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Form factor decomposition of the off-shell four-gluon amplitudes

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- The QCD gluon amplitudes, on- and off-shell.

- One-loop gluon amplitudes in the string-inspired formalism.

- Recalculation of the one-loop three-gluon vertex.

- Form-factor decomposition of the one-loop four-gluon vertex.

On-shell vs. off-shell

Multi-gluon amplitudes in QCD pose two very different computational challenges:

- On-shell matrix elements: Tremendous progress in recent years - generalized unitarity methods, twistors, BCFW (Britto-Cachazo-Feng-Witten) recursion, ... particularly for the massless and/or SUSY cases.
- Off-shell one-particle irreducible amplitudes ("vertices"): No comparable recent progress in perturbation theory, but important input from lattice calculations.

What do we know about the off-shell gluon amplitudes?

- J. S. Ball and T. W. Chiu 1980:
  Studied the off-shell gluon amplitudes for the gluon loop in Feynman gauge; analyzed the Ward identities and derived a form factor ("Ball-Chiu") decomposition of the three-gluon vertex.
- J. M. Cornwall and J. Papavassiliou 1989:
  Constructed a "gauge invariant three-gluon vertex" through the pinch technique.
- J. Papavassiliou 1993:
  Studied the structure of the four-gluon vertex.
- A. I. Davydychev, P. Osland and O. Tarasov 1996:
  Treated the gluon loop in arbitrary covariant gauge, and also the massless fermion loop case.
- A. I. Davydychev, P. Osland and L. Saks 2001:
  Studied the massive fermion loop case.
- M. Binger and S.J. Brodsky 2006:
  Studied the scalar, fermion and gluon loop cases in various dimensions using the background field method.
- J.A. Gracey 2011:
  Three-gluon vertex at two loops for some momentum configurations.

The Ward identities for the off-shell gluon amplitudes

Off-shell, the Ward identities for the gluon amplitudes are inhomogeneous and map N - point to $N-1$ - point.
E. g. for the four-point case (J. Papavassiliou 1993):

$$p_1^{\mu}\Gamma^{abcd}_{\mu\nu\alpha\beta}(p_1,p_2,p_3,p_4) = f_{abe}\Gamma^{cde}_{\nu\alpha\beta}(p_1+p_2,p_3,p_4) + \text{perm.}$$

At the one-loop level:

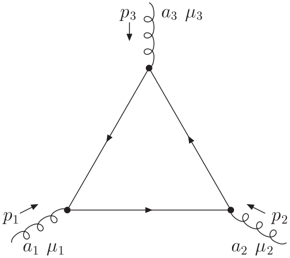
- These identities hold for the scalar and spinor loop unambiguously.
- For the gluon loop, they take the same form if one uses either the pinch technique or, equivalently, the background field method with quantum Feynman gauge.
- Other gauge fixings of the gluon loop will generally lead to a more complicated right-hand side involving ghosts.

The QCD three-gluon vertex

The three-gluon vertex at tree level:

$$igf^{a_1 a_2 a_3}\left[\eta_{\mu_1\mu_2}(p_1 - p_2)_{\mu_3} + cycl.\right]$$

Corrected at one-loop by the 1PI 3 gluon amplitude with a scalar, spinor or gluon loop. Feynman diagram (for the spinor loop):



The Ball-Chiu decomposition (1980)

$$
\begin{aligned}
\Gamma_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3) \quad = \quad & f^{abc}\Big\{A(p_1^2,p_2^2;p_3^2)g_{\mu_1\mu_2}(p_1-p_2)_{\mu_3}+B(p_1^2,p_2^2;p_3^2)g_{\mu_1\mu_2}(p_1+p_2)_{\mu_3} \\
& -C(p_1^2,p_2^2;p_3^2)[(p_1p_2)g_{\mu_1\mu_2}-p_{1\mu_2}p_{2\mu_1}](p_1-p_2)_{\mu_3} \\
& +\frac{1}{3}S(p_1^2,p_2^2,p_3^2)(p_{1\mu_3}p_{2\mu_1}p_{3\mu_2}+p_{1\mu_2}p_{2\mu_3}p_{3\mu_1}) \\
& +F(p_1^2,p_2^2;p_3^2)[(p_1p_2)g_{\mu_1\mu_2}-p_{1\mu_2}p_{2\mu_1}][p_{1\mu_3}(p_2p_3)-p_{2\mu_3}(p_1p_3)] \\
& +H(p_1^2,p_2^2,p_3^2)\Big(-g_{\mu_1\mu_2}[p_{1\mu_3}(p_2p_3)-p_{2\mu_3}(p_1p_3)]+\frac{1}{3}(p_{1\mu_3}p_{2\mu_1}p_{3\mu_2}-p_{1\mu_2}p_{2\mu_3}p_{3\mu_1})\Big) \\
& +[\text{cyclic permutations of } (p_1,\mu_1),(p_2,\mu_2),(p_3,\mu_3)]\Big\}
\end{aligned}
$$

- Universal tensor decomposition, valid for scalar, spinor and gluon loop, and also for higher loop corrections.
- Only the coefficient functions A, B, C, F, H, S change.
- At tree level, $A = 1$, the other functions vanish.
- The tensor structures multiplying F, H are manifestly transversal.

The string-inspired formalism

Bern-Kosower master formula (Z. Bern and D. Kosower 1991)

$$\Gamma^{a_1\cdots a_N}[p_1,\varepsilon_1;\dots;p_N,\varepsilon_N] = (-ig)^N \mathrm{tr}(T^{a_1}\dots T^{a_N})\int_0^\infty dT(4\pi T)^{-D/2}e^{-m^2T}$$
$$\times\int_0^T d\tau_1\int_0^{\tau_1}d\tau_2\dots\int_0^{\tau_{N-2}}d\tau_{N-1}$$
$$\times\exp\left\{\sum_{i,j=1}^N\left[\frac{1}{2}G_{Bij}p_i\cdot p_j - i\dot G_{Bij}\varepsilon_i\cdot p_j+\frac{1}{2}\ddot G_{Bij}\varepsilon_i\cdot\varepsilon_j\right]\right\}\Big|_{\mathrm{lin}(\varepsilon_1\dots\varepsilon_N)}$$

As it stands, this is a parameter integral representation for the (color-ordered) N - gluon vertex, with momenta p_i and polarizations ε_i, induced by a scalar loop, in D dimensions.
Here m and T are the loop mass and proper-time, τ_i the location of the ith gluon, and

$$G_{Bij}=|\tau_i-\tau_j|-\frac{(\tau_i-\tau_j)^2}{T},\ \dot G_B(\tau_1,\tau_2)=\mathrm{sign}(\tau_1-\tau_2)-2\frac{(\tau_1-\tau_2)}{T},\ \ddot G_B(\tau_1,\tau_2)=2\delta(\tau_1-\tau_2)-\frac{2}{T}\,.$$

The Bern-Kosower rules

In the string-derived Bern-Kosower formalism, the master formula is a generating functional for the full on-shell N - gluon amplitudes for the scalar, spinor and gluon loop, through the

Bern-Kosower rules:

1. For fixed N, expand the generating exponential.
2. Use suitable integrations-by-parts (IBPs) to remove all second derivatives $\ddot{G}_{Bij}$.
3. Apply two types of pattern-matching rules:
   - The "tree replacement rules" generate the contributions of the missing reducible diagrams.
   - The "loop replacement rules" generate the integrands for the spinor and gluon loop from the one for the scalar loop.

The worldline path integral approach

M. J. Strassler, NPB 385 (1992) 145:

- Rederived the master formula and the loop replacement rules using worldline path integral representations of the gluonic effective actions. E.g. for the scalar loop

$$\Gamma[A] = \mathrm{tr} \int_0^\infty \frac{dT}{T}\, e^{-m^2 T} \int \mathcal{D}x(\tau)\, \mathcal{P} e^{-\int_0^T d\tau \left(\frac{1}{4}\dot{x}^2 + ig\dot{x}\cdot A(x(\tau)) \right)}$$

  where $A_\mu = A^a_\mu T^a$ and $\mathcal{P}$ denotes path ordering.

- This also shows that the master formula and the loop replacement rules hold off-shell.

M. J. Strassler, SLAC-PUB-5978 (unpubl.): noted that the IBP generates automatically

- abelian field strength tensors $f_i^{\mu\nu} \equiv p_i^\mu \varepsilon_i^\nu - \varepsilon_i^\mu p_i^\nu$ in the bulk and
- color commutators $[T^{a_i}, T^{a_j}]$ as boundary terms.
- Those fit together to produce full nonabelian field strength tensors

$$F_{\mu\nu} \equiv F^a_{\mu\nu} T^a = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) T^a + ig[A^b_\mu T^b, A^c_\nu T^c] \tag{1}$$

  in the low-energy effective action.

Thus we see the emergence of gauge invariant tensor structures at the integrand level.

Integration-by-parts algorithms

Removing all $\ddot{G}_{Bij}$ by IBP can be done in many ways!

- M.J. Strassler 1992: started to investigate this ambiguity at the four-point level.
- C. Schubert 1998: found an algorithm that preserves the full permutation symmetry.
- N. Ahmadiniaz, C. Schubert, V.M. Villanueva, JHEP 01 (2013) 132: two IBP algorithms that work for arbitrary N and lead to explicit form-factor decompositions of the off-shell N - gluon amplitudes:
  - The first algorithm uses only local total derivative terms and leads to a representation that matches term-by-term with the low-energy effective action ("Q-representation").
  - The second algorithm uses both local and nonlocal total derivative terms and leads to the transversality of all bulk terms at the integrand level ("S-representation").
- N. Ahmadiniaz, C. Schubert, NPB 869 (2013) 417: for $N = 3$ the second algorithm generates the Ball-Chiu decomposition.
- N. Ahmadiniaz, C. Schubert, in preparation: form factor decompositions of the four-gluon amplitudes.

The three-gluon vertex in the Q-representation

For $N = 3$, the Q-representation is (for the scalar loop)

$$\Gamma = \frac{g^3}{(4\pi)^{\frac{D}{2}}} \mathrm{tr}(T^{a_1}[T^{a_2}, T^{a_3}])(\Gamma^3 + \Gamma^2 + \Gamma^{\mathrm{bt}})$$

$$\begin{aligned}
\Gamma^3 &= -\int_0^\infty \frac{dT}{T^{\frac{D}{2}}} e^{-m^2 T} \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 \, Q_3^3 \exp\left\{ \sum_{i,j=1}^{3} \frac{1}{2} G_{Bij} p_i \cdot p_j \right\} \\
\Gamma^2 &= \Gamma^3 (Q_3^3 \to Q_3^2) \\
\Gamma^{\mathrm{bt}} &= -\int_0^\infty \frac{dT}{T^{\frac{D}{2}}} e^{-m^2 T} \int_0^T d\tau_1 \dot{G}_{B12} \dot{G}_{B21} \left[\varepsilon_3 \cdot f_1 \cdot \varepsilon_2 \, e^{G_{B12} p_1 \cdot (p_2 + p_3)} + \mathrm{cycl.} \right]
\end{aligned}$$

$$\begin{aligned}
Q_3^3 &= \dot{G}_{B12} \dot{G}_{B23} \dot{G}_{B31} \mathrm{tr}\,(f_1 f_2 f_3) \\
Q_3^2 &= \frac{1}{2} \dot{G}_{B12} \dot{G}_{B21} \mathrm{tr}\,(f_1 f_2) \dot{G}_{B3k} \varepsilon_3 \cdot p_k + 2\,\mathrm{perm.}
\end{aligned}$$

Here the lower index on a Q refers to $N = 3$, the upper index to the "cycle content".
Dummy indices like k are to be summed from 1 to $N = 3$.

Loop replacement rules

From Scalar Loop to Spinor Loop:

$$
\begin{aligned}
\dot{G}_{Bij}\dot{G}_{Bji} &\to \dot{G}_{Bij}\dot{G}_{Bji} - G_{Fij}G_{Fji} \\
\dot{G}_{B12}\dot{G}_{B23}\dot{G}_{B31} &\to \dot{G}_{B12}\dot{G}_{B23}\dot{G}_{B31} - G_{F12}G_{F23}G_{F31}
\end{aligned}
$$

where $G_{Fij} = \mathrm{sign}(\tau_i - \tau_j)$.

From Scalar Loop to Gluon Loop:

$$
\begin{aligned}
\dot{G}_{Bij}\dot{G}_{Bji} &\to \dot{G}_{Bij}\dot{G}_{Bji} - 4G_{Fij}G_{Fji} \\
\dot{G}_{B12}\dot{G}_{B23}\dot{G}_{B31} &\to \dot{G}_{B12}\dot{G}_{B23}\dot{G}_{B31} - 4G_{F12}G_{F23}G_{F31}
\end{aligned}
$$

The generated integrand for the gluon loop corresponds to the background field method with quantum Feynman gauge.

Comparison with the effective action

The low energy expansion of the one-loop QCD effective action induced by a loop particle of mass m has the form

$$\Gamma[F] = \int_0^\infty \frac{dT}{T} \frac{\mathrm{e}^{-m^2 T}}{(4\pi T)^{D/2}} \,\mathrm{tr} \int dx_0 \sum_{n=2}^{\infty} \frac{(-T)^n}{n!} O_n[F]$$

where $O_n(F)$ is a Lorentz and gauge invariant expression of mass dimension $2n$. To lowest orders,

$$\begin{aligned} O_2 &= c_2 g^2 F_{\mu\nu} F_{\mu\nu} \,, \\ O_3 &= c_3^3\, ig^3\, F_{\kappa\lambda} F_{\lambda\mu} F_{\mu\kappa} + c_3^2 g^2 D_\lambda F_{\mu\nu} D^\lambda F^{\mu\nu} \end{aligned}$$

We recognize the correspondences

$$\Gamma^3 \leftrightarrow F_\kappa{}^\lambda F_\lambda{}^\mu F_\mu{}^\kappa = f_\kappa^\lambda f_\lambda^\mu f_\mu^\kappa + \text{higher point terms}$$

$$\Gamma^2 \leftrightarrow (\partial + ig \underbrace{A) F (\partial} + igA) F$$

$$\Gamma^{\mathrm{bt}} \leftrightarrow (f + ig \underbrace{[A, A])(f} + ig[A, A])$$

The three-gluon vertex in the S-representation

$$\tilde{\Gamma} = \frac{g^3}{(4\pi)^{\frac{D}{2}}} \mathrm{tr}(T^{a_1}[T^{a_2}, T^{a_3}])(\tilde{\Gamma}^3 + \tilde{\Gamma}^2 + \tilde{\Gamma}^{\mathrm{bt}})$$

$$\begin{aligned}
\tilde{\Gamma}^3 &= -\int_0^\infty \frac{dT}{T^{\frac{D}{2}}} e^{-m^2 T} \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2\, S_3^3 \exp\Big\{ \sum_{i,j=1}^3 \frac{1}{2} G_{Bij} p_i \cdot p_j \Big\} \\
\tilde{\Gamma}^2 &= \tilde{\Gamma}^3 (S_3^3 \to S_3^2) \\
\tilde{\Gamma}^{\mathrm{bt}} &= -\int_0^\infty \frac{dT}{T^{\frac{D}{2}}} e^{-m^2 T} \int_0^T d\tau_1 \dot{G}_{B12} \dot{G}_{B21} \Big\{ \Big[\varepsilon_3 \cdot f_1 \cdot \varepsilon_2 + \frac{1}{2} \mathrm{tr}(f_1 f_2) \rho_3 - \frac{1}{2} \mathrm{tr}(f_3 f_1) \rho_2 \Big] \\
&\qquad \times e^{G_{B12} p_1 \cdot (p_2 + p_3)} + \mathrm{cycl.} \Big\}
\end{aligned}$$

$$\begin{aligned}
S_3^3 &= \dot{G}_{B12} \dot{G}_{B23} \dot{G}_{B31} \mathrm{tr}\,(f_1 f_2 f_3) \\
S_3^2 &= \frac{1}{2} \dot{G}_{B12} \dot{G}_{B21} \mathrm{tr}\,(f_1 f_2) \dot{G}_{B3k} \frac{r_3 \cdot f_3 \cdot p_k}{r_3 \cdot p_3} + 2\,\mathrm{perm.}
\end{aligned}$$

where $\rho_i := \frac{r_i \cdot \varepsilon_i}{r_i \cdot p_i}$ with $r_i \cdot p_i \neq 0$ but arbitrary otherwise. Now all bulk terms are transversal, and choosing

$$r_1 = p_2 - p_3\,,\, r_2 = p_3 - p_1\,,\, r_3 = p_1 - p_2$$

we get a term-by-term match with the Ball-Chiu decomposition.

The three-gluon vertex in the S-representation

$$H(p_1^2,p_2^2,p_3^2) = -\frac{d_0 g^2}{2(4\pi)^{D/2}}\Gamma(3-\frac{D}{2}) I^D_{3,B}(p_1^2,p_2^2,p_3^2)$$

$$A(p_1^2,p_2^2;p_3^2) = \frac{d_0 g^2}{4(4\pi)^{D/2}}\Gamma(2-\frac{D}{2})\Big[I^D_{\mathrm{bt,B}}(p_1^2) + I^D_{\mathrm{bt,B}}(p_2^2)\Big]$$

$$B(p_1^2,p_2^2;p_3^2) = \frac{d_0 g^2}{4(4\pi)^{D/2}}\frac{1}{2}\Gamma(2-\frac{D}{2})\Big[I^D_{\mathrm{bt,B}}(p_1^2) - I^D_{\mathrm{bt,B}}(p_2^2)\Big]$$

$$F(p_1^2,p_2^2;p_3^2) = \frac{d_0 g^2}{2(4\pi)^{D/2}}\Gamma(3-\frac{D}{2})\frac{I^D_{2,B}(p_1^2,p_2^2,p_3^2) - I^D_{2,B}(p_2^2,p_1^2,p_3^2)}{p_1^2-p_2^2}$$

$$C(p_1^2,p_2^2;p_3^2) = \frac{d_0 g^2}{2(4\pi)^{D/2}}\Gamma(2-\frac{D}{2})\frac{I^D_{\mathrm{bt,B}}(p_1^2) - I^D_{\mathrm{bt,B}}(p_2^2)}{p_1^2-p_2^2}$$

$$S(p_1^2,p_2^2;p_3^2) = 0$$

Replacement rules $\rightarrow$ spinor and gluon loop cases similar.

Note that all non-transversal terms are boundary terms, the bulk has become transversal.

Integrals in term of the standard *Feynman/Schwinger* parameter $\alpha_{1,2,3}$

$$I^D_{3,B}(p_1^2,p_2^2,p_3^2) = \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3) \times \frac{(1-2\alpha_1)(1-2\alpha_2)(1-2\alpha_3)}{\left(m^2+\alpha_1\alpha_2 p_1^2+\alpha_2\alpha_3 p_2^2+\alpha_1\alpha_3 p_3^2\right)^{3-\frac{D}{2}}}$$

$$I^D_{2,B}(p_1^2,p_2^2,p_3^2) = \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3) \times \frac{(1-2\alpha_2)^2(1-2\alpha_1)}{\left(m^2+\alpha_1\alpha_2 p_1^2+\alpha_2\alpha_3 p_2^2+\alpha_1\alpha_3 p_3^2\right)^{3-\frac{D}{2}}}$$

$$I^D_{bt,B}(p^2) = \int_0^1 d\alpha \frac{(1-2\alpha)^2}{\left(m^2+\alpha(1-\alpha)p^2\right)^{2-\frac{D}{2}}}$$

The four-gluon vertex (Q-representation): bulk terms

$$\Gamma^{a_1 a_2 a_3 a_4} = g^4 \mathrm{tr}(T^{a_1} \dots T^{a_4}) \int_0^\infty dT (4\pi T)^{-D/2} e^{-m^2 T} \times \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 Q_4 \exp\left\{ \sum_{i,j=1}^4 \frac{1}{2} G_{Bij} p_i \cdot p_j \right\}$$

$$\begin{aligned}
Q_4 &= Q_4^4 + Q_4^3 + Q_4^2 - Q_4^{22} \\
Q_4^4 &= \dot{G}(1234) + \dot{G}(1243) + \dot{G}(1324) \\
Q_4^3 &= \dot{G}(123)T(4) + \dot{G}(234)T(1) + \dot{G}(341)T(2) + \dot{G}(412)T(3) \\
Q_4^2 &= \dot{G}(12)T(34) + \dot{G}(13)T(24) + \dot{G}(14)T(23) + \dot{G}(23)T(14) \\
&\quad + \dot{G}(24)T(13) + \dot{G}(34)T(12) \\
Q_4^{22} &= \dot{G}(12)\dot{G}(34) + \dot{G}(13)\dot{G}(24) + \dot{G}(14)\dot{G}(23)
\end{aligned}$$

$$\begin{aligned}
\dot{G}(i_1 i_2 \cdots i_n) &:= \dot{G}_{Bi_1 i_2} \dot{G}_{Bi_2 i_3} \cdots \dot{G}_{Bi_n i_1} \left(\frac{1}{2}\right)^{\delta_{n,2}} \mathrm{tr}(f_{i_1} f_{i_2} \cdots f_{i_n}) \\
T(i) &:= \sum_r \dot{G}_{Bir} \varepsilon_i \cdot p_r \\
T(ij) &:= \sum_{r,s} \left\{ \dot{G}_{Bir} \varepsilon_i \cdot p_r \dot{G}_{js} \varepsilon_j \cdot p_s + \frac{1}{2} \dot{G}_{Bij} \varepsilon_i \cdot \varepsilon_j \left[\dot{G}_{Bir} p_i \cdot p_r - \dot{G}_{Bjr} p_j \cdot p_r \right] \right\}
\end{aligned}$$

The four-gluon vertex: boundary terms

Now there are single boundary terms and double boundary terms.
Recursive structure at the integrand level:

- Each single boundary term, say for the limit $3 \to 4$, matches some bulk term in the Q-representation of the three-gluon vertex, with momenta $(p_1, p_2, p_3 + p_4)$, and $f_3 = p_3 \otimes \varepsilon_3 - \varepsilon_3 \otimes p_3$ replaced by $\varepsilon_3 \otimes \varepsilon_4 - \varepsilon_4 \otimes \varepsilon_3$.
- Each double boundary term, say for the limit $1 \to 2, 3 \to 4$, matches the bulk term in the Q-representation of the two-point function, with momenta $(p_1 + p_2, p_3 + p_4)$, and the double replacement

$$
\begin{aligned}
f_1 &= p_1 \otimes \varepsilon_1 - \varepsilon_1 \otimes p_1 \to \varepsilon_1 \otimes \varepsilon_2 - \varepsilon_2 \otimes \varepsilon_1 \\
f_2 &= p_2 \otimes \varepsilon_2 - \varepsilon_2 \otimes p_2 \to \varepsilon_3 \otimes \varepsilon_4 - \varepsilon_4 \otimes \varepsilon_3
\end{aligned}
$$

- Effectively, a boundary term always completes a f_i to a full nonabelian field strength tensor,

$$
\partial_\mu A_\nu - \partial_\nu A_\mu \to \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]
$$

- This recursive structure is compatible with the replacement rules.
- The S-representation looks similar, but has the bulk terms written completely in terms of the f_i, and involves the choice of four vectors r_i with $r_i \cdot p_i \neq 0$.

Off-shell one-loop four-gluon vertex in $\mathcal{N} = 4$ SYM

In $\mathcal{N} = 4$ SYM the one-loop two - and three - gluon amplitudes vanish (this relates to the finiteness of the theory). The one-loop four-gluon vertex becomes extremely simple: all boundary terms cancel out, and the bulk term involves only the scalar box integral:

$$\Gamma^{a_1 a_2 a_3 a_4} = 4g^4 \mathrm{tr}\,(T^{a_1} T^{a_2} T^{a_3} T^{a_4}) F^4_{\mathrm{ss}} B(1234) + \mathrm{non-cyclic\ permutations}$$

Here $B(1234)$ is the off-shell scalar box integral with momenta $p_1, \ldots, p_4$, and

$$\begin{aligned} F^4_{\mathrm{ss}} &= \mathrm{tr}\,(f_1 f_2 f_3 f_4) + \mathrm{tr}\,(f_1 f_2 f_4 f_3) + \mathrm{tr}\,(f_1 f_3 f_2 f_4) \\ &\quad - \frac{1}{4}\mathrm{tr}\,(f_1 f_2)\mathrm{tr}\,(f_3 f_4) - \frac{1}{4}\mathrm{tr}\,(f_1 f_3)\mathrm{tr}\,(f_2 f_4) - \frac{1}{4}\mathrm{tr}\,(f_1 f_4)\mathrm{tr}\,(f_2 f_3) \end{aligned}$$

This invariant is well-known to string theorists!

Summary and Outlook

- The string-inspired formalism makes it possible to generate form factor decompositions of the N - gluon vertex without analyzing the Ward identities.
- At the one-loop level, the parameter integrals appearing in the form factors for the scalar, spinor and gluon loop cases are all obtained directly from the Bern-Kosower master formula.
- We have carried out this program explicitly for the three- and four-point cases.
- In particular, we have obtained a natural four-point generalization of the Ball-Chiu decomposition.

Thanks for your attention!

We welcome your questions, suggestions and comments !