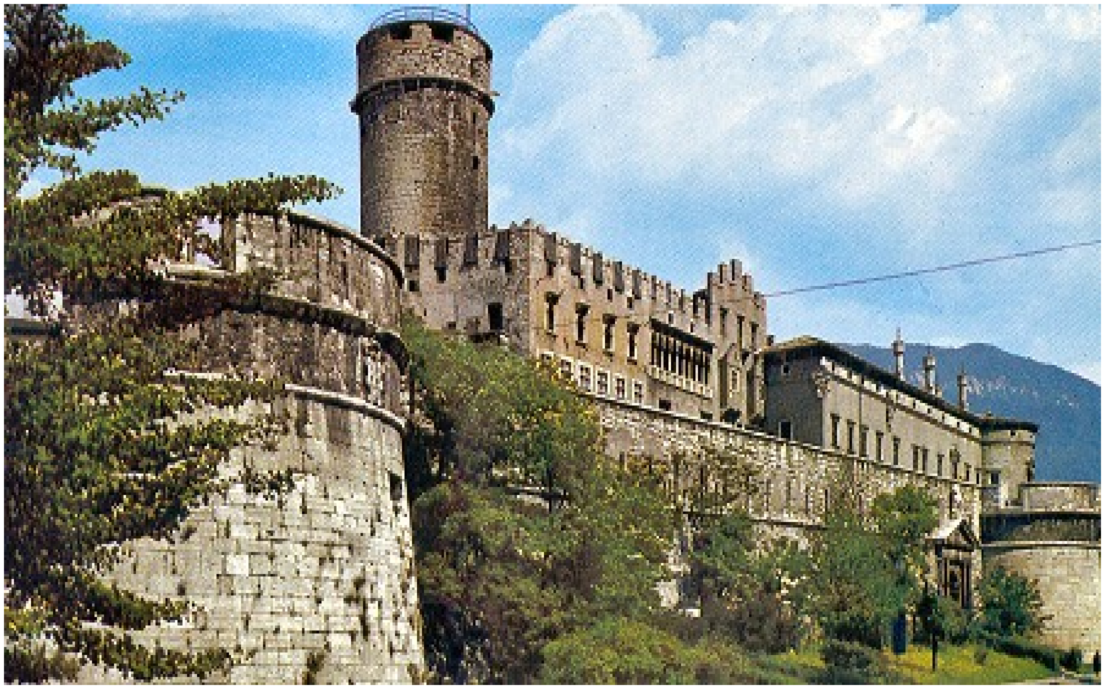
The dynamical gluon mass in the massless bound-state formalism



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Based on **Phys. Rev. D 87, 034008 (2013)**
D. I. and **J. Papavassiliou**

Outline

- Motivation
- Pole vertices
- Schwinger-Dyson formalism
- Massless Bound-State formalism
- Formal equivalence
- Conclusions

Motivation

- Study the mechanism of dynamical gluon mass generation in pure Yang-Mills theories.

J. M. Cornwall, Phys. Rev. D26, 1453 (1982)

- Gauge invariant generation through the triggering of the Schwinger mechanism.

J. S. Schwinger, Phys. Rev.125, 397 (1962)

- Crucial ingredient: Special non-perturbative vertices V.

Two approaches:

- Schwinger-Dyson (SD) formalism: Global properties of V
- Massless bound-state formalism: Dynamical ingredients of V

Pole vertices

- The implementation of the Schwinger mechanism requires the existence of a very special type of nonperturbative vertices V, called pole vertices.
- Contain massless poles of nonperturbative origin ($\sim 1/q^2$).
- Make possible that $\Delta^{-1}(0) \neq 0$.
- Allow for a gauge invariant generation of a gluon mass (preserve the STIs).
- Are completely longitudinally coupled, act as a composite Nambu-Goldstone-like bosons.

R. Jackiw and K. Johnson, Phys. Rev. D 8, 2386 (1973)
E. Eichten and F. Feinberg, Phys. Rev. D 10, 3254 (1974)
E. C. Poggio, E. Tomboulis and S. H. Tye, Phys. Rev. D 11, 2839 (1975)

Global properties of the pole vertices

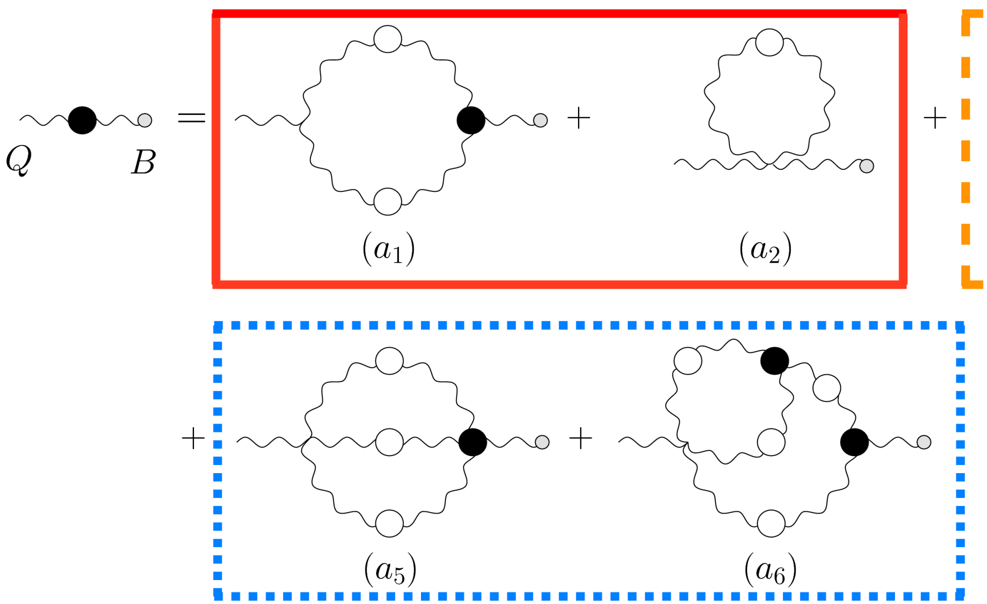
- Completely longitudinally coupled condition (three-gluon pole vertex)

$$P^{\alpha\rho}(q)P^{\mu\sigma}(r)P^{\nu\tau}(p)V_{\rho\sigma\tau}(q,r,p) = 0 \quad ; \quad P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

- Gauge invariance (STIs preserved) requires that in the presence of masses

$$q^\alpha \widetilde{V}_{\alpha\mu\nu}(q,r,p) = m^2(r^2)P_{\mu\nu}(r) - m^2(p^2)P_{\mu\nu}(p)$$

Schwinger-Dyson formalism (PT-BFM scheme)



$$\Delta^{-1}(q^2)P_{\mu\nu}(q) = \frac{q^2 P_{\mu\nu}(q) + i\sum_{i=1}^{6}(a_i)_{\mu\nu}}{1+G(q^2)}$$

- $F^{-1}(q^2) \sim 1 + G(q^2)$ Ghost dressing function: $F(q^2) = q^2 D(q^2)$

D. Binosi and J. Papavassiliou, **Phys. Rept. 479 (2009)**
A. C. Aguilar, D. Binosi and J. Papavassiliou, **Phys. Rev. D 78, 025010 (2008)**

- Assume the existence of the pole vertices.

Mass equation (Landau gauge)

Gluon propagator SDE + Global properties of V

D. Binosi, D. I. and J. Papavassiliou,
Phys. Rev. D86, 085033 (2012)

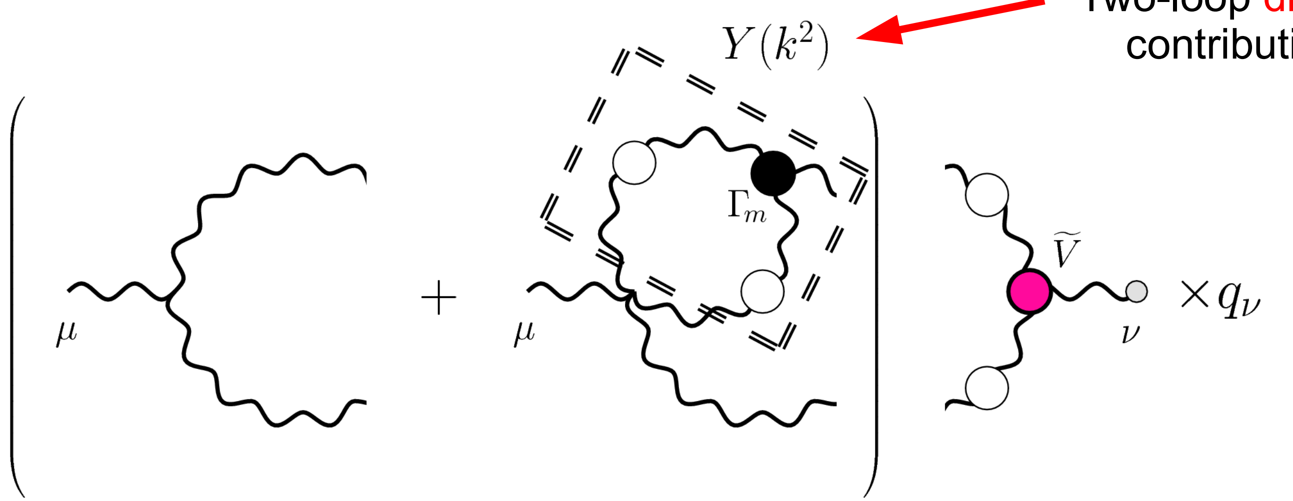
$$m^2(q^2) = \frac{ig^2 C_A}{1+G(q^2)}\frac{1}{q^2}\int_k m^2(k^2)\Delta_{\gamma\rho}(k)\Delta^{\rho}_{\mu}(k+q)\mathcal{K}^{\gamma\mu}_{SD}(q,k)$$

SD kernel

$$\mathcal{K}^{\gamma\mu}_{SD}(q,k) \;=\; g^{\gamma\mu}[(k+q)^2-k^2]\left\{1+\frac{3}{4}ig^2C_A[Y(k+q)+Y(k)]\right\} + \frac{3}{4}ig^2C_A(q^2g^{\gamma\mu}-2q^{\gamma}q^{\mu})[Y(k+q)-Y(k)].$$

Two-loop dressed contributions

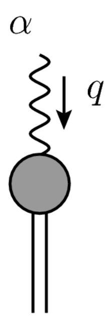
$$\widetilde{m}^2(q^2) = \frac{1}{q^2}\,q^{\mu}\times$$



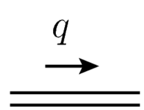
Massless Bound-State Formalism

D. I. and J. Papavassiliou,
Phys. Rev. D87, 034008 (2013)

- Describe the gluon mass in terms of quantities appearing in the physics of bound-states.



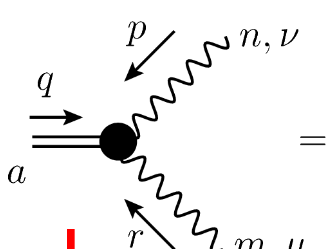
$= I_\alpha(q) = q_\alpha I(q^2)$



$= i/q^2$

Massless bound-state propagator

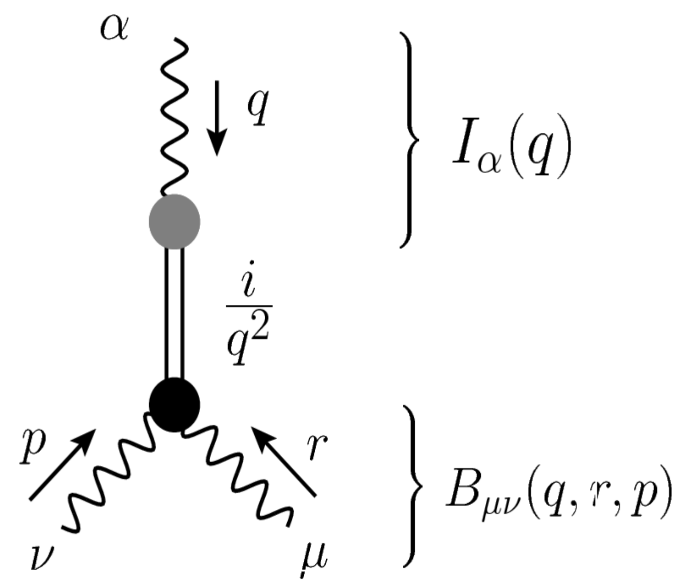
Transition amplitude



$= i f^{amn} B_{\mu\nu}(q, r, p)$

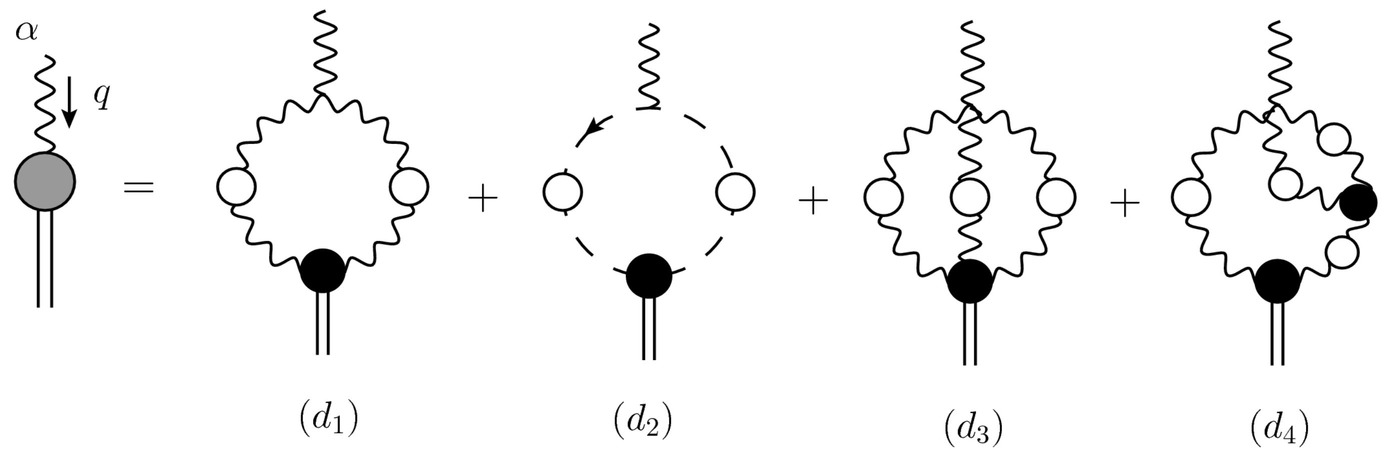
Effective vertices

- Express the pole vertices V in terms of these quantities.

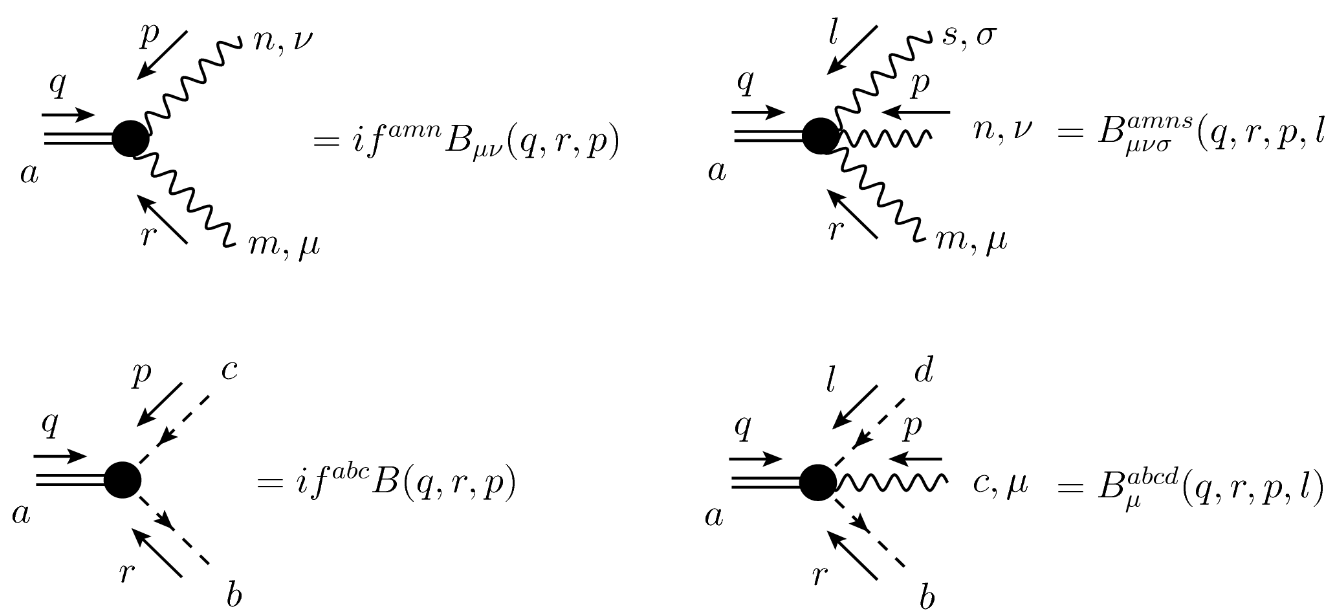


$$V_{\alpha\mu\nu}(q, r, p) = I_\alpha(q)\left(\frac{i}{q^2}\right)B_{\mu\nu}(q, r, p)$$

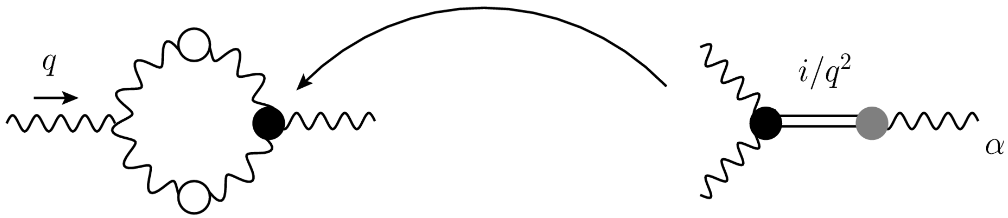
- Diagrammatic representation of the transition amplitude



- Different effective vertices

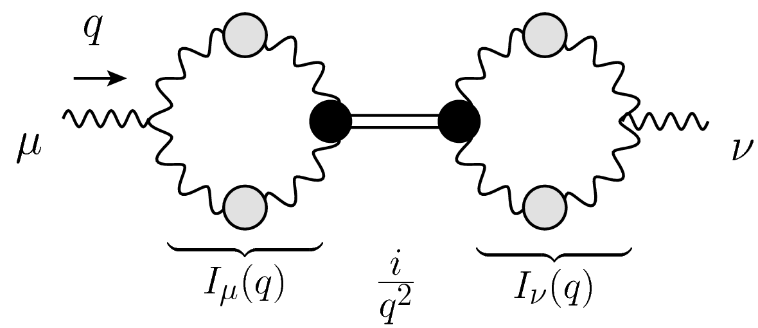


- Relating the gluon mass with the transition amplitude.



Insert the pole vertices V in the SDE of the gluon propagator

Get the "squared" diagrams



Generalize to all diagrams appearing in the SDE

$$m^2(q^2) = g^2 I^2(q^2)$$

Possitive definite gluon mass

- Fundamental and exact relation of the massless bound-state formalism

$$q^{\alpha}\widetilde{V}_{\alpha\mu\nu}(q,r,p) = m^2(r^2)P_{\mu\nu}(r) - m^2(p^2)P_{\mu\nu}(p)$$

WI of the pole vertex

$$P^{\rho\mu}(r)P^{\sigma\nu}(p)q^{\alpha}\widetilde{V}_{\alpha\mu\nu}(q,r,p) = [m^2(r^2) - m^2(p^2)]P^{\rho\mu}(r)P^{\sigma}_{\mu}(p)$$

$$\widetilde{V}_{\alpha\mu\nu}(q,r,p) = i\frac{q_\alpha}{q^2}\widetilde{I}(q^2)B_{\mu\nu}(q,r,p)$$

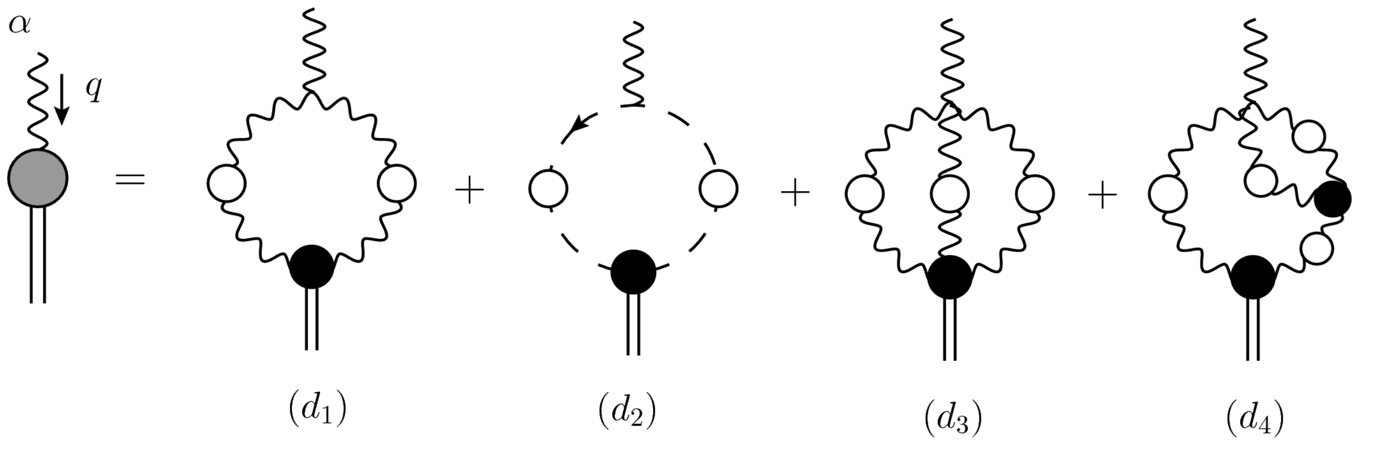
Representation of the pole vertex

$$P^{\rho\mu}(r)P^{\sigma\nu}(p)q^{\alpha}\widetilde{V}_{\alpha\mu\nu}(q,r,p) = iF^{-1}(q^2)I(q^2)B_1(q,r,p)P^{\rho\mu}(r)P^{\sigma}_{\mu}(p)$$

Equating both results

$$iI(q^2)B_1(q,r,p) = F(q^2)[m^2(r^2) - m^2(p^2)]$$

Formal equivalence of the two approaches



Diagrammatic representation

- In the Landau gauge $(d_3)=0$.
- Since the ghost propagator is massless $(d_2)=0$.
- Only (d_1) and (d_4) contribute.

$$
\begin{aligned}
I(q^2) &= \frac{q^\alpha}{q^2}[(d_1)+(d_4)]_\alpha = \frac{i}{q^2}C_A\int_k k^2\Delta_\mu^\rho(k)\Delta^{\mu\sigma}(k+q)B_{\rho\sigma} \\
&+ \frac{3}{2}\frac{g^2C_A^2}{q^2}\int_k[(kq)g_{\mu\gamma}+q_\mu q_\gamma]Y(k^2)\Delta^{\mu\sigma}(k+q)\Delta^{\rho\gamma}(k)B_{\rho\sigma}.
\end{aligned}
$$

Expression for the transition amplitude

- Only B_1 survives in the Landau gauge.

$$
I(q^2) = \frac{i}{q^2}C_A\int_k \Delta_{\gamma\rho}(k)\Delta_\mu^\rho(k+q)\underbrace{\mathcal{K}_{SD}^{\gamma\mu}(q,k)}B_1
$$

SD kernel of the mass equation !!

- Use the relations derived in the massless bound-state formalism.

$$iI(q^2)B_1(q,r,p) = F(q^2)[m^2(r^2) - m^2(p^2)]$$

$$I^2(q^2) = \frac{iC_A}{1+G(q^2)}\frac{1}{q^2}\int_k \Delta_{\gamma\rho}(k)\Delta^{\rho}_{\mu}(k+q)\mathcal{K}^{\gamma\mu}_{SD}(q,k)m^2(k^2)$$

$$m^2(q^2) = g^2 I^2(q^2)$$

Same equation as in the SD formalism !!

SDE vs Massless Bound-State Formalism

SDE formalism

$$m^2(q^2) = \frac{ig^2 C_A}{1+G(q^2)}\frac{1}{q^2}\int_k m^2(k^2)\Delta_{\gamma\rho}(k)\Delta^{\rho}_{\mu}(k+q)\mathcal{K}^{\gamma\mu}_{SD}(q,k)\,.$$

Massless bound-state formalism

$$m^2(q^2) = g^2 I^2(q^2)$$

$$m^2 = I \times B$$

Same dynamical equations for the effective gluon mass

Conclusions

- The gauge invariant generation of a gluon mass relies on the existence of massless bound-state excitations, which trigger the Schwinger mechanism.

- The effect of these massless bound-state excitations is transmited to the gluon propagator through a new kind of nonperturbative vertices V, called pole vertices.

- Two different approaches: SD formalism, massless bound-state formalism.

- In the massless bound-state formalism we have established a direct conection between the gluon mass and quantities appearing in the physics of bound-states.

- We have demonstrated the formal equivalence of both formalisms.