Progress in the Quantum Monte Carlo calculations for medium mass hypernuclei

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Collaborators

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- Stefano Gandolfi (LANL)
- F. Catalano (Uppsala)
Open questions...

The *fine tuning* of the hyperon-nucleon interaction is essential to understand the behavior of matter in extreme conditions.

Our favorite example: Neutron stars!

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Far away from any possible perturbative treatment...

Neutron star structure

Equation of state

Internal composition still largely unknown

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Modeling the Complete Gravitational Wave Spectrum of Neutron Star Mergers

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In the context of neutron star mergers, we study the gravitational wave spectrum of the merger remnant using numerical relativity simulations. Postmerger spectra are characterized by a main peak frequency \( f_1 \) related to the particular structure and dynamics of the remnant hot by hypermassive neutron star. We show that \( f_1 \) is correlated with the tidal coupling constant \( \ell_1 \) that characterizes the binary tidal interactions during the late-inspiral merger. The relation \( f_1(\ell_1) \) depends very weakly on the binary total mass, mass ratio, equations of state, and thermal effects. This observation opens up the possibility of developing a model of the gravitational spectrum of every merger unifying the late-inspiral and postmerger descriptions.

**Introduction.**—Direct gravitational wave (GW) observations of binary neutron stars (BNS), late-inspiral merger and postmerger by ground-based GW interferometric experiments, can lead to the strongest constraints on the equation of state (EOS) of matter at supranuclear densities [1–7]. There are two ways to set such constraints (GW observations of BNS mergers can also constrain the source redshift [8,9]): (I) measure the binary phase during the last minutes of coalescence using matched filtered searches [1,3–5] and (II) measure the postmerger GW spectrum frequencies using burst searches [6,7].

![Graph showing gravitational wave frequency vs. time](image)
So many different models!

Table A.1. Parameters of the EOS and of NS models based on them.

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So many different models!

All masses compatible with the 2$M_\odot$ constraint

Fortin, M., Zdunik, J. L., Haensel, P., & Bejger, M.  
_Astronomy & Astrophysics, 576, A68_ (2015)
So many different models!

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Radii roughly divided in two groups.
So many different models!

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CONSTRAINING EoS

Nuclear Interactions

? ~ 18 orders of magnitude in between…
Possible description of the YN interaction

**NON RELATIVISTIC:**
write a Hamiltonian using a model potential and try to solve a many-body Schroedinger equation.

- The potential energy is **not an observable**: several different equivalent descriptions are possible.
- The interaction can be based on some more or less phenomenological scheme (fit the existing experimental data, rely on some systematic meson exchange model), or can be inferred from EFT systematic expansions.
- Only **accurate many-body calculations** can help distinguishing among different realisations of the potential.

**RELATIVISTIC:**
write a Lagrangian including relevant fields, and try to solve the field theoretical problem (usually RMF calculations are performed).
Some hints from LQCD……

Fig. 10. Left: The central potential in the $^1S_0$ channel of the $\Lambda N$ system in 2 + 1 flavor QCD as a function of $r$. Right: The central potential in the $^1S_0$ channel of the $\Sigma N(I = 3/2)$ system as a function of $r$.

Fig. 11. Left: The central potential (circle) and the tensor potential (triangle) in the $^3S_1-^3D_1$ channel of the $\Lambda N$ system as a function of $r$. Right: The central potential (circle) and the tensor potential (triangle) in the $^3S_1-^3D_1$ channel of the $\Sigma N(I = 3/2)$ system as a function of $r$.

NB: The potential is NOT an observable! Features like the hard core depend e.g. on the method used to reconstruct the kinetic energy.

S. Aoki et al. (HAL-QCD collaboration)
Model Hyperon-nucleon interaction

In order to gain some understanding, we need to set up some scheme.

OUR CHOICE

- **NON RELATIVISTIC APPROACH** (should be fine if the central density is not too large)
- **YN INTERACTION CHOSEN TO FIT EXISTING SCATTERING DATA** (with a hard-core)
- **PHENOMENOLOGICAL YNN THREE-BODY FORCES** with the fewest possible parameters to be adjusted to reproduce light hypernuclei binding energies
- **ALL OF THE OTHER RESULTS ARE PREDICTIONS WITH NO OTHER ADJUSTABLE PARAMETERS** obtained from an *accurate solution of the Schroedinger equation*. 
Model Hyperon-nucleon interaction

Model interaction (Bodmer, Usmani, Carlson):

\[ V_{\Lambda i}(r) = v_0(r) + v_0(r)\varepsilon(P_x - 1) + \frac{1}{4}v_\sigma T_\pi^2(m_\pi r)\sigma_\Lambda \cdot \sigma_i \]

from Kaon exchange terms (not considered explicitly in our calculations)

Two-body potential: accurately fitted on p-\(\Lambda\) scattering data

Two-body potential: accurately fitted on p-\(\Lambda\) scattering data

\[ V_{\Lambda ij} = V_{\Lambda ij}^{2\pi} + V_{\Lambda ij}^D \]

\[
\begin{align*}
V_{\Lambda ij}^{2\pi} &= C_{2\pi}^{SW} O_{\Lambda ij}^{2\pi,SW} + C_{2\pi}^{PW} O_{\Lambda ij}^{2\pi,PW} \\
V_{\Lambda ij}^D &= W^D T_\pi^2(m_\pi r_{\Lambda i}) T_\pi^2(m_\pi r_{\Lambda j}) \left[ 1 + \frac{1}{6} \sigma_\Lambda \cdot (\sigma_i + \sigma_j) \right]
\end{align*}
\]
Model Hyperon-nucleon interaction

Model interaction (Bodmer, Usmani, Carlson):

\[ V_{\Lambda i}(r) = v_0(r) + v_0(r)\varepsilon(P_x - 1) + \frac{1}{4} v_\sigma T_\pi^2(m_\pi r) \sigma_\Lambda \cdot \sigma_i \]

Two-body potential: accurately fitted on p-\(\Lambda\) scattering data

\[ V_{\Lambda i j} = V_{\Lambda i j}^{2\pi} + V_{\Lambda i j}^D \]

Parameters to be determined from calculations


Input from experiment

We need to fit the three body interaction against some experimental data. There are available several measurements of the binding energy of $\Lambda$-hypernuclei, i.e. nuclei containing a $\Lambda$ hyperon. The idea is to compute such binding energies. We can then compute the hyperon separation energy:

\[
B_\Lambda = B_{hyp} - B_{nuc}
\]

where $B_{hyp}$ is the total binding energy of a hypernucleus with $A$ nucleons and one $\Lambda$, and $B_{nuc}$ is the total binding energy of the corresponding nucleus with $A$ nucleons. This number can be used to gauge the coefficients in the nucleon-$\Lambda$ interaction.
Many-Body theory: projection Monte Carlo

We compute ground state energies of nuclei by means of projection Monte Carlo methods. The ground state of a many-body system is computed by applying an “imaginary time propagator” to an arbitrary state that has to be non-orthogonal to the ground state (power method):

$$\langle R | \Psi(\tau) \rangle = \langle R | e^{-(\hat{H} - E_0)\tau} | R' \rangle \langle R' | \Psi(0) \rangle$$

In the limit of “short” $\tau$ (let us call it “$\Delta \tau$”), the propagator can be broken up as follows (Trotter-Suzuki formula):

$$\langle R | e^{-(\hat{H} - E_0)\Delta \tau} | R' \rangle \sim e^{-\frac{(R - R')^2}{2m\Delta \tau}} e^{-\left(\frac{\mathcal{V}(R) + \mathcal{V}(R')}{2} - E_0\right)\Delta \tau}$$

- Kinetic term
- Potential term (“weight”)

Sample a new point from the Gaussian kernel

Create a number of copies proportional to the weight

If the weight is small, the points are canceled.

$M = \langle \text{int}[\mathcal{W}(R, R', \Delta \tau) + 1\text{ and()}] \rangle$
Many-nucleon systems

PROBLEM
for realistic many-nucleon Hamiltonians, propagators must be evaluated on wave functions that have a number of components exponentially growing with $A \leq 12$ by the Argonne based group (GFMC calculations by Pieper, Wiringa, Carlson, Schiavilla...). These calculations include two- and three-nucleon interactions.

Very accurate results have been obtained in the years for the ground state and some excitation properties of nuclei with $A \leq 12$ by the Argonne based group (GFMC calculations by Pieper, Wiringa, Carlson, Schiavilla...). These calculations include two- and three-nucleon interactions.

Argonne v$^{18}$ with Illinois-7
GFMC Calculations
10 January 2014

- IL7: 4 parameters fit to 23 states
- 600 keV rms error, 51 states
- ~60 isobaric analogs also computed

Courtesy of R. Wiringa, ANL
The computational cost of GFMC can be reduced by introducing a way of sampling over the space of states, rather than summing explicitly over the full set. For simplicity let us consider only one of the terms in the interaction. We start by observing that:

\[ \sum_{i<j} v(r_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j = \frac{1}{2} \sum_{i;\alpha,j;\beta} \sigma_{i;\alpha} A_{i;\alpha,j;\beta} \sigma_{j;\beta} = \sum_{n=1}^{3A} \lambda_n \hat{O}_n \]

Then, we can linearize the operatorial dependence in the propagator by means of an integral transform:

\[ e^{-\frac{1}{2} \lambda \hat{O}_n^2 \Delta \tau} = \frac{1}{\sqrt{2\pi}} \int dx e^{-\frac{x^2}{2}} e^{-x \sqrt{\lambda \Delta \tau} \hat{O}_n} \]

Auxiliary Field Diffusion Monte Carlo (AFDMC)

The operator dependence in the exponent has become linear.

In the Monte Carlo spirit, the integral can be performed by sampling values of $x$ from the Gaussian $e^{-\frac{x^2}{2}}$. For a given $x$ the action of the propagator will become:

$$e^{-x\sqrt{\lambda \Delta \tau} \hat{O}_n} |S\rangle = \prod_{k=1}^{3A} e^{-x\sqrt{\lambda \Delta \tau} \phi_n^k \sigma_k} |S\rangle$$

In a space of spinors, each factor corresponds to a rotation induced by the action of the Pauli matrices.

The sum over the states has been replaced by sampling rotations!
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Experimental data in the table from:
• A.Gal, E.V. Hungerford, and D.J.Millener, Rev.Mod.Phys 88, 035004 (2016)
• P.H. Pile et al., Phys. Rev. Lett. 66, 2585 (1991) [s-orbit $^{40}$Ca]
• T.Gogami, Ph.D. Thesis (2014) [s-orbit $^{52}$V]
Hypernuclei

$^{48}\text{Ca}(e, e' K^+) ^{\Lambda} K$

$^{40}\text{Ca}(e, e' K^+) ^{\Lambda} K$

$A^{-2/3}$
Can we really constrain $\Lambda$NN interaction from hyper nuclear data?

In hypernuclei it is possible that the $\Lambda$NN interaction is not well constrained, especially in the isospin triplet channel:

![Diagram: $\Lambda$NN interaction in isospin singlet and triplet channels]

We are doing the exercise of re-projecting the interaction in the isospin singlet and triplet channels and try to explore the dependence of the hypernuclei binding energy on the relative strength.

\[ v^{2\pi,P} = -\frac{C_P}{6} \{ X_{i\lambda}, X_{\lambda j} \} \vec{\tau}_i \cdot \vec{\tau}_j \]
\[ v^{2\pi,S} = C_S O_{i j \lambda}^{2\pi,S} \vec{\tau}_i \cdot \vec{\tau}_j \]

\[ v^{\tau\tau}_{ij\lambda} = -3v^{P}_{ij\lambda} \hat{P}^{T=0}_{ij} + C_T v^{P}_{ij\lambda} \hat{P}^{T=1}_{ij} \]
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Pauli repulsion

We are doing the exercise of re-projecting the interaction in the isospin singlet and triplet channels and try to explore the dependence of the hypernuclei binding energy on the relative strength.

Let's consider $C_T = 1$ gives the original potential, but we can choose an arbitrary value. $C_T < 1 \Rightarrow$ more repulsion

$$v_{ij\lambda}^{\tau\tau} = -3v_{ij\lambda}^{P} \hat{P}_{ij}^{T=0} + C_T v_{ij\lambda}^{P} \hat{P}_{ij}^{T=1}$$

$$v_{ij\lambda}^{\tau\tau} = \frac{3}{4} (C_T - 1)v_{ij\lambda}^{P} + \frac{1}{4} (3 + C_T)v_{ij\lambda}^{P} \vec{\tau}_i \cdot \vec{\tau}_j$$
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Pauli repulsion

must be negative on average to give repulsion
Can we really constrain the interaction from hyper nuclear data?

Francesco Catalano, Diego Lonardoni, FP
Charge symmetry breaking

Obviously one also has to consider CSB interactions.

This adds a further parameter in the interaction:

\[ v^{CSB}_{\lambda i} = C \tau T^2_\pi (r_{\lambda i}) \tau_i \]


- Notice that no spin dependence has been added here. Is it important?
- CSB interaction can be measured in light nuclei.
CSB displays some isospin dependence, at last in the s state, while in p-state new results seem to question that.

This effect in the many-body system might be partly reabsorbed in the isospin dependent 3-body force.
Conclusions

• Our philosophy in attacking the problem of the hyperon-nucleon interaction: we do not want to add more information than the one that experiments can give us. Having too many parameters will result in a substantially arbitrary prediction of the EoS, and consequently adjustable predictions on the Neutron Star structures.

• AFDMC calculations are evolving. Better accuracy, better performance. This reflects on the work on hypernuclei Accessible systems: definitely A=90. For heavier systems one can possibly use alternative approaches.

• At this point there is real need of accurate experiments on hypernuclei in order to be able to gain more insight on NS interior at densities $> 2\rho_0$. 
Results: hyper-neutron matter

\[ \lambda\text{-neutron matter} \]

\[ \begin{align*}
\mu_{\Lambda}(\rho_b, x_\Lambda) &= \mu_n(\rho_b, x_\Lambda) \\
E_{\text{HNM}}(\rho_b, x_\Lambda) &= E_{\text{HNM}}(\rho_b, x_\Lambda)
\end{align*} \]

PNM \rightarrow \text{hyperon fraction} \rightarrow \text{energy per particle} \rightarrow E_{\text{HNM}} = E_{\text{HNM}}(\rho_b, x_\Lambda)

\[ \epsilon_{\text{HNM}}(\rho_b, x_\Lambda) \]

equilibrium condition: chemical potentials

equal: \[ x_\Lambda = x_\Lambda(\rho_b) \]
Neutron star structure
Neutron star structure

![Graph showing neutron star structure with M in $M_\odot$ and $\rho_c$ in $fm^{-3}$](graph.png)

- PSR J0348+0432
- PSR J1614-2230
- PNM

- Masses: 0.0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.6, 2.0, 2.4, 2.8
- Densities: 0.0, 0.2, 0.4, 0.6, 0.8, 1.0
Neutron star structure

![Graph showing neutron star structure](image)

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 Parameters used in the calculations:

- $\rho_c$ [fm$^{-3}$]
- $M$ [M$_\odot$]

- Comparison between PNM and $\Delta N$ models

- $\Delta N + \Delta NN$ (I) model

The graphs illustrate the mass-radius relationship for different neutron stars, highlighting the impact of various models on the neutron star structure.
Neutron star structure

\[ M \left[ M_\odot \right] = \rho c \left[ \text{fm}^{-3} \right] \]

\[ \text{PNM} \]

\[ \text{PSR J1614-2230} \]

\[ \text{PSR J0348+0432} \]

\[ \Delta N \]

\[ \Delta N + \Delta NN \]

\[ \Delta N + \Delta NN \ (I) \]

\[ \Delta N + \Delta NN \ (II) \]
Within this model the repulsion needed to correctly describe hypernuclear binding energy is so strong that **hyperons would not be present in 2M_☉ stars!**
Results: hyper-neutron matter

\[ M \sim 0.66(2) M_\odot \]

PSR J1614-2230

PSR J0348+0432

\[ 2.45(1) M_\odot \]

Results: hyper-neutron matter

$M \ [M_\odot]$  

$R \ [\text{km}]$

PNM  

$\Delta N + \Delta NN \ (I)$

$\Delta N$

$\gamma N$ interaction only (from $\Delta N$ scattering data)

PSR J1614-2230

PSR J0348+0432

Neutron star structure

Results: hyper-neutron matter $M \approx 0.66M_\odot$.

$\Lambda N + \Lambda NN$ interaction (old VMC based parametrization)

$\Lambda N$ interaction only (from $\Lambda N$ scattering data)

$M \approx 1.36(5)M_\odot$ for PSR J1614-2230

$M \approx 2.45(1)M_\odot$ for PSR J0348+0432

Neutron star structure

Results: hyper-neutron matter

\[ M \sim 0.66 \pm 0.02 M_\odot \]

\[ R \sim 10^{12}\text{ km} \]

\[ \Lambda N + \Lambda NN \text{ (I)} \]

\[ \Lambda N \text{ interaction only (from } \Lambda N \text{ scattering data)} \]

\[ \Lambda N + \Lambda NN \text{ (old VMC based parametrization)} \]

\[ \Lambda N + \Lambda NN \text{ interaction (more accurate description of hypernuclear BE)} \]

PSR J1614-2230

PSR J0348+0432