Spin-isospin excitations
in relativistic nuclear field theory

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• Relativistic Nuclear Field Theory (RNFT) solutions to the nuclear hierarchy problem

• Multiple meson exchange and re-scattering: emergent collective vibrations (phonons)

• Spin-isospin nuclear response: Gamow-Teller (GTR), spin-dipole etc.

• High-order pion-exchange contributions to the nucleonic self-energy and GTR

• Deuteron transfer modes: precursors of deuteron condensate in N=Z nuclei

• Conclusions and perspectives
• Nuclear scales: Hierarchy problem

Non-relativistic nuclear theory:

\[ H = K + V \]

center of mass internal degrees of freedom: next energy scale

QFT: interaction String theory: merging strings

• No connection between the scales in the traditional NS models
Relativistic Nuclear Field Theory (RNFT):

- **RNFT as a solution**: microscopic, universal, connecting scales from Quantum Hadrodynamics to emergent collective phenomena
- Lagrangian for mesons and nucleons constrained by QCD symmetries and sum rules
- Lorentz covariance: ~5-10% accuracy at the excitation energy of interest (grows with energy)
- Spin-orbit and tensor “forces” are naturally included
- Fewer parameters; hidden correlations minimized (4-10 universal parameters)
- Natural extension to the inclusion of the delta isobar, to higher excitation energy ~200-300 MeV and to hypernuclei
- Non-perturbative self-consistent response theory with high-order NN correlations: time(energy)-dependent interaction
Systematic expansion in the RNFT: single-nucleon self-energy

Quantum Hadrodynamics (QHD)

\[ \sum_{\text{RHF}} = \omega + \rho + \pi + \sigma = m \approx \chi_{\text{EFT}} \]

\begin{align*}
\sum_{\text{e}} &= \sum_{\text{L}} + \sum_{\text{T}} + \ldots \\
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\end{align*}

- Explicit or implicit (Fierz transformation)
- Relativistic Hartree-Fock (RHF)
- Adjusting masses and couplings at HF level (Covariant DFT, P. Ring et al.)
- Relativistic Brueckner-HF (from bare interaction: S. Shen, H. Liang et al.)

Quasiparticle-vibration coupling (QVC)
Beyond Hartree-Fock: quasiparticle-vibration coupling (QVC)

Additional “potential”
  = “self-energy” =
  = “mass operator”
with energy dependence

One-body propagator $G$: Dyson equation

\[
G (\varepsilon) = G_0 (\varepsilon) + G_0 (\varepsilon) \left[ \Sigma_{HF} + \Sigma^e (\varepsilon) \right] G (\varepsilon)
\]

\[
\Sigma^{(e)\eta_1\eta_2} (\varepsilon) = \sum_{\eta=\pm 1} \sum_{k,\mu} \frac{\gamma^{\eta_1\eta_1\eta} \gamma^{\eta_2\eta}_\mu k_1 k \gamma^{\eta_2\eta_2\eta*}_\mu k_2 k}{\varepsilon - \eta (E_k + \Omega_\mu - i\delta)} \quad \eta = \pm 1
\]

forward / backward components in Nambu space: superfluidity included
**Nuclear response: toward a unified description of high-frequency oscillations and low-energy spectroscopy**

**Bethe-Salpeter Equation (here in Dirac-Nambu space):**

$n$-th order correlated propagator:

\[
R^{(n)} = \begin{cases} 
R^{e(n)} & \text{if } n = 3, 3p3h (\text{‘3body’}) \\
R^{(n)} & \text{otherwise}
\end{cases}
\]

**“Conventional” NFT (Copenhagen-Milano-...):**

\[
\Phi^{(2)} = \Phi^{(n+1)} = \begin{cases} 
R^{e(n)} & \text{if } n = 3, 3p3h (\text{‘3body’}) \\
R^{(n)} & \text{otherwise}
\end{cases}
\]

\[
\Phi^{(n+1)} = \Phi^{(n)}
\]

E.L. PRC 91, 034332 (2015)
Isospin transfer response function: proton-neutron relativistic quasiparticle time blocking approximation (pn-RQTBA)

Response

\[ R(\omega) = \tilde{R}^0(\omega) + \tilde{R}^0(\omega) \overline{W}(\omega) R(\omega) \]

Interaction

\[ \overline{W}(\omega) = V_\rho + V_\pi + V_{\delta\pi} + \Phi(\omega) - \Phi(0) \]

Subtraction to avoid double counting (if CDFT-based)
NO subtraction for pionic modes!

CDFT Lagrangian

\[ V_\rho(1, 2) = g_\rho^2 \overline{\tau}_1 \overline{\tau}_2 (\beta \gamma^\mu)_1 (\beta \gamma^\mu)_2 D_\rho(\mathbf{r}_1, \mathbf{r}_2) \]

\[ V_\pi(1, 2) = -\left( \frac{f_\pi}{m_\pi} \right)^2 \overline{\tau}_1 \overline{\tau}_2 (\Sigma_1 \nabla_1)(\Sigma_2 \nabla_2) D_\pi(\mathbf{r}_1, \mathbf{r}_2), \]

\[ V_{\delta\pi}(1, 2) = g' \left( \frac{f_\pi}{m_\pi} \right)^2 \overline{\tau}_1 \overline{\tau}_2 \Sigma_1 \Sigma_2 \delta(\mathbf{r}_1 - \mathbf{r}_2) \]

Static: R(Q)RPA

Free-space coupling: fixed strength

Dynamic (retardation), 2-nd order:

Quasiparticle-vibration coupling

in time blocking approximation

\[ \Phi^{\eta}_{k_1 k_4, k_2 k_3}(\omega) = \]

\[ = \sum_{\mu \xi} \delta_{\eta \xi} \delta_{k_1 k_3} \sum_{k_6} \frac{\gamma^{\eta; -\xi}_{\mu; k_6 k_2} \gamma^{\eta; -\xi^*}_{\mu; k_6 k_4}}{\eta \omega - E_{k_1} - E_{k_6} - \Omega_\mu} + \delta_{k_2 k_4} \sum_{k_5} \frac{\gamma^{\eta; \xi}_{\mu; k_1 k_5} \gamma^{\eta; \xi^*}_{\mu; k_2 k_5}}{\eta \omega - E_{k_5} - E_{k_2} - \Omega_\mu} \]

\[ - \left( \frac{\gamma^{\eta; \xi}_{\mu; k_1 k_3} \gamma^{\eta; -\xi^*}_{\mu; k_2 k_4}}{\eta \omega - E_{k_3} - E_{k_2} - \Omega_\mu} + \frac{\gamma^{\eta; \xi^*}_{\mu; k_3 k_1} \gamma^{\eta; -\xi}_{\mu; k_4 k_2}}{\eta \omega - E_{k_1} - E_{k_4} - \Omega_\mu} \right) \]

Spin-isospin excitations in proton-neutron relativistic time blocking approximation (pn-RTBA)

Gamow-Teller resonance

\[ P = \sum_i \sigma^{(i)} \tau^{(i)} \]

Spin-dipole resonance

\[ P^\lambda_\pm = \sum_i r(i) [\sigma(i) \otimes Y_1(i)]_\lambda t_\pm(i) \]

\[ \Delta L = 1 \]
\[ \Delta T = 1 \]
\[ \Delta S = 1 \]
\[ \lambda = 0, 1, 2 \]


Gamow-Teller resonance in open-shell nuclei: superfluid pairing and phonon coupling (pn-RQTBA)

Overall strength

Low-energy part

$^{68}\text{Ni}$  $^{70}\text{Ni}$  $^{72}\text{Ni}$  $^{74}\text{Ni}$  $^{76}\text{Ni}$  $^{78}\text{Ni}$

$S_{GT}(E')$ (MeV$^{-1}$)

$E'$ (MeV)

$\Sigma B_{GT}(E')$

$\Delta E$ (MeV)

$T_{1/2}$ (s)

EXP

pnRQRPA

pnRQTBA ($\Omega_{\text{max}}=10$ MeV)

pnRQTBA ($\Omega_{\text{max}}=30$ MeV)

Gamow-Teller resonance: comparison to data


Phonon energy

Convergence study at (30 MeV; J=6)

Phonon angular momentum

Convergence

Phonon energy

Convergence study at (30 MeV; J=6)
Spin-dipole resonance in neutron-rich nuclei

Earlier studies:
W.H. Dickhoff et al., PRC 23, 1154 (1981)

Existence of low-lying unnatural parity states indicates that nuclei are close to the pion condensation point. However, it is not clear which observables are sensitive to this phenomenon.

Only nuclear matter and doubly-magic nuclei were studied...

Now: In some exotic nuclei 2-states are found at very low energy. Similar situation with 0-, 4-, 6-,... states.

2- Soft “pionic” mode
h11/2 => g7/2 (M. Sasano)

2- Soft “pionic” mode
g9/2 => f5/2 (M. Sasano)
Low-lying states in $\Delta T=1$ channel and nucleonic self-energy

In spectra of neighboring odd-odd nuclei we see low-lying (collective) states with natural and unnatural parities: $0^+, 0^-, 1^+, 1^-, 2^+, 2^-, 3^+, 3^-, \ldots$ Their contribution to the nucleonic self-energy is expected to affect single-particle states:

\[
(N,Z) \leftrightarrow (N+1,Z-1)
\]

**Nucleonic self-energy beyond mean-field:**

- **Isoscalar**
- **Isovector**

**Underlying Mechanism for pn-pairing or $T=0$ pairing?**

**Matrix element in Nambu space**

\[
\sum_{\eta_1,\eta_2} \langle \varepsilon \rangle = \sum_{\eta=\pm 1} \sum_{k,\mu} \frac{\gamma_{\mu;k_1k} \gamma_{\mu;k_2k}^{*}}{\varepsilon - \eta(E_k + \Omega_\mu - i\delta)}
\]
Single-particle states in 100-Sn: effects of pion dynamics

Truncation scheme: phonons below 20 MeV
Phonon basis: \( \Delta T=0 \) phonons: 2+, 3-, 4+, 5-, 6+
\( \Delta T=1 \) phonons: 0±, 1±, 2±, 3±, 4±, 5±, 6±

Next step: isovector ground state correlations (backward going diagrams), in progress.

Influence of spin-isospin phonons on GTR

Existence of low-energy isospin-flip modes which can couple to single-nucleon degrees of freedom → additional terms in the effective interaction:

\[ \Phi \]

Effect on the nuclear response:

- Preliminary results for $^{78}$Ni (coupling to $0^\pm \rightarrow 6^\pm$):

\[ S_{GT}(E^*) \text{ (MeV)} \]

\[ E^* \text{ (MeV)} \]

$\sim 81\%$ vs $86\%$ of the total GT strength

Additional decrease of the half-life

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*Slide from the Talk of Caroline Robin at DNP APS Meeting (Vancouver, October 2016)*
Proton-neutron dynamical pairing mechanism:

Work in progress…

Proton-neutron particle-particle (deuteron transfer) channel

Ground state of $^{58}$Cu

No deuteron condensate at realistic values of $g'$, but the low-lying phonons are collective and can mediate $T=1$ and $T=0$ proton-neutron dynamical pairing.

Transition amplitudes of the low-energy $1^+$ peak:

$$
\begin{align*}
&g' = 0.60 \\
&E = -5.800 \\
&2p\,1/2\; 2p\,1/2 & 0.3748 & \text{pn} \\
&2p\,3/2\; 2p\,1/2 & 0.2041 & \text{pn} \\
&2p\,1/2\; 2p\,3/2 & 0.1657 & \text{pn} \\
&2p\,3/2\; 2p\,3/2 & 0.1218 & \text{pn} \\
&1f\,5/2\; 1f\,5/2 & 0.0213 & \text{pn} \\
&1f\,5/2\; 2p\,3/2 & 0.0148 & \text{pn} \\
&2p\,3/2\; 1f\,5/2 & 0.0109 & \text{pn} \\
\end{align*}
$$

$$
\begin{align*}
&g' = 0.33 \\
&E = -7.100 \\
&2p\,3/2\; 2p\,1/2 & 0.3720 & \text{pn} \\
&2p\,1/2\; 2p\,3/2 & 0.2075 & \text{pn} \\
&2p\,1/2\; 2p\,1/2 & 0.1693 & \text{pn} \\
&2p\,3/2\; 2p\,3/2 & 0.0750 & \text{pn} \\
&1f\,5/2\; 1f\,5/2 & 0.0605 & \text{pn} \\
\end{align*}
$$

$$
\begin{align*}
&g' = 0.00 \\
&E = -9.700 \\
&2p\,3/2\; 2p\,3/2 & 0.6422 & \text{pn} \\
&1f\,5/2\; 1f\,5/2 & 0.1376 & \text{pn} \\
&2p\,1/2\; 2p\,1/2 & 0.0570 & \text{pn} \\
&2p\,1/2\; 2p\,3/2 & 0.0413 & \text{pn} \\
&2p\,3/2\; 2p\,1/2 & 0.0366 & \text{pn} \\
\end{align*}
$$

Proton-neutron particle-particle (deuteron transfer) channel

\[ ^{56}\text{Ni} \rightarrow ^{58}\text{Cu} \]

No deuteron condensate at realistic values of \( g' \), but the low-lying phonons are collective and can mediate \( T=1 \) and \( T=0 \) proton-neutron dynamical pairing

Proton-neutron dynamical pairing mechanism:

\[ \Phi^{(1)}_\zeta = \]

\[ \begin{align*}
\zeta^{pn} & \quad p \quad n' \\
& \quad p' \\
\zeta^{pn} & \quad n'' \quad n \\
& \quad n' \\
\end{align*} \]


Work in progress…
Outlook

Summary:

- Relativistic NFT offers a powerful framework for the treatment of meson-nucleon correlations beyond the Hartree-Fock approximation.

- Non-perturbative self-consistent response theory based on QHD and including high-order correlations is available for a large class of nuclear excited states.

- Effects of isospin dynamics are introduced into RNFT. In particular, the pion exchange is included non-perturbatively in both shell structure and response calculations.

Perspectives:

- Dynamical proton-neutron pairing (phonons from deuteron transfer channel);

- Higher-order and complex ground-state correlations;

- Deformations;

- Toward ab initio description: renormalization of meson-nucleon coupling vertices, G-matrix etc.
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Relativistic description of pairing vibrations

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