Impact of pairing correlations on the chemical composition of the inner crust of a neutron star

A. Pastore, M. Shelley, C. Diget

Department of Physics, University of York, Heslington, York, YO10 5DD, UK

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Neutron Stars

Several neutron stars detected in the universe

Vela Nebula

- $10^4$ years ago explosion
- Rotating neutron star (pulsar)
- Radius $\approx 10$ Km
- Density $\approx 5 - 10 \rho_0$ ($\rho_0 \rightarrow$ density of a nucleus)
The crust of a Neutron Star ($\approx 0.5 - 1$ Km)

The structure evolves with the density

- Neutron crust $\rho < \rho_0$
- Crystalline structure: isolated nuclei, nuclei + neutron gas

Inner crust of neutron star

Isolated nuclei in a crystalline structure surrounded by neutron gas

Interesting aspects

- Nucleus-gas interaction
- Neutron superfluidity
- Thermal evolution
How to determine the chemical composition?

Approximations

- Spherical cells → Wigner Seitz (WS) cells
- Non-interacting WS cells
- Uniform e distribution

We need to minimise the energy

$$E = Z(m_p + m_e) + (N - A)m_n + E_{nuclear} + K_e + E_L$$

- $E_{nuclear}$: nuclear binding energy
- $K_e$ electron kinetic energy (ultra-relativistic)
- $E_L$ lattice energy

How to calculate $E_{nuclear}$

Nuclear physics input → calculate $E_{nuclear}$ for a wide range of densities, asymmetries and temperatures.
Proton pairing → small shift on energy minima, smaller differences between minima (more mixing? )

No spurious Box-effects

Shell effects → Strutinsky correction

No neutron pairing
HFB methods

DFT models

Solve HFB equations in a WS cell

\[ \sum_{n'} (h_{n'nlj}^q - \mu_{F,q}) U_{n'lj}^{i,q} + \sum_{n'} \Delta_{nn'lj}^q V_{n'lj}^{i,q} = E_{ij}^q U_{nlj}^{i,q} \]

\[ \sum_{n'} \Delta_{nn'lj}^q U_{n'lj}^{i,q} - \sum_{n'} (h_{n'nlj}^q - \mu_{F,q}) V_{n'lj}^{i,q} = E_{ij}^q V_{nlj}^{i,q} \]

- Microscopic functionals + pairing (no approx.)
- Boundary conditions → continuum effects

- No assumption on density shapes (spherical symmetry!)
Boundary conditions I

\[ M. \text{ Baldo et al. Nucl. Phys. A775, 235-244 (2006)} \]
Boundary conditions II

Compare $E/A$ in pure neutron matter and in a box at same $k_F$

Fig. 2. We represent several quantities versus the W-S radius: (a) the binding energy $B_{WS-hom.}(\rho_{unb.}, R_{WS})$ corresponding to $N_{zone}=6$, 7 and 8 (b) the difference $B_{hom.}(\rho_{unb.}) - B_{WS-hom.}(\rho_{unb.}, R_{WS})$ (dots) and the function $f(\rho_{unb.}, R_{WS})$ which fit the smooth component (solid lines), and (c) the difference between the dots presented in (b) and the function $f(\rho_{unb.}, R_{WS})$.

[J. Margueron et al., arXiv:0711.0106 (2007)]
Boundary conditions III

Compare $E/A$ in pure neutron matter and in a box at same $k_F$

We need large boxes!
- We solve HFB equations in a large box $R_b = 80$ fm
- Approx. constant error $7$keV/particle $\rightarrow$ constant shift
- Long CPU time....
Keeping box fixed....

- We solve HFB equations
- We solve HF equations
- We solve HF+BCS equations

The error is *mainly* due to the treatment of pairing!
Pairing filed in the crust

$\Delta(r)$ is not flat

- Nuclear cluster enhances/reduces $\Delta(r)$!!
- Need careful treatment BCS or HFB (LDA is not good!)
We work in $R_{box} = 80$ fm to calculate $E_{nuclear}$. $\beta$-equilibrium is imposed $a posteriori$ to determine $R_{WS}$. External neutron gas inert [need constant $\rho$!!]

$$E_{nuclear} = \int_0^{R_{WS}} dr \mathcal{H}(\rho(r))$$

$$N = \int_0^{R_{WS}} dr \rho(r)$$
HFB: large scale calculations

How to reduce CPU time?

For fixed $\rho_b$ one needs to determine

- Need to explore 2D space $(Z, R_{WS})$ for fixed $\rho_b$

Approximation

We treat $V_{pe}$ as *perturbation*: correction on total energy and $\mu_p$
HFB: large scale calculations

Coulomb interaction

- Proton potential treated fully-self-consistent (direct + Slater)
- Electron $\rightarrow$ uniform gas
- Proton-electron potential

$$V_{p-e} = \frac{Ze^2}{2R_{WS}} \left[ \left( \frac{r}{R_{WS}} \right)^2 - 3 \right]$$
Solution:

We treat $V_{pe}$ as *perturbation* on final result ($E_{tot}$ and $\mu_p$). Error of $\sigma \approx 2/3$ keV/particle.

Error estimate:

- Error on total energy $E_{ep} \approx 2$ keV/p
- Error on $\mu_p \rightarrow$ inaccuracy in $\beta$-equilibrium $\rightarrow 2$ keV/p
Skyrme HFB+\(\delta\)-pairing (\(\Delta \approx 3\) MeV)

We perform HFB calculations in function of \(Z\) and \(N\) (total box) in steps of 10 (50) neutrons.

We span \(\rho \in [10^{-4} - 3 \times 10^{-2} \text{ [fm}^{-3}\text{]}] \rightarrow 800 \times (Z_{\text{max}} - Z_{\text{min}}) \approx 18000\).

Simple *parametric* script.
Gaussian Process Emulators (GPEs)

- Emulate unknown outputs of a simulation
- Use Bayesian inference (i.e. not the same as basic interpolation)
- Probability of output being in certain region is also used by emulator
- Outputs of simulation are expected to vary smoothly with simulation inputs
- Outputs are modelled as a random Gaussian process in parameter space defined by simulation inputs
- In 1D, works by fitting set of polynomials to simulation output
GPE applied to sample of 10 data points (left) and 20 data points (right).

- Dotted line is true unknown function, $y(x)$.
- Points plus error bars → outputs of a simulation
- Solid lines → outputs GPE plus 95% confidence interval.
GPE results

- We have confidence interval given by GPE and based on our error input!
- $\rho_b$ becomes a continuous variable!! (as in semi-classic)
- GPE is not necessarily linked to HFB!
Cluster configurations

- We have determined the cluster configuration
- We add all $Z$ values falling in the confidence interval associated with the minimum
Conclusions and Perspectives

Conclusions

- Pairing correlations impact chemical composition
- Strong shell effects → Need microscopic description
- Very difficult problem to solve

GPE methods

Advanced GPE methods → make the problem solvable with controlled approximations.

Perspectives

- Full HFB treatment (no \textit{ad hoc} corrections)
- Very large boxes → better treatment of continuum
- Systematic calculations (\textit{i.e.} vary functionals/pairing....)
Some problems at the drip-line....

- Inconsistent treatment at the drip-line: HFB vs Semi-classic (same interaction!)

<table>
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<th>Force</th>
<th>$\bar{n}_{\text{drip}}$ (fm$^{-3}$)</th>
<th>Z</th>
<th>N</th>
<th>$e$ (MeV)</th>
<th>$P$ (MeV fm$^{-3}$)</th>
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<td>BSk19</td>
<td>$2.63464 \times 10^{-4}$</td>
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<td>93</td>
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