The impact of the ghost-gluon vertex on the ghost Schwinger-Dyson equations

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Based on:
A. C. A., D. Ibáñez and J. Papavassiliou, Phys. Rev. D87, 114020 (2013)

Outline of the talk

- *Motivation*
- *The nonperturbative structure of the ghost-gluon vertex*
- *The ghost SDE*
- *The SDE for the ghost-gluon vertex*
- *Kinematic limits :*
  - *Soft gluon limit*
  - *Soft ghost limit*
- *Coupled system: ghost-gluon vertex and ghost SDEs*
- *Comparison with the lattice results: SU(3) and SU(2)*
- *Conclusion*

Motivation

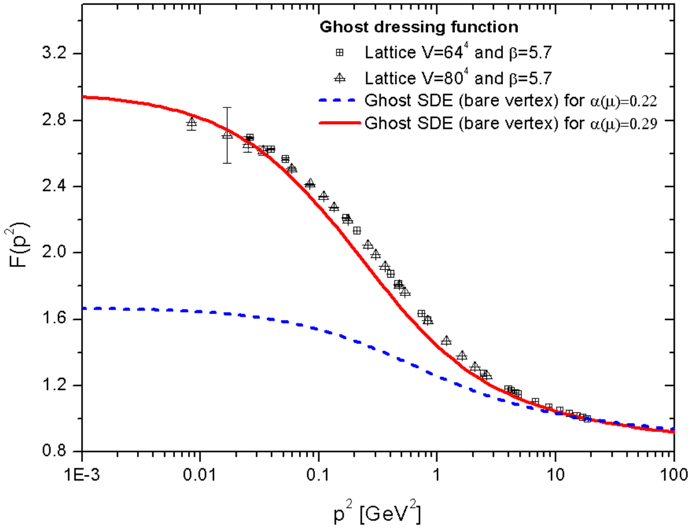
- *Lattice results and various analytic approaches find a massless ghost propagator with an infrared finite dressing function, $F(p^2)$.*

- *Interpretation: The finiteness of $F(p^2)$ is associated with the massiveness of the gluon propagator, which saturates the logarithms present in $F(p^2)$.*

- *Qualitative agreement between lattice and SDE.*

- *Ghost SDE results are suppressed in comparison to the lattice, when we use the **bare ghost-gluon vertex** in the structure of the ghost SDE.*

- *Lattice is reproduced with artificial increase of the gauge coupling value from α_s (4.3 GeV) = 0.22 to α_s (4.3 GeV) = 0.29*



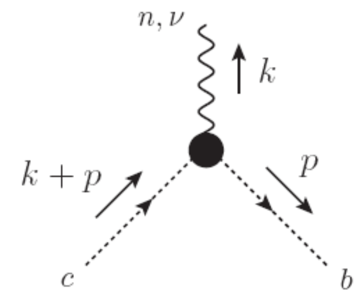
- *Improvement: Study a **SDE that controls the nonperturbative behavior of the ghost-gluon vertex**, in the Landau gauge, and coupled to the ghost SDE*

- *Other analytical studies:*

OPE-Based **D. Dudal, O. Oliveira and J. Rodriguez-Quintero**, Phys. Rev. D 86, 105005 (2012).
P. Boucaud, et al. JHEP 1112, 018 (2011).

SDE **M. Q. Huber and L. von Smekal**, arXiv:1211.6092 [hep-th].

The gluon-ghost vertex



- *Let us denote the full ghost-gluon vertex as*

$$\Gamma_{\nu}^{nbc}(-k,-p,r) = g f^{nbc}\Gamma_{\nu}(-k,-p,r)\,, \quad r = k+p\,,$$

where k is momentum of the gluon and p of the anti-ghost.

- *The most general tensorial structure of the vertex is given by*

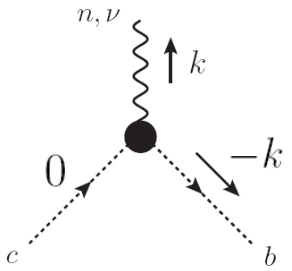
$$\Gamma_{\nu}(-k,-p,r) = A(-k,-p,r)\,p_{\nu} + B(-k,-p,r)\,k_{\nu}\,;$$

at tree-level, the two form factors assume the values:

$$\begin{cases} A^{[0]}(-k,-p,r) = 1 \\ B^{[0]}(-k,-p,r) = 0. \end{cases} \Rightarrow \Gamma_{\nu}^{[0]} = p_{\nu}$$

$$\Gamma_\nu(-k,-p,r) = A(-k,-p,r)\,p_\nu + B(-k,-p,r)\,k_\nu\,;$$

Taylor limit



- *When r=0 (zero ghost momentum) → p= -k , in this limit full vertex becomes*

$$\Gamma_\nu(-k,k,0) = -[A(-k,k,0) - B(-k,k,0)]k_\nu$$

- *The Taylor theorem states that, to all orders, in perturbation theory,*

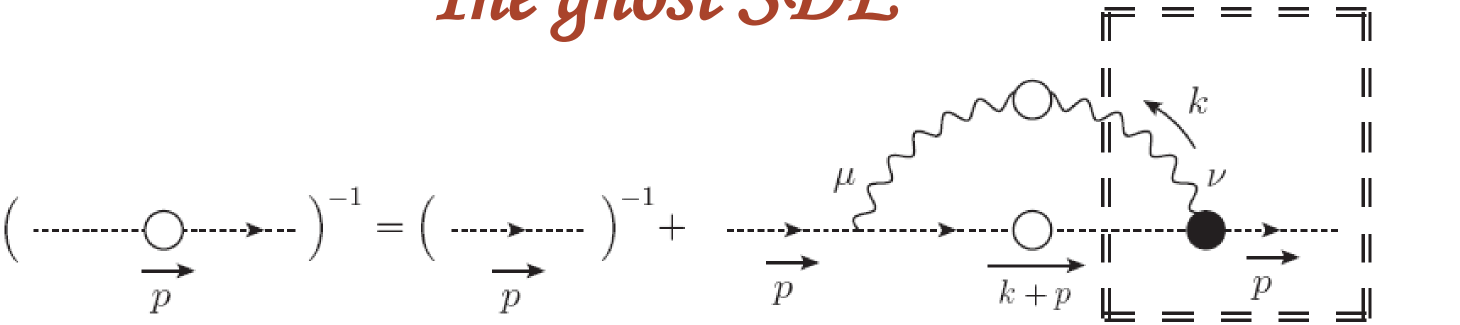
J. C. Taylor, Nucl. Phys. B 33, 436 (1971).

$$A(-k,k,0) - B(-k,k,0) = 1;$$

- *The vertex assumes the tree-level value in this particular kinematic configuration, i.e.*

$$\Gamma_\nu(-k,k,0) = -k_\nu.$$

The ghost SDE



- *The ghost SDE can be written as:*

$$iD^{-1}(p^2) = ip^2 - g^2 C_{\rm A} \int_k \Gamma_\mu^{[0]}(k, -k-p, p)\Delta^{\mu\nu}(k)\Gamma_\nu(-k,-p,k+p)D(k+p)\,,$$

- *Due to the transversality of* $\Delta_{\mu\nu}$ (k), *any reference to the form factor B disappears from the ghost SDE which reduces to*

$$F^{-1}(p^2) = 1 + ig^2 C_{\rm A} \int_k \left[1 - \frac{(k\cdot p)^2}{k^2 p^2}\right] \underline{A(-k,-p,k+p)}\Delta(k)D(k+p)\,,$$

Function of 2 momenta p, k and the angle between them

$$D(q^2) = \frac{F(q^2)}{q^2}$$

- *We will consider the behavior of A(-k, -p, k + p) for vanishing external momentum p .*

- *We keep the first term of the Taylor expansion of A(-k, -p, k + p) around p = 0 → Converting A into a function of a single variable.*

- *Notice that limit p → 0 is taken only inside the argument of the form factor A, but not in the rest of the terms appearing in the ghost SDE*

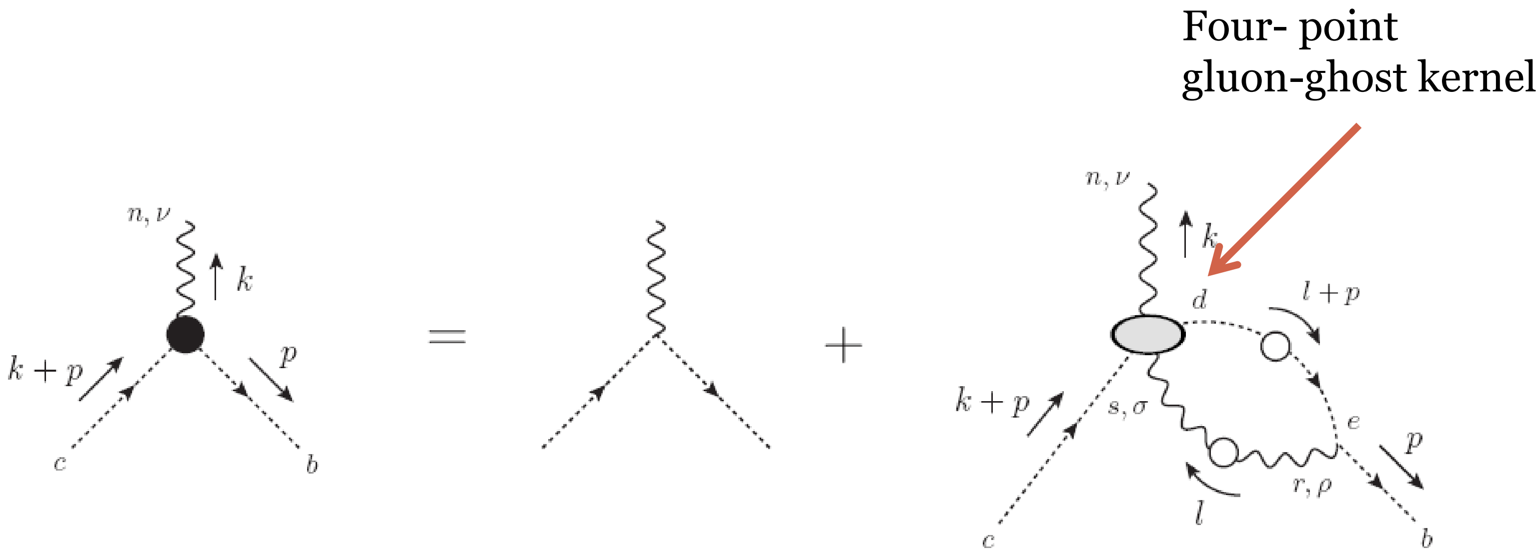
- *The approximate version of the SDE becomes*

$$F^{-1}(p^2) = 1 + ig^2 C_{\mathrm{A}} \int_k \left[1 - \frac{(k\cdot p)^2}{k^2 p^2}\right] \underline{A(-k,0,k)}\Delta(k)D(k+p)\,.$$

Soft ghost limit

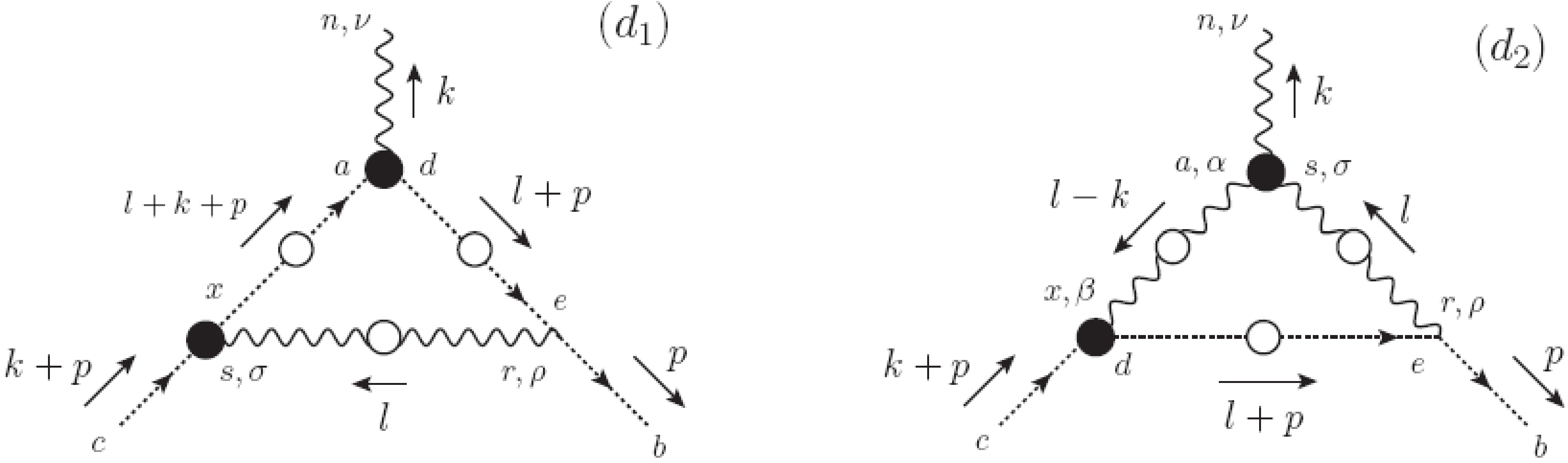
The SDE for the ghost-gluon vertex

- *The diagrammatic of SDE for the ghost-gluon vertex is given by:*



- *Relevant quantity is the* ***four-point ghost-gluon kernel.***

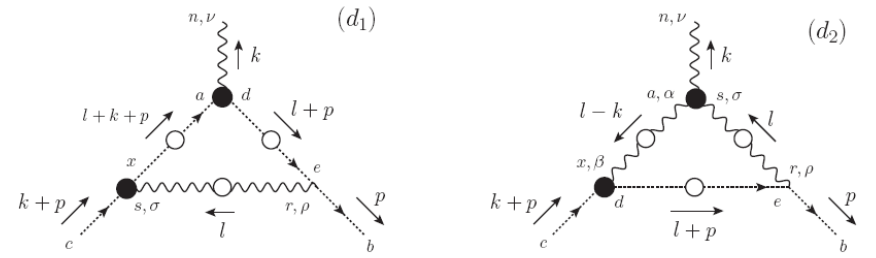
- *The ghost-gluon kernel will be replaced by its "one-loop dressed" approximation, i.e.*



$$\Gamma_\nu(-k,-p,k+p) = p_\nu - \frac{i}{2} g^2 C_{\mathrm{A}} [(d_1)_\nu - (d_2)_\nu] \,,$$

$$\begin{aligned}
(d_1)_\nu &= \int_l \Gamma_\rho^{[0]} \Delta^{\rho\sigma}(l) \Gamma_\sigma D(l+k+p) \Gamma_\nu D(l+p) \,, \\
(d_2)_\nu &= \int_l \Gamma_\rho^{[0]} \Delta^{\rho\sigma}(l) \Gamma_{\nu\sigma\alpha} \Delta^{\alpha\beta}(l-k) \Gamma_\beta D(l+p) \,.
\end{aligned}$$

- *In the two diagrams:*

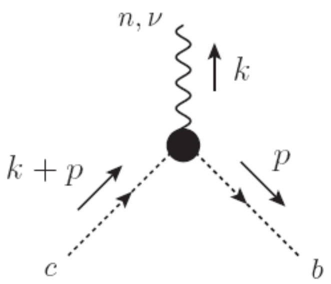


1. *We will keep the fully dressed gluon and ghost propagators,*
2. *We will replace the fully dressed three-gluon vertex by tree-level expression*

$$\Gamma_{\alpha\mu\nu}(q,r,p) \to \Gamma^{[0]}_{\alpha\mu\nu}(q,r,p) = (r-p)_\alpha g_{\mu\nu} + (p-q)_\mu g_{\nu\alpha} + (q-r)_\nu g_{\alpha\mu} \,.$$

3. *Additional approximations will be imposed on the fully dressed ghost-gluon vertices for each configuration we study.*

Kinematic configurations

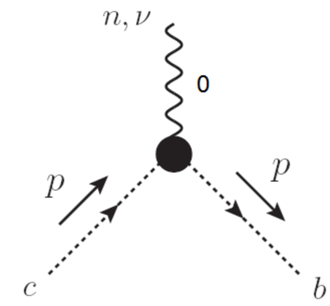


- *We will study the form factor A in two special kinematic configurations:*

1. ***The soft gluon limit****, in which the momentum carried by the gluon leg is zero (k = 0)*

2. ***The soft ghost limit****, where the momentum of the anti-ghost leg vanishes (p = 0).*

Soft gluon configuration

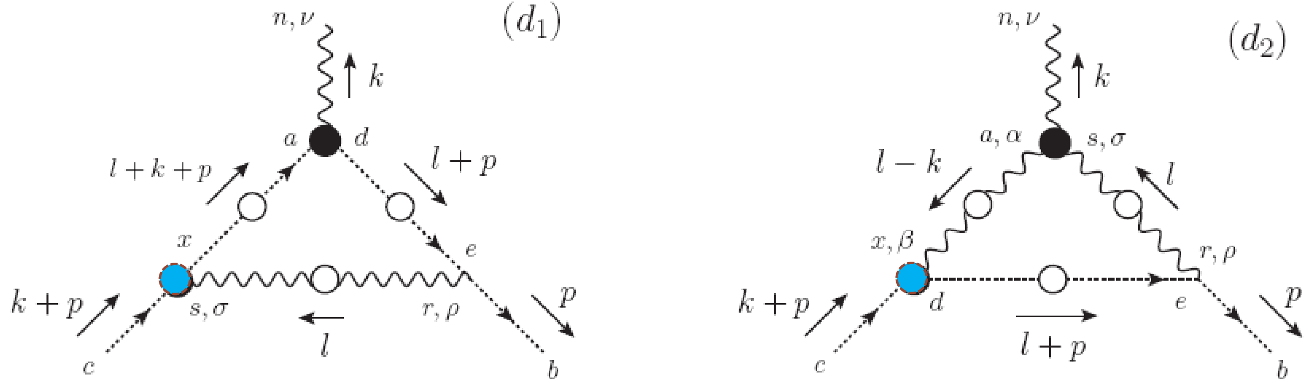


- *In this limit the momentum carried by the gluon leg is zero (k = 0)*

- *The form factor A* becomes a function of only p. i.e.

$$\Gamma_\nu(0,-p,p) = A(p)p_\nu; \quad A(p) \equiv A(0,-p,p)\,.$$

- *We use for the ghost-gluon vertices (●) their tree level expression → linear integral equation*



$$(d_1) = \int_l \frac{(l \cdot p)}{(l+p)^2 p^2} [(l \cdot p)^2 - l^2 p^2] D^2(l) \Delta(l+p) A(l) \, .$$

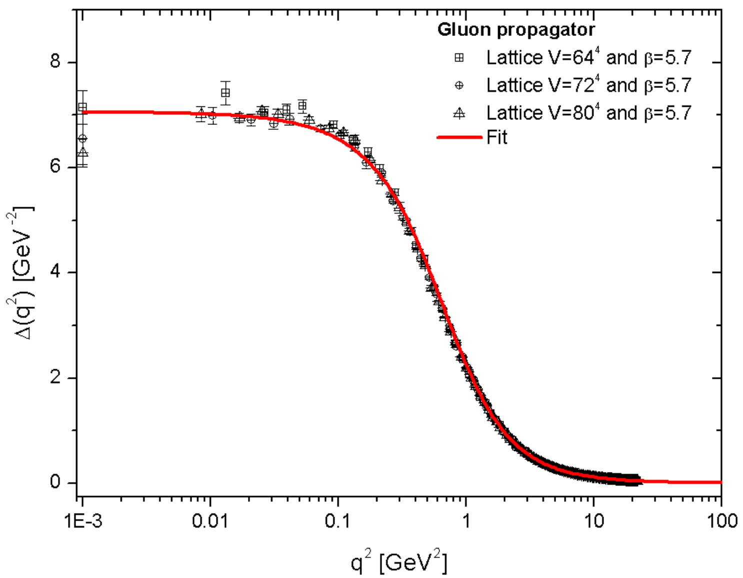
$$(d_2) = 2 \int_l \frac{(l \cdot p)}{l^2 p^2} [l^2 p^2 - (l \cdot p)^2] \Delta^2(l) D(l+p) \, .$$

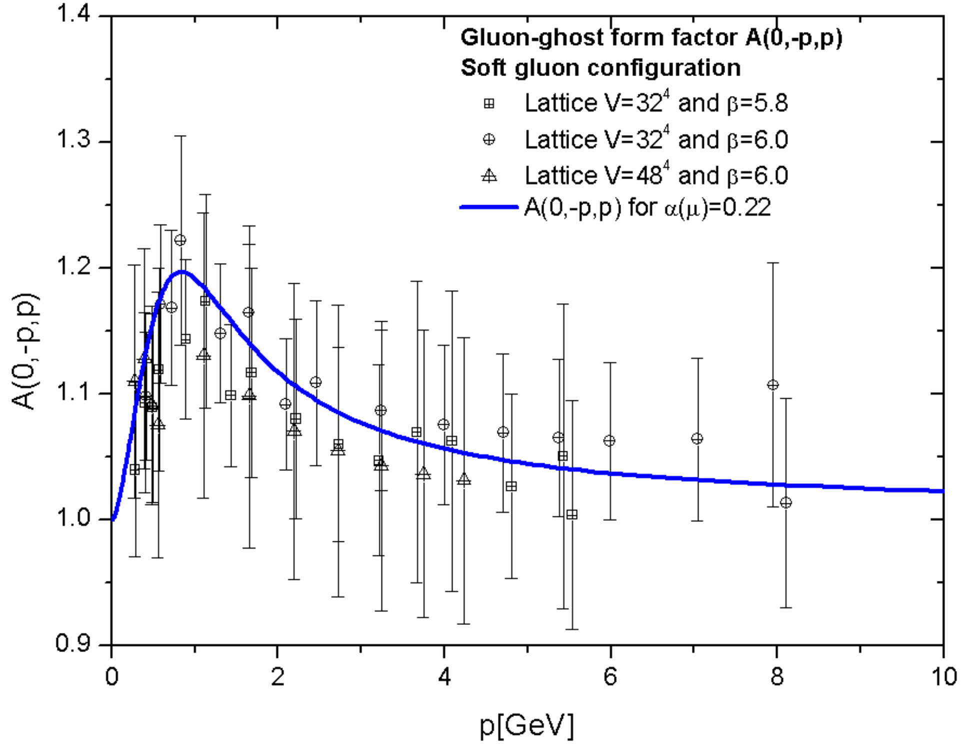
- *The form factor A(p) is given by*

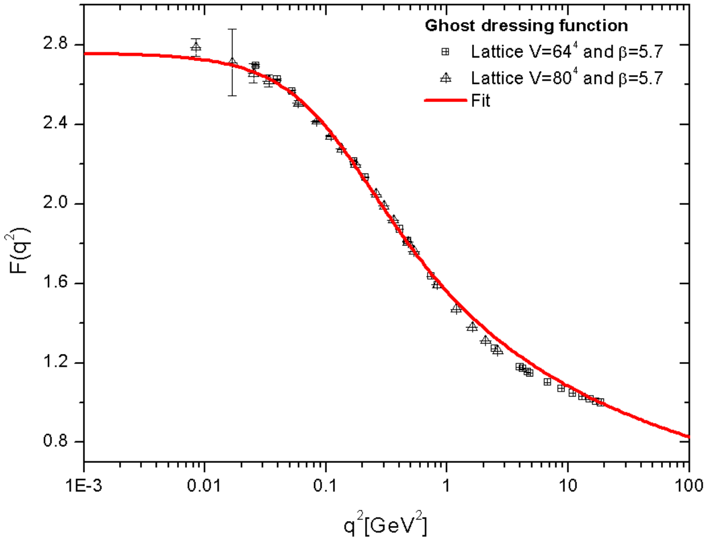
$$A(p) = 1 - \frac{i}{2} g^2 C_A [(d_1) - (d_2)];$$

- Linear integral equation
- *When* $p^2=0$ → $A(0)=1$ *(tree level value)*

Numerical results: Soft gluon







Notice that:

✓**A (0,-p,p)** *develops peak around p=830 MeV*.

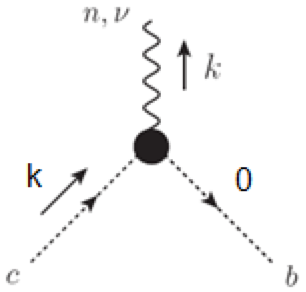
✓**A(0)=1** *and in the UV limit the form factors gradually approaches to 1.*

E.-M. Ilgenfritz, et al, Braz. J. Phys.37, 193 (2007).
A.Sternbeck, hep-lat/0609016.

I. L. Bogolubsky, et al. PoS LATTICE, 290 (2007).

A. Cucchieri, A. Maas and T. Mendes,
Phys. Rev. D 77, 094510 (2008).

Soft ghost limit



- *In this limit the momentum of the anti-ghost leg vanishes (p = 0).*

- *The form factor A* becomes a function of only k , i.e.

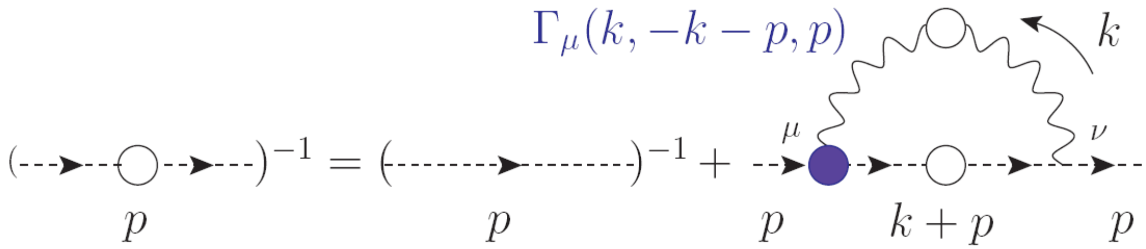
$$\lim_{p\to 0} A(-k,-p,k+p) = A(-k,0,k) \equiv A(k)\,.$$

- The soft ghost limit coincides with that of the Taylor kinematics, i.e.

$$A(-k,0,k) = \underline{A(-k,k,0)},$$

→ *Taylor limit*

- To see that rewrite the SDE of the ghost propagator, dressing the left ghost-gluon vertex instead of the right,



$$F^{-1}(p^2) = 1 + ig^2 C_{\rm A} \int_k \left[1 - \frac{(k\cdot p)^2}{k^2 p^2}\right] A(k,-k-p,p)\Delta(k)D(k+p).$$

- *Since A is quadratic in k and p →* $A(k,-k-p,p) = A(-k,k+p,-p).$

- *Evidently we must get an identical result for F(p²), independent of the vertex we choose to dress. Then*

$$A(-k,-p,k+p) = A(-k,k+p,-p),$$

- *In the limit of p →0 this reduces to*

$$A(-k,0,k) = A(-k,k,0),$$ *Taylor limit*

- *Dressing all the propagators and using tree level values for the vertices, we arrive at*

$$(d_1)_\nu = -\frac{1}{d-1} p_\nu \int_l \frac{(l\cdot k)}{l^2k^2}[l^2k^2 - (l\cdot k)^2] D(l) D(l+k) \Delta(l)\,,$$

$$(d_2)_\nu = \frac{2}{d-1} p_\nu \int_l \frac{[l^2k^2 - (l\cdot k)^2]}{l^2k^2(l+k)^2}[(l+k)^2(l\cdot k) - (l\cdot k)^2 - (d-2)l^2k^2] D(l) \Delta(l) \Delta(l+k)\,,$$

- *The form factor is given by*

$$A(k) = 1 - \frac{i}{2} g^2 C_A [(d_1) - (d_2)];$$

The coupled system

- The final system of equations for A in the soft ghost limit and the ghost SDE is given by

$$F^{-1}(x) = 1 - \frac{\alpha_s C_{\rm A}}{2\pi^2} \int_0^\infty dy y\, \Delta(y) A(y) \int_0^\pi d\theta \sin^4\theta \left[\frac{F(z)}{z} - \frac{F(z')}{z'} \right] .$$

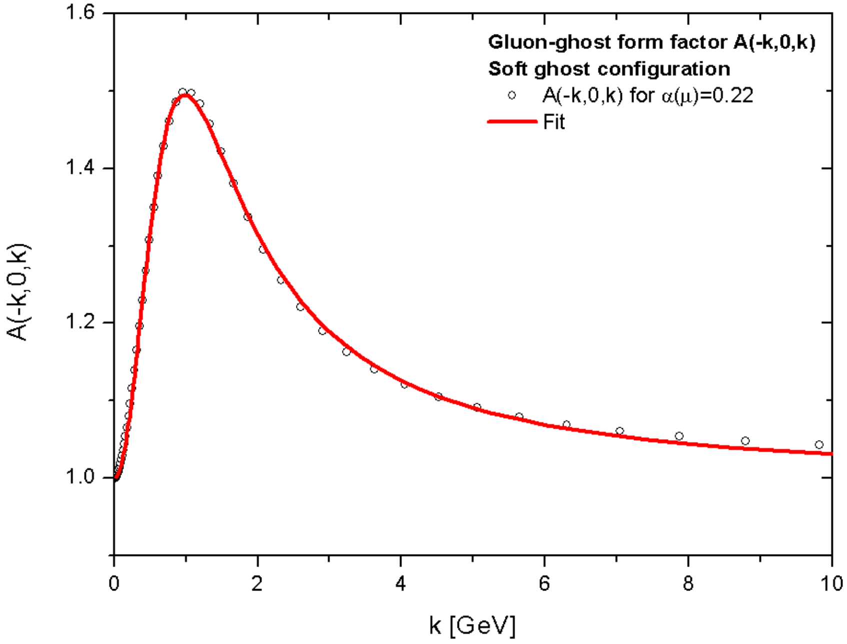
$$\begin{aligned} A(y) &= 1 - \frac{\alpha_s C_{\rm A}}{12\pi^2} \int_0^\infty dt \sqrt{yt}\, F(t)\Delta(t) \int_0^\pi d\theta' \sin^4\theta' \cos\theta' \left[\frac{F(u)}{u} \right] \\ &+ \frac{\alpha_s C_{\rm A}}{6\pi^2} \int_0^\infty dt\, F(t)\Delta(t) \int_0^\pi d\theta' \sin^4\theta' \left[\frac{\Delta(u)}{u} \right] \left[yt(1+\sin^2\theta') - (y+t)\sqrt{yt}\cos\theta' \right] , \end{aligned}$$

where

$$x = p^2$$
$$y = k^2$$
$$z = (k+p)^2,\; z' = (k+\mu)^2$$

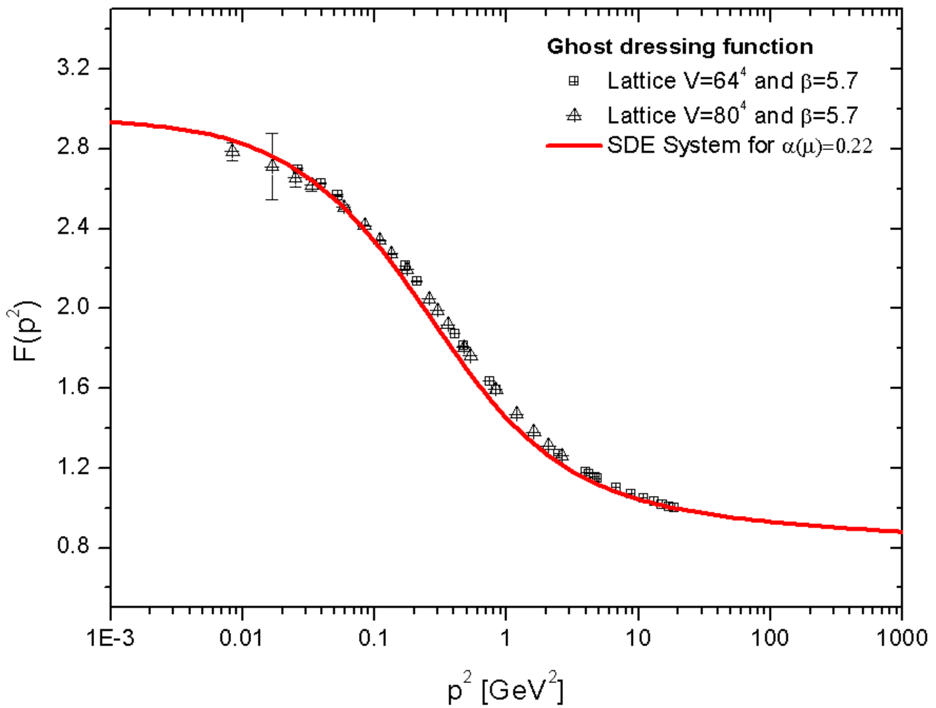
SU(3) *results for the system*

A.C. A., D. Ibáñez and J. Papavassiliou, Phys. Rev. D87, 114020 (2013)



Ghost SDE

$\alpha(\mu)=0.29 \rightarrow \alpha(\mu)=0.22$



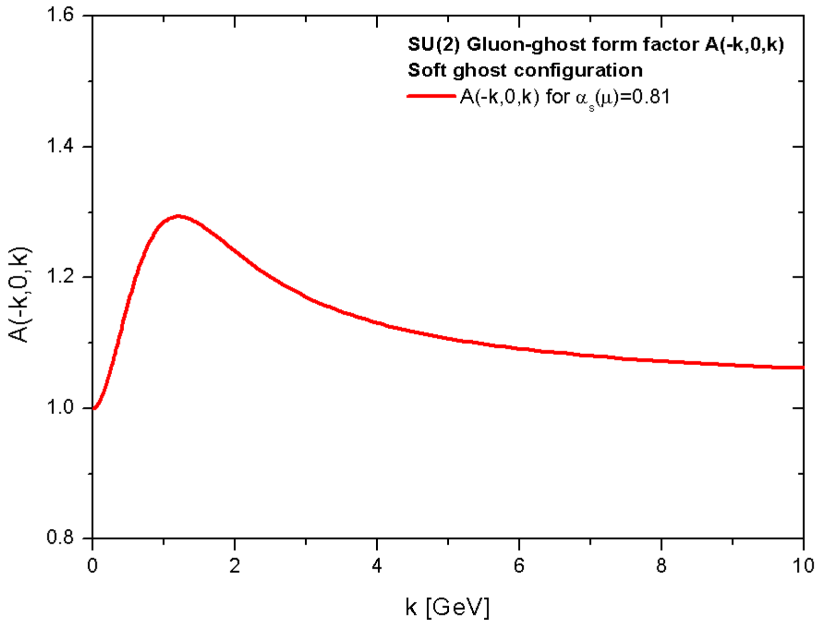
✓**A (-k,0,k)** *develops a sizable peak at p=1 GeV* (*50% bigger than of tree value)*

✓ *In the IR and UV limits the form factors approaches to 1.*

I. L. Bogolubsky, et al. PoS LATTICE, 290 (2007).

SU(2) *results for the system*

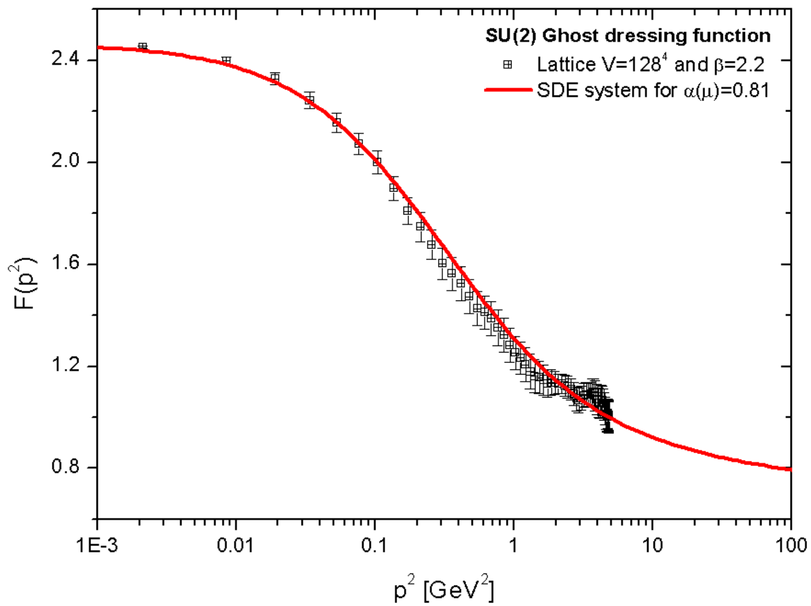
A.C. A., D. Ibáñez and J. Papavassiliou, Phys. Rev. D87, 114020 (2013)



Same qualitative behavior found in the SU(3)

Ghost SDE
$\alpha(\mu)=0.99 \rightarrow \alpha(\mu)=0.81$

- ✓*Peak at p=1.2 GeV*
- ✓ *In the IR and UV limits the form factors approaches to 1.*



A. Cucchieri, A. Maas and T. Mendes,
Phys. Rev. D 77, 094510 (2008).

A. Cucchieri and T. Mendes,
PoS LAT2007 , 297 (2007);

Conclusions

- We study the “one-loop dressed" approximation for the ghost-gluon vertex SDE.
- The vertex SDE has been evaluated for two special kinematic congurations: soft gluon and soft ghost.
- Soft ghost limit were coupled to the ghost SDE.
- Rather good agreement with the lattice data available for the vertex
- The A form factor accounts for the missing strength in the kernel of the ghost SDE → Allowing to reproduce the ghost lattice results with the standard value of α_s.