 5-body force.

deuteron: $-2.2$ MeV, the energy of $3H$: $-8.5$ MeV
$V=5V_0 \exp\left(-\left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2\right)/(5r_0^2)\right)$

Just now, I will use ACC method to extrapolate resonant state. $V_0$ and $R_0$ are parameters.
\[ V = 5V_0 \exp\left(-\left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2\right)/(5r_0^2)\right) \]

\[ R_0 = 3.0 \text{ fm} \quad \text{Energy of } ^5\text{H} \]

\begin{array}{|c|c|c|}
\hline
V_0 & \text{Rimas} & \text{Emiko} \\
\hline
-3.0 & -11.16 \text{ MeV} & -11.57 \text{ MeV} \\
-2.5 & -10.27 & -10.57 \\
-2.0 & -9.39 & -9.65 \\
-1.55 & -8.65 & -8.93 \\
\hline
\end{array}

We are happy to see our results are consistent with each other.

Just yesterday, I succeeded in making code of $^5\text{H}$ with MT13+ three-body force.

\[ V_{ijk}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^{2} W_n(T) e^{-\left(r_{ij}^2 + r_{jk}^2 + r_{ki}^2\right)/b_n^2} P_{ijk}(T) \]

- Add artificial binding $\lambda V(\rho)$ potential to the Hamiltonian ($\lambda V(\rho)$ should not affect the thresholds)
- Calculate several binding energies of the system $E_i(\lambda_i)$
- Determine accurately $\lambda_0$ such that $E(\lambda_0) = 0$ (or $E(\lambda_0) = E_{\text{threshold}}$)
- Extrapolate $E_{\text{res}} = E(\lambda = 0)$ using $E_i(\lambda_i)$ and $\lambda_0$ values, knowing that:

$$k_{\text{res}}(\lambda \to \lambda_0) \sim i\sqrt{\lambda - \lambda_0}; \quad k_{\text{res}} = \frac{m}{\hbar^2}(E - E_{\text{threshold}})$$

Trajectory of the S-matrix pole with $\lambda$

Algebraic branching point $\lambda = \lambda_0$

Analytic behavior in the vicinity of the branching point

$$k_{\text{res}}(\lambda \to \lambda_0) \sim i\sqrt{\lambda - \lambda_0}$$

- Add artificial binding \( \lambda V(\rho) \) potential to the Hamiltonian (\( \lambda V(\rho) \) should not affect the thresholds)
- Calculate several binding energies of the system \( E_i(\lambda_i) \)
- Determine accurately \( \lambda_0 \) such that \( E(\lambda_0) = 0 \) (or \( E(\lambda_0) = E_{\text{threshold}} \))
- Extrapolate \( E_{res} = E(\lambda = 0) \) using \( E_i(\lambda_i) \) and \( \lambda_0 \) values, knowing that:
  \[ k_{res}(\lambda \rightarrow \lambda_0) \sim i\sqrt{\lambda - \lambda_0} \]
- Extrapolation using Padé approximant

\[ k_{res}(\lambda) = P[m, n] = \frac{a_1\sqrt{\lambda - \lambda_0} + a_2 \left( \sqrt{\lambda - \lambda_0} \right)^2 + \cdots + a_m \left( \sqrt{\lambda - \lambda_0} \right)^m}{1 + b_1\sqrt{\lambda - \lambda_0} + b_2 \left( \sqrt{\lambda - \lambda_0} \right)^2 + \cdots + b_n \left( \sqrt{\lambda - \lambda_0} \right)^n} \]

Resonance trajectory for \( L=1, S=1 \) state of \(^4\text{H}\)

Resonance trajectory for \( J=1/2^+ \) state of \(^5\text{H}\)
Currently, our result is similar with

\begin{align*}
E &= 1.7 \text{ MeV} \quad \Gamma = 2.2 \text{ MeV} \\
\end{align*}
\[ V_{4n} = W e^{-\rho/\rho_0}; \quad \rho = \sqrt{2(r_1^2 + r_2^2 + r_3^2 + r_4^2)} \]

\[ \rho_0 = 2.5 \text{ fm} \]
Future plan

AV8’ potential +three-body force

CSM method?

What is value for $W_1(T=3/2)$ to reproduce the data of 5H? What is decay width?

I will answer the experimental result of $^6\Lambda{}^4\text{H}$ as six-body problem.
Thank you!