Motivations

- The existence of gauge-invariant bound states of gluons implied by confinement
- However, glueball states have not yet been convincingly identified in experiments
- Glueball-like states can play a key role also in strongly interacting dynamics beyond the standard model
- A calculation from first principles using lattice techniques can serve as a guidance to theoretical models and experimental searches
Outline

1. Lattice setup
2. Glueballs in QCD-like theories
3. Glueballs and (near-)conformal dynamics
4. Conclusions and outlook
The Lattice
Lattice action for full QCD

Path integral

\[ Z = \int (\mathcal{D}U_\mu(i)) (\det M(U_\mu))^N \ e^{-S_g(U_{\mu\nu}(i))} \]

with

\[ U_\mu(i) = P\exp \left( ig \int_i^{i+a\hat{\mu}} A_\mu(x)dx \right) \]

and

\[ U_{\mu\nu}(i) = U_\mu(i)U_\nu(i + \hat{\mu})U_\mu^\dagger(i + \hat{\nu})U_\nu^\dagger(i) \]

Gauge part

\[ S_g = \beta \sum_{i,\mu} \left( 1 - \frac{1}{N} \text{Re} \ Tr(U_{\mu\nu}(i)) \right) \]

with \( \beta = 2N/g_0^2 \)
Trial operators $\Phi_1(t), \ldots, \Phi_n(t)$ with the quantum numbers of the state of interest

\[
C_{ij}(t) = \langle 0 | (\Phi_i(0))^\dagger \Phi_j(t) | 0 \rangle = \langle 0 | (\Phi_i(0))^\dagger e^{-Ht} \Phi_j(0) e^{Ht} | 0 \rangle = \sum_n \langle 0 | (\Phi_i(0))^\dagger | n \rangle \langle n | e^{-Ht} \Phi_j(0) e^{Ht} | 0 \rangle = \sum_n e^{-\Delta E_n t} \langle 0 | (\Phi_i(0))^\dagger | n \rangle \langle n | \Phi_j(0) | 0 \rangle = \sum_n c^*_i n c_j n e^{-\Delta E_n t} = \delta_{ij} \sum_n |c_i n|^2 e^{-a m_n t} \to \infty \delta_{ij} |c_{i1}|^2 e^{-a m_1 t}
\]
Variational principle

1. Find the eigenvector $v$ that minimises

$$am_1(t_d) = -\frac{1}{t_d} \log \frac{v_i^* C_{ij}(t_d)v_j}{v_i^* C_{ij}(0)v_j}$$

for some $t_d$

2. Fit $v(t)$ with the law $A e^{-m_1 t}$ to extract $m_1$

3. Find the complement to the space generated by $v(t)$

4. Repeat 1-3 to extract $m_2, \ldots, m_n$

Sources of systematics

- Need a good overlap of the eigenvectors with the state of interest
- Need a large variational basis including all possible states overlapping with the required one
- Need to keep under control finite size and lattice artefacts
- Care should be taken in assigning the spin
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Glueballs in the quenched approximation

(From Morningstar and Peardon, hep-lat/9901004)
QCD at large $N$

Generalisation of QCD: SU($N$) gauge theory (possibly enlarged with $N_f$ fermions in the fundamental representation)

Taking the limit $g^2 \to 0$, $N \to \infty$, $\lambda = g^2 N$ fixed simplifies the theory and one can see that:

- Quark loop effects $\propto 1/N \Rightarrow$ The $N = \infty$ limit is quenched
- Mixing glueballs-mesons $\propto 1/\sqrt{N} \Rightarrow$ No mixing between glueballs and mesons at $N = \infty$
- Meson decay widths $\propto 1/N \Rightarrow$ Mesons do not decay at $N = \infty$
- OZI rule $\propto 1/N \Rightarrow$ OZI rule exact at $N = \infty$

$\hookrightarrow$ The simpler large $N$ phenomenology can explain features of QCD phenomenology in a quenched setup that removes most of the practical computational difficulties for QCD (and SU(3))
The lattice approach allows us to go beyond perturbative and diagrammatic arguments. For a given observable

1. Continuum extrapolation
   - Determine its value at fixed $a$ and $N$
   - Extrapolate to the continuum limit
   - Extrapolate to $N \to \infty$ using a power series in $1/N^2$

2. Fixed lattice spacing
   - Choose $a$ in such a way that its value in physical units is common to the various $N$
   - Determine the value of the observable for that $a$ at any $N$
   - Extrapolate to $N \to \infty$ using a power series in $1/N^2$

Study performed for various observables both at zero and finite temperature for $2 \leq N \leq 8$
Glueball masses at large $N$

Masses at $N = \infty$

$$0^{++} \quad \frac{m}{\sqrt{\sigma}} = 3.28(8) + \frac{2.1(1.1)}{N^2}$$

$$0^{++*} \quad \frac{m}{\sqrt{\sigma}} = 5.93(17) - \frac{2.7(2.0)}{N^2}$$

$$2^{++} \quad \frac{m}{\sqrt{\sigma}} = 4.78(14) + \frac{0.3(1.7)}{N^2}$$

Accurate $N = \infty$ value, normal $\mathcal{O}(1/N^2)$ correction
Glueball spectrum at $aT_c = 1/6$

[BL, Rago and Rinaldi, JHEP 1008 (2010) 119]
Meson spectrum (at fixed $a$)

[Bali et al., JHEP 06 (2013) 071]
Large $N$ vs. experiments

![Graph showing ground states and excited states compared to experimental data.](image)

[Ref: Bali et al., JHEP 06 (2013) 071]
Back to QCD

[Gregory et al., JHEP 1210 (2012) 170]
The spectrum for a QCD-like theory

- At high fermion masses the theory is nearly-quenched
- At low fermion masses the relevant degrees of freedoms are the pseudoscalar mesons
All spectral mass ratios depend very mildly on $m$ below the locking scale.
Locking near the Banks-Zaks point

If this scenario is valid beyond BZ, at all scales $\ll \Lambda$

\[
\begin{align*}
    m_V/m_{PS} &\simeq 1 + \epsilon \\
    m_{PS} &\gg \sigma^{1/2} \\
    m_G/\sigma^{1/2} &\simeq [m_G/\sigma^{1/2}]^{(YM)}
\end{align*}
\]
Spectrum in SU(2) $N_f = 2$ Adj

The pseudoscalar is always higher in mass than the $0^{++}$ glueball, as predicted by the locking scenario at high fermion mass.

These theories naturally provide a light scalar ⇒ is this a route to understanding the mechanism of electroweak symmetry breaking?
Lattice calculations are an (increasingly more) useful tool to understand the fate of glueballs in QCD

- Need to control better mixing with scattering and meson states

Valuable information can be provided by lattice calculations in the large $N$ limit

- Need to take the continuum limit

The dynamics of glueball is heavily influenced by the proximity to the conformal window

- New classes of theories need to be explored in order to see if strongly interacting BSM dynamics is a viable mechanism of electroweek symmetry breaking