Mu-e scattering:
Measuring the leading hadronic contribution to \((g-2)_{\mu}\)

Massimo Passera
INFN Padova

LFC17
ECT* Trento
14 Sep 2017
Outline

- Status of the muon g-2
- Hadronic corrections to the muon g-2: a new approach
- Muon-electron scattering: proposal for a new experiment
Status of the muon g-2
Uhlenbeck and Goudsmit in 1925 proposed:

\[
\vec{\mu} = g \frac{e}{2mc} \vec{s}
\]
\[
g = 2 \text{ (not 1!)}
\]

Dirac 1928:

\[
(i \partial_\mu - eA_\mu) \gamma^\mu \psi = m\psi
\]

A Pauli term in Dirac’s eq would give a deviation…

\[
a \frac{e}{2m} \sigma_{\mu\nu} F^{\mu\nu} \psi \quad \rightarrow \quad g = 2(1 + a)
\]

…but there was no need for it! g=2 stood for ~20 yrs.
Kusch and Foley 1948:

\[ \mu_{e}^{\text{exp}} = \frac{e\hbar}{2mc} \left( 1.00119 \pm 0.00005 \right) \]

Schwinger 1948 (triumph of QED!):

\[ \mu_{e}^{\text{th}} = \frac{e\hbar}{2mc} \left( 1 + \frac{\alpha}{2\pi} \right) = \frac{e\hbar}{2mc} \times 1.00116 \]

Keep studying the lepton–γ vertex:

\[ \bar{u}(p') \Gamma_{\mu} u(p) = \bar{u}(p') \left[ \gamma_{\mu} F_{1}(q^{2}) + \frac{i\sigma_{\mu\nu} q^{\nu}}{2m} F_{2}(q^{2}) + \ldots \right] u(p) \]

A pure “quantum correction” effect!

\[ F_{1}(0) = 1 \quad F_{2}(0) = \alpha_{l} \]
The muon g-2: experimental status

Today: $a_\mu^{\text{EXP}} = (116592089 \pm 54_{\text{stat}} \pm 33_{\text{sys}}) \times 10^{-11}$ [0.5ppm].

Future: new muon g-2 experiments at:
- **Fermilab E989**: aims at $\pm 16 \times 10^{-11}$, ie 0.14ppm. Data taking starting in November. First result expected in 2018 with a precision comparable to that of BNL E821.
- **J-PARC proposal**: phase-1 start with 0.37ppm (TDR 2016).

Are theorists ready for this (amazing) precision? **Not yet!**
The muon g-2: the QED contribution

\[ a_\mu^{\text{QED}} = (1/2)(\alpha/\pi) \]  
Schwinger 1948

+ 0.765857426 (16) \((\alpha/\pi)^2\)

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

+ 24.05050988 (28) \((\alpha/\pi)^3\)

Remiddi, Laporta, Barbieri …; Czarnecki, Skrzypek; MP '04; Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

+ 130.8780 (60) \((\alpha/\pi)^4\)


+ 752.85 (93) \((\alpha/\pi)^5\) COMPLETED!

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012 & 2015

Adding up, I get:

\[ a_\mu^{\text{QED}} = 116584718.944 (21)(77) \times 10^{-11} \]

from coeffs, mainly from 4-loop unc \(\downarrow\) from \(\delta \alpha(Rb)\)

with \(\alpha=1/137.035999049(90) [0.66 \text{ ppb}]\)
The muon g-2: the electroweak contribution

- **One-loop term:**

\[
a_{\mu}^{\text{EW}}(1\text{-loop}) = \frac{5G_{\mu}m_{\mu}^2}{24\sqrt{2}\pi^2} \left[ 1 + \frac{1}{5} \left( 1 - 4\sin^2\theta_W \right)^2 + O \left( \frac{m_{\mu}^2}{M_{Z,W,H}^2} \right) \right] \approx 195 \times 10^{-11}
\]

1972: Jackiw, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;
Studenikin et al. '80s

- **One-loop plus higher-order terms:**

\[a_{\mu}^{\text{EW}} = 153.6 (1) \times 10^{-11}\]

with \(M_{\text{Higgs}} = 125.6 (1.5) \text{ GeV}\)

Hadronic loop uncertainties and 3-loop nonleading logs.

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.
The muon g-2: the Hadronic LO contribution (HLO)

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)}$$

$$a_{\mu}^{HLO} = \frac{1}{4\pi^3} \int_{4m^2_\pi}^{\infty} ds K(s) \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m^2_\pi}^{\infty} \frac{ds}{s} K(s) R(s)$$

$$a_{\mu}^{HLO} = 6870 (42)_{\text{tot}} \times 10^{-11}$$

$$= 6926 (33)_{\text{tot}} \times 10^{-11}$$

$$= 6949 (37)_{\exp (21)}_{\text{rad}} \times 10^{-11}$$

Radiative Corrections are crucial.

Lots of progress in lattice calculations.

F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

M. Davier, arXiv:1612.02743

Hagiwara et al, JPG 38 (2011) 085003
HNLO: Vacuum Polarization

$O(\alpha^3)$ contributions of diagrams containing hadronic vacuum polarization insertions:

\[ a_{\mu}^{\text{HNLO}(\text{vp})} = -98 \times 10^{-11} \]

Krause '96, Alemany et al. '98, Hagiwara et al. 2011
HNLO: Light-by-light contribution

Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.

This term had a troubled life! Latest values:

\[
\begin{align*}
 a_\mu^{\text{HNLO}}(\text{lbl}) &= +80 \,(40) \times 10^{-11} & \text{Knecht & Nyffeler '02} \\
 a_\mu^{\text{HNLO}}(\text{lbl}) &= +136 \,(25) \times 10^{-11} & \text{Melnikov & Vainshtein '03} \\
 a_\mu^{\text{HNLO}}(\text{lbl}) &= +105 \,(26) \times 10^{-11} & \text{Prades, de Rafael, Vainshtein '09} \\
 a_\mu^{\text{HNLO}}(\text{lbl}) &= +102 \,(39) \times 10^{-11} & \text{Jegerlehner, arXiv:1511.04473}
\end{align*}
\]

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

Improvements expected in the \(\pi^0\) transition form factor A. Nyffeler 1602.03398

The HLbL contribution can be expressed in terms of observables in a dispersive approach. Colangelo et al, 2014, 15 & 17; Pauk & Vanderhaeghen 2014.

Progress on the lattice: +53.5(13.5)x10^{-11}. Statistical error only, finite-volume and finite lattice-spacing errors being studied. Omitted subleading disconnected graphs still need to be computed.
The muon g-2: the Hadronic NNLO contributions (HNNLO)

- **HNNLO: Vacuum Polarization**

\[ a_\mu^{\text{HNNLO}(\text{vp})} = 12.4 (1) \times 10^{-11} \]

Kurz, Liu, Marquard, Steinhauser 2014

- **HNNLO: Light-by-light**

\[ a_\mu^{\text{HNNLO}(\text{lbl})} = 3 (2) \times 10^{-11} \]

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014
Comparisons of the SM predictions with the measured g-2 value:

\[
a_\mu^{\text{EXP}} = 116592091 (63) \times 10^{-11}
\]

E821 – Final Report: PRD73 (2006) 072 with latest value of \( \lambda = \mu / \mu_p \) from CODATA’10

<table>
<thead>
<tr>
<th>( a_\mu^{\text{SM}} \times 10^{11} )</th>
<th>( \Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>116 591 761 (57)</td>
<td>330 (85) \times 10^{-11}</td>
<td>3.9 [1]</td>
</tr>
<tr>
<td>116 591 818 (51)</td>
<td>273 (81) \times 10^{-11}</td>
<td>3.4 [2]</td>
</tr>
<tr>
<td>116 591 841 (58)</td>
<td>250 (86) \times 10^{-11}</td>
<td>2.9 [3]</td>
</tr>
</tbody>
</table>

with the recent “conservative” hadronic light-by-light \( a_\mu^{\text{HNLO}}(\text{lbl}) = 102 (39) \times 10^{-11} \) of F. Jegerlehner arXiv:1511.04473, and the hadronic leading-order of:

A new approach to $a_\mu^{\text{HLO}}$

C. Carloni Calame, MP, L. Trentadue, G. Venanzoni

PLB 2015 - arXiv:1504.02228
New space-like proposal for HLO

- At present, the leading hadronic contribution $a_\mu^{\text{HLO}}$ is computed via the time-like formula:

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds \ K(s) \sigma^0_{\text{had}}(s)$$

$$K(s) = \int_0^1 dx \ \frac{x^2(1-x)}{x^2 + (1-x)(s/m_\mu^2)}$$

- Alternatively, exchanging the $x$ and $s$ integrations in $a_\mu^{\text{HLO}}$

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx \ (1-x) \Delta\alpha_{\text{had}}[t(x)]$$

$$t(x) = \frac{x^2m_\mu^2}{x-1} < 0$$

which involves $\Delta\alpha_{\text{had}}(t)$, the hadronic contribution to the running of $\alpha$ in the space-like region. It can be extracted from scattering data!
New space-like proposal for HLO (2)

**Time-like**

- $e^+e^- \rightarrow$ hadrons

**Space-like**

- $\Delta \alpha_{\text{had}} \left( \frac{x^2 m^2}{x-1} \right) \times 10^4$

- Smooth integrand

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F. Jegerlehner, arXiv:1511.04473

Carloni Calame, MP, Trentadue, Venanzoni, PLB 2015

M. Passera    LFC17 Trento   Sep 14 2017
New space-like proposal for HLO (3)

- $\Delta \alpha_{\text{had}}(t)$ can be measured via Bhabha scattering:

  $x_{\text{peak}} \approx 0.914$
  $t_{\text{peak}} \approx -0.108 \text{ GeV}^2$

  $|t| \times 10^3 \text{ (GeV}^2\text{)}$

- The peak occurs at $x_{\text{peak}} = 0.914$, $t_{\text{peak}} = -0.108 \text{ GeV}^2 \approx -(330 \text{ MeV})^2$

Carloni Calame, MP, Trentadue, Venanzoni, PLB 2015
Muon-electron scattering

Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni

EPJC 2017 - arXiv:1609.08987
**Δα_{had}(t)** can also be measured via the elastic scattering $μ e → μ e$.

- We propose to scatter a 150 GeV muon beam, available at CERN’s North Area, on a fixed electron target. Modular apparatus, 20 layers of low Z material (Be or C) paired to Si strip planes.
With CERN’s 150 GeV muon beam M2, which has an average of \( \sim 1.3 \times 10^7 \mu/s \), incident on 20 Be layers, each 3 cm thick, and 2 years of data taking with a running time of \( 2 \times 10^7 \text{ s/yr} \), one can reach an int. luminosity of \( \mathcal{L}_{\text{int}} \sim 1.5 \times 10^7 \text{ nb}^{-1} \) \((\sigma_{\text{LO}} \sim 245 \mu\text{b})\).
For a 150 GeV muon beam, the scan region extends up to $x=0.932$, i.e., beyond the peak! (the peak is at $x=0.914$)

The integrand in the remaining region $x \in [0.932,1]$ accounts for \(\sim 13\%\) of the $a_{\mu}^{\text{HLO}}$ integral. It cannot be reached by our experiment but it can be determined using pQCD & time-like data, and/or lattice QCD results.

Same detector for signal and normalization ($x \leq 0.3$, $\Delta \alpha_{\text{had}}(t) \leq 10^{-5}$) leads to cancellation of detector effects at first order.

With $\mathcal{L}_{\text{int}} \sim 1.5 \times 10^7 \text{ nb}^{-1}$ we estimate that we can reach a statistical sensitivity of $\sim 0.3\%$ on $a_{\mu}^{\text{HLO}}$, i.e., $\sim 20 \times 10^{-11}$!

It looks like an ideal process!
Systematic effects must be known at the level of ≤ 10ppm!

**Systematics**

1. Acceptance
2. Tracking
3. Trigger
4. PID
5. Effects of $E_e$ energy cut
6. Signal/Background:
   - It requires a dedicated event generator.
7. Uncertainty in the location of interaction vertices: Segmented/active target to resolve the vertex position
8. Uncertainty in the muon beam momentum:
   - Scattering kinematics to determine the beam momentum
9. Effects of Multiple Scattering (must be known at ~1%):
   - It requires dedicated work on simulation and measurements (test beam).
10. Theoretical uncertainty on the mu-e cross section (see later)

All the systematic effects must be known to ensure an error on the cross section < 10ppm

Affordable by means of GEANT4 based simulations

G. Venanzoni, Fermilab, June 5th 2017
Test Beam at CERN — Sep 2017

Test Beam

Check Geant4 MSC prediction and populate the 2D ($\theta_e$, $\theta_\mu$) scattering plane

- 27 Sep-3 October 2017 allocated at CERN in "H8 Beam Line"
- 5 Si strips planes: 2 before (upstream) and 3 after the target
- Max rate 10 kHz
- Beam energy in the range 90 - 190 GeV

U. Marconi, CSN1, Roma, May 12 2017
• Build up and test a full scale prototype.

High-energy resolution

U. Marconi, CSN1, Torino, Sep 12 2017
Muon-electron scattering: SM theory

- To extract $\Delta \alpha_{\text{had}}(t)$ from the measured cross section, the SM prediction must be known at NNLO!

- NLO QED corrections known & checked. Pavia group: MC ready!
  
  Carloni Calame, Padova $\mu e$ theory kick-off workshop, Sep 4-5 2017

- NNLO QED corrections unknown.

- NLO hadronic contributions unknown.

- Dedicated high-precision MC tools needed.

- Possible interplay with lattice calculations.
State-of-the-art methods required to calculate the 2-loop diagrams.

Examples of 2-loop diagrams:

Bonciani, Ferroglia, Gehrmann & von Manteuffel

Mastrolia, MP, Primo & Schubert, work in progress.
The aim of the workshop is to explore the opportunities offered by a recent proposal for a new experiment at CERN to measure the scattering of high-energy muons on atomic electrons of a low-Z target through the process $\mu e \rightarrow \mu e$. The focus will be on the theoretical predictions necessary for this scattering process, its possible sensitivity to new physics signals, and the development of new high-precision Monte Carlo tools. This kickoff workshop is intended to stimulate new ideas for this project.

It is organized and hosted by INFN Padova and the University.

Organizing Committee
Carlo Carloni Calame - INFN Pavia
Pierpaolo Mastrolia - U. Padova
Guido Montagna - U. Pavia
Oreste Nicosini - INFN Pavia
Paride Paradisi - U. Padova
Massimo Passera - INFN Padova (Chair)
Fulvio Piccinini - INFN Pavia
Luca Trentadue - U. Parma

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https://agenda.infn.it/internalPage.py?pageId=0&confId=13774
SCIENTIFIC PROGRAMS

Probing Physics Beyond SM with Precision
Ansgar Denner U Würzburg, Stefan Dittmaier U Freiburg, Tilman Plehn U Heidelberg
February 26-March 9, 2018

Bridging the Standard Model to New Physics with the Parity Violation Program at MESA
Jens Erler UNAM, Mikhail Gorshteyn, Hubert Spiesberger JGU
April 23-May 4, 2018

Modern Techniques for CFT and AdS
Bartlomiej Czech IAS Princeton, Michal P. Heller
MPI for Gravitational Physics, Alessandro Vichi EPFL
May 28-June 8, 2018

TOPOCAL WORKSHOPS

The Evaluation of the Leading Hadronic Contribution to the muon anomalous magnetic moment
Massimo Passera INFN Padua, Luca Trentadue U Parma,
Carlo Carloni Calame INFN Pavia Graziano Venanzoni INFN Frascati
February 19-23, 2018

Challenges in Semileptonic B Decays
Paolo Gambino U Turin, Andreas Kronfeld Fermilab,
Marcello Rotondo INFN-LNF Frascati,
Christof Schwanda OEW Vienna
April 16-20, 2018

Tension in LCDM Paradigm
Cora Dvorkin U Harvard, Silvia Galli IAP Paris,
Fabio Iocco ICTP-SAIFR, Federico Marinacci MIT
May 14-18, 2018
Conclusions

Muon g-2: $\Delta a_\mu \sim 3.5\, \sigma$. New upcoming experiment: QED & EW ready. Lots of progress in the hadronic sector, but not yet ready!

New proposal for an experiment at CERN to measure the leading hadronic contribution to the muon g-2 via $\mu$-e elastic scattering.

In collaboration with:


JOIN US!
The End
\[ L = 1.5 \times 10^7 \text{ nb}^{-1} \]
\[ \sigma_{LO} = 245.04 \text{ \( \mu \)b} \]
\[ E_{\ell} > 1 \text{ GeV} \]

\[ dN/dx \left( 10^{11} \right) \]

\[ L = 1.5 \times 10^7 \text{ nb}^{-1} \]
\[ \sigma_{LO} = 245.04 \text{ \( \mu \)b} \]
\[ E_{\ell} > 1 \text{ GeV} \]

\[ dN/dt \left( 10^{11} \text{ GeV}^{-2} \right) \]