GLUON PROPAGATOR WITH DYNAMICAL QUARKS

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Based on:

A. C. Aguilar, D. Binosi and J. P., Phys.Rev. D86, 014032 (2012); arXiv:1304.5936 [hep-ph].

QCD-TNT 3: From quarks and gluons to hadronic matter: A brigde too far ?

September 2-6, 2013 - Trento , Italy,

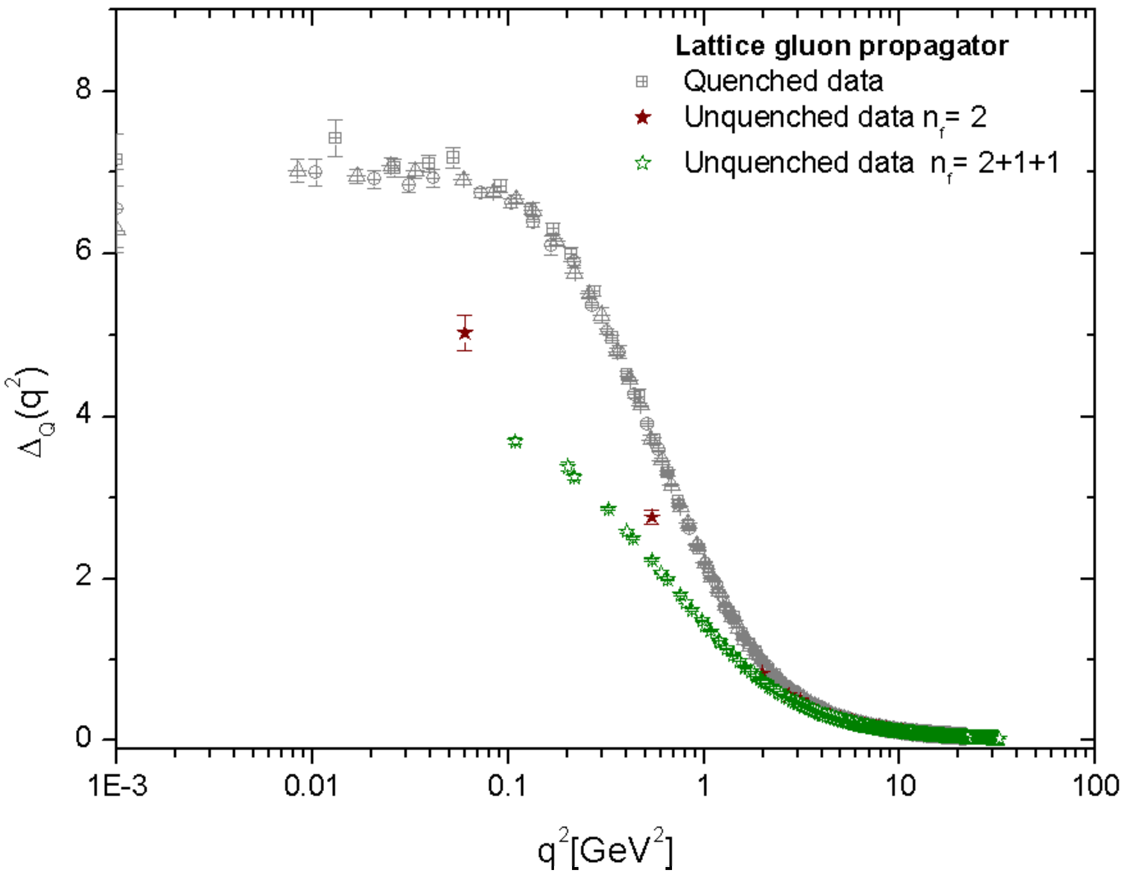
Outline of the talk

- Motivation
- Main dynamical features of gluon propagator
- Inclusion of quark loops
- The effect of unquenching
- Comparison with lattice

Motivation

- In recent years, fruitful **synergy** between **lattice and SDEs.**
- **Most SDE studies** focus on Green's functions of **pure Yang-Mills**
- Majority of **lattice simulations** works in the **quenched limit** (no dynamical quarks)
- Must make the **transition to real-world QCD**
- **New unquenched lattice data** for gluon and ghost propagators
- **New SDE-based algorithm** for **estimating the quark-loop effects** on the gluon propagator.

Lattice results



Quark	Current Mass
"up/down"	41.2 MeV
"strange"	95 MeV
"charm"	1.51 GeV

I.L.Bogolubsky, et al , PoS **LAT2007**, 290 (2007)
A. Ayala, et. al **Phys.Rev. D86 (2012) 074512**

- These lattice results suggest that in the presence of dynamical quarks:

1. Gluon propagator continues to saturate in the deep IR.

2. Overall suppression in the IR and intermediate regions.

3. Interpreting the saturation as a result of the gluon mass generation, i.e.

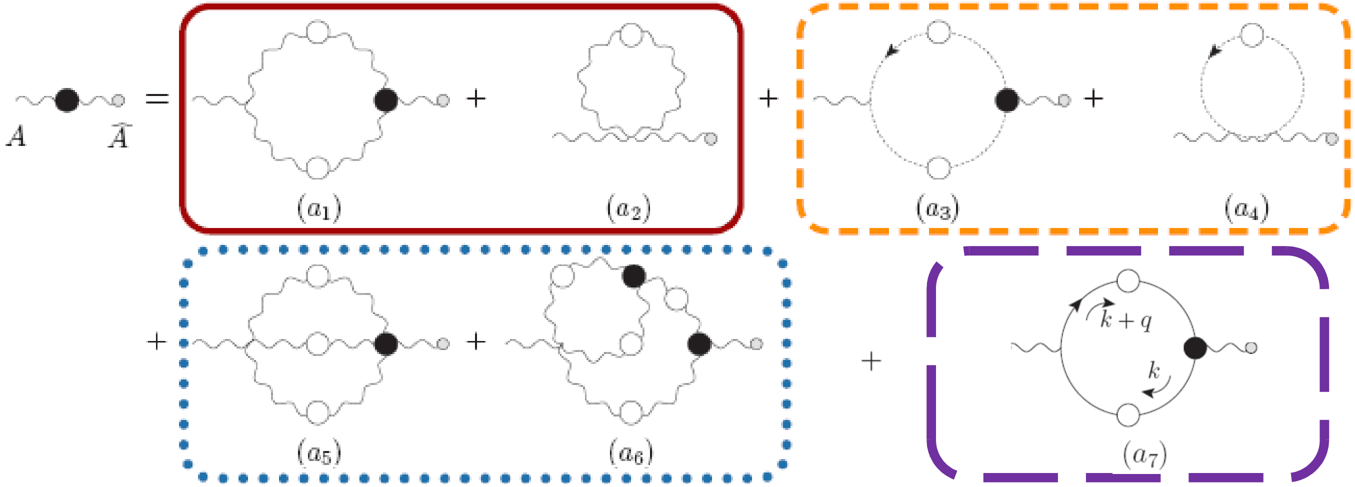
$$\Delta^{-1}(0) = m^2(0)$$

$\Downarrow$

Inclusion of quarks $\Rightarrow$ **Heavier gluon mass**

- We want to understand these features using the SDE

The gluon SDE (Landau gauge)



Quenched

$$\widetilde{\Delta}^{-1}(q^2) = q^2 + \sum_{i=1}^{6} a_i$$

Unquenched

$$\widetilde{\Delta}_{N_f}^{-1}(q^2) = \widetilde{\Delta}^{-1}(q^2) + a_7$$

$$\Delta(q^2) = [1 + G(q^2)]\widetilde{\Delta}(q^2);$$

Infrared finiteness ⟺ Gluon mass generation

- IR finiteness means:

$$\Delta^{-1}(q^2) = q^2 J(q^2) \quad \longrightarrow \quad \Delta_m^{-1}(q^2) = q^2 J_m(q^2) - m^2(q^2),$$

- **Coupled system of integral equations**

$$J_m(q^2) = 1 + \int_k \mathcal{K}_1(k, q, m^2, \Delta),$$
$$m^2(q^2) = \int_k \mathcal{K}_2(k, q, m^2, \Delta),$$

- In the limit $q^2 \rightarrow 0$

$$\mathcal{K}_2(q^2, m^2, \Delta_m) \neq 0$$

because of the inclusion of the massless poles.

The complete gluon mass equation

D. Binosi, D. Ibanez and J. P, **Phys. Rev. D86, 085033 (2012)**

$$m^2(q^2) = -g^2 C_A D(q^2) \int_k m^2(k^2) \Delta_\rho^\mu(k) \Delta^{\nu\rho}(k+q) \mathcal{K}_{\mu\nu}(k,q).$$

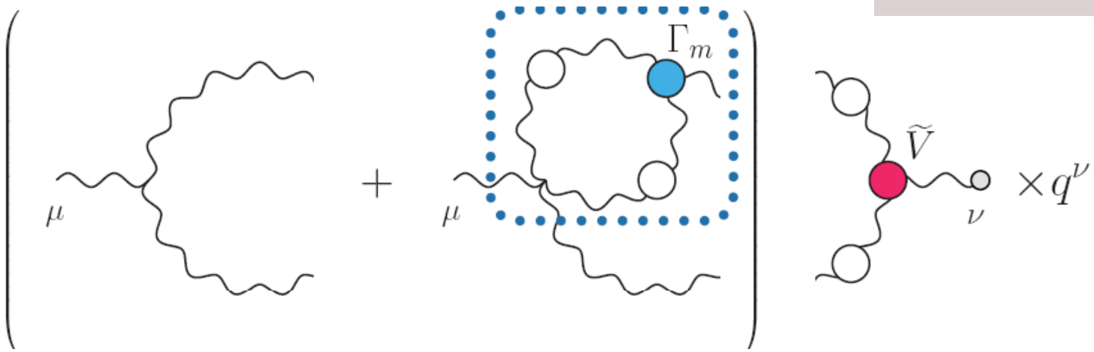
where $\mathcal{K}_{\mu\nu}(k,q) \; = \; [(k+q)^2 - k^2]\{1 - [Y(k+q) + Y(k)]\} g_{\mu\nu} + [Y(k+q) - Y(k)](q^2 g_{\mu\nu} - 2q_\mu q_\nu).$

$1 + G(q^2) \approx F^{-1}(q^2)$

$D(q^2) = \dfrac{F(q^2)}{q^2}$

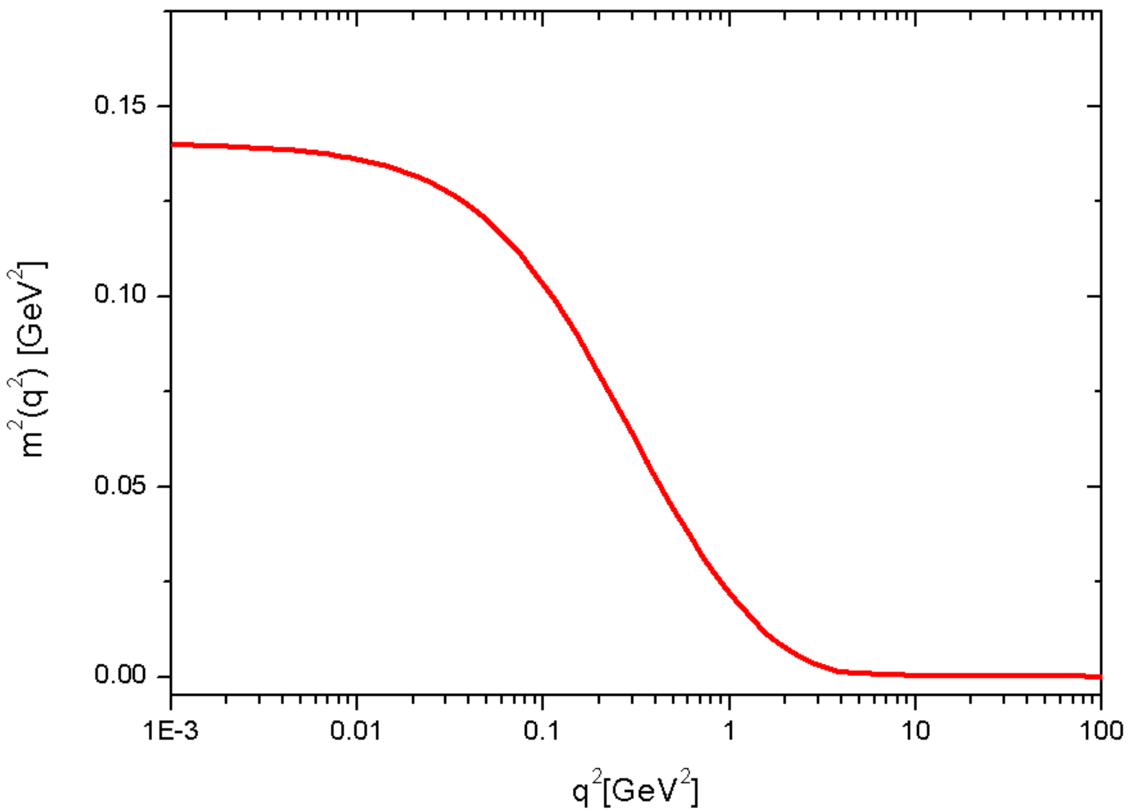
- Diagramatically

$m^2(q^2) = D(q^2) \;\; q^\mu \;\; \times$



- The solution depends on a subtle interplay between the shape of the full $\Delta(q^2)$ and the kernel $\mathcal{K}_{\mu\nu}$ **(k,q)**.

Solution of the mass equation



- Positive definite and monotonically decreasing gluon mass

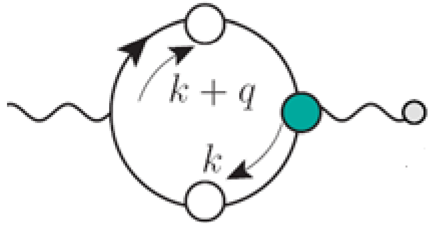
- Solution normalized to coincide with lattice value $\Delta^{-1}(0) \approx 0.14\ \mathrm{GeV}^{-2}$, namely $m = 375$ MeV.

Unquenching the gluon propagator

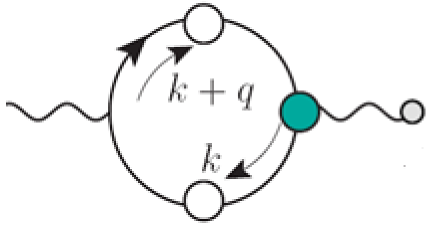
- We will assume that the main bulk of the quark contribution comes from the diagram a_7 (fully dressed quark loop), i.e.

$$\widetilde{\Delta}_{N_f}^{-1}(q^2) = \widetilde{\Delta}^{-1}(q^2) + a_7 + \text{"subleading corrections"}$$

$$a_7 = X(q^2) =$$



Leading effects: the quarks loop

$X(q^2) =$ 

$$X(q^2) = -\frac{g^2}{6}\int_k \mathrm{Tr}\left[\gamma^{\mu} S(k)\widehat{\Gamma}_{\mu}(k,-k-q,q)S(k+q)\right]$$

- The PT-BFM quark-gluon vertex satisfies the Ward identity

$$p_3^{\mu}\widehat{\Gamma}_{\mu}(p_1,p_2,p_3) = S^{-1}(-p_1) - S^{-1}(p_2)$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - q_{\mu}q_{\nu}/q^2,$$

and the quark loop is transverse, i.e. $X_{\mu\nu}(q) = X(q^2)P_{\mu\nu}(q)$

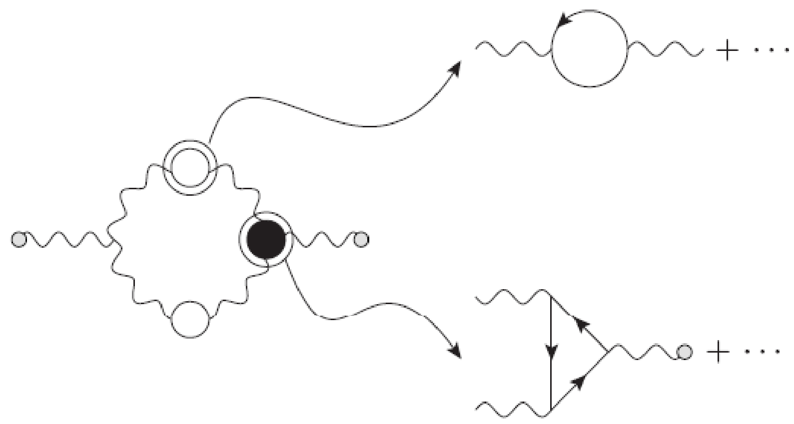
- Moreover, we have that

$$X(0) = 0$$

- **No direct** influence on the value of $\Delta(0)$;
- However modifies it **indirectly,** due to the **change in the overall shape of $\Delta(q^2)$** throughout the entire range of momenta.

A.C. Aguilar and J.P., Phys. Rev. D 81, 034003 (2010).

Subleading contributions



• There will be a nonlinear propagation of the changes induced due to X(q), which will also affect the original subset of purely Yang-Mills graphs (a_1 –a_6)
→ **Internal gluon propagator and the three-gluon vertex gets modified.**

• We assume that the inclusion of two light quark flavors (m = 300 MeV) may be considered as a **"perturbation"** to the quenched case.

• Our **operating assumption** is that these effects may be **relatively small** compared to those originating from graph a_7 (quark loop)

The unquenching formula

Δ_{Nf} (q^2) may be expressed as a **deviation** from $\Delta(q^2)$:

$$\Delta_{N_f}(q^2) = \frac{\Delta(q^2)}{1 + X(q^2)F^2(q^2) + \lambda^2(q^2)}$$

where

$$\lambda^2(q^2) = m^2_{N_f}(q^2) - m^2(q^2),$$

measures the **difference induced** to the gluon mass **due to the inclusion of quarks.**

A.C.Aguilar, D.Binosi and J.P., **Phys.Rev. D86, 014032 (2012)**

System of Equations

- Now, since the dynamical equation for $\lambda^2(q^2)$ is known, we will solve the system

A. C. Aguilar, D. Binosi and J. P., arXiv:1304.5936 [hep-ph].

$$\begin{cases} \Delta_{N_f}(q^2) = \dfrac{\Delta(q^2)}{1 + X(q^2)F^2(q^2) + \lambda^2(q^2)} \\ m^2_{N_f}(q^2) = \displaystyle\int_k \mathcal{K}_2(k, q, \Delta_{N_f}), \end{cases}$$

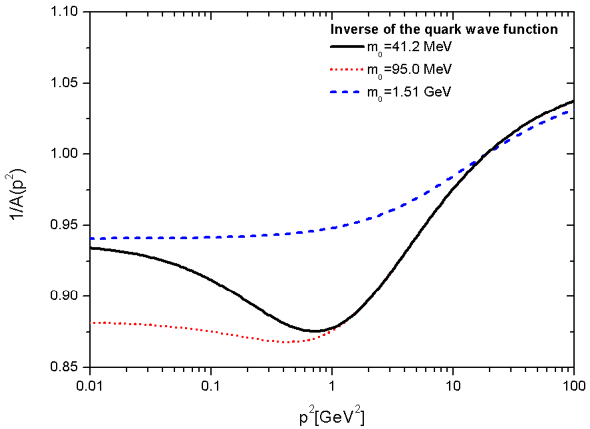
$$\lambda^2(q^2) = m^2_{N_f}(q^2) - m^2(q^2),$$

- Mass equation is **not linear**, and has a **unique solution**.

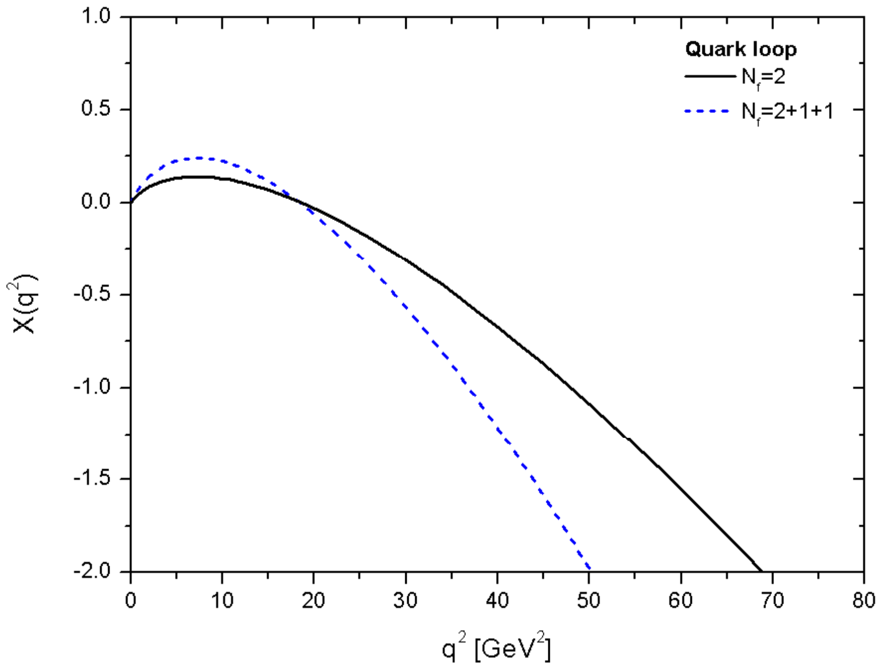
Calculating the quark loop - $X(q^2)$

$$S^{-1}(k) = -i\,[A(k)\not{k} - B(k)] = -iA(k)\,[\not{k} - \mathcal{M}(k)]$$

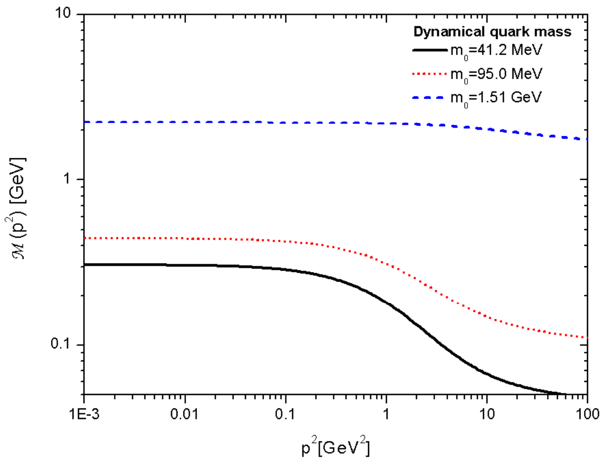
Quark wave function



Quark loop



Quark Masses

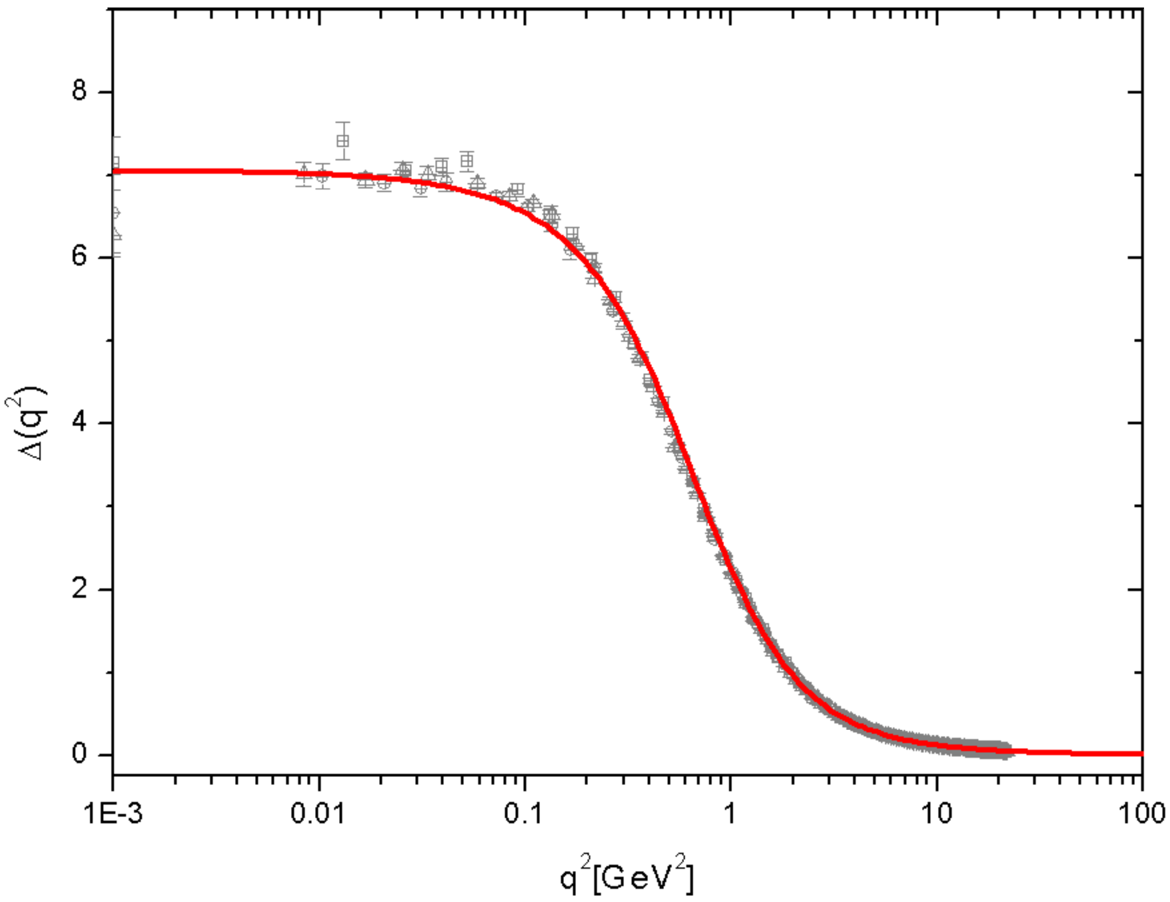


Quark	Current Mass	*Dynamical mass*
"up/down"	41.2 MeV	307 MeV
"strange"	95 MeV	445 MeV
"charm"	1.51 GeV	2.25 GeV

N_f	α_s (4.3 GeV)
0	0.222
2	0.285
2+1+1	0.331

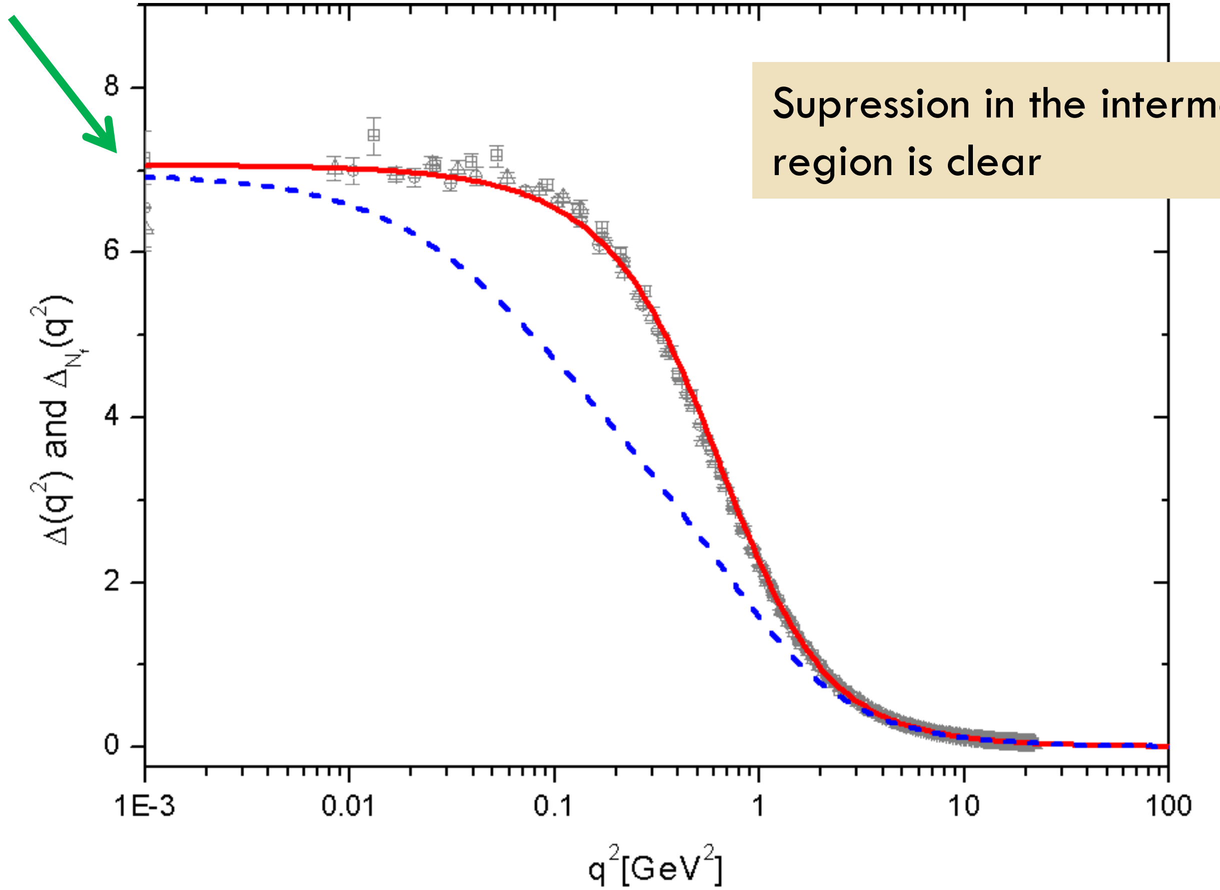
- The case where $\lambda(q^2) = 0$ → **gluon mass equation turned off**

A. C. Aguilar, D. Binosi and J. P., Phys. Rev. D86, 014032 (2012)



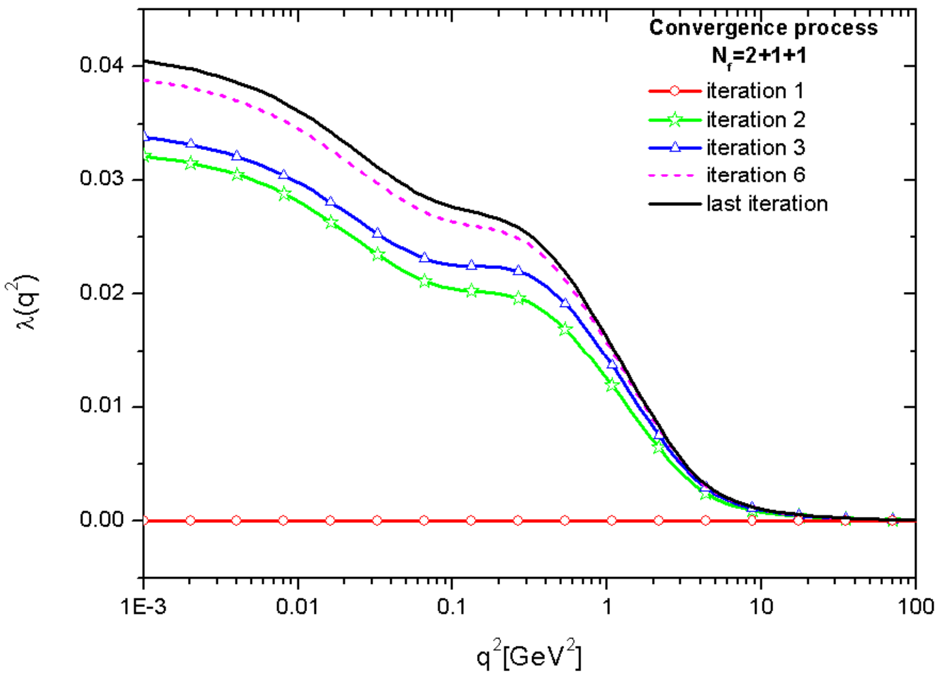
Since X(0)=0 → saturation point remains intact

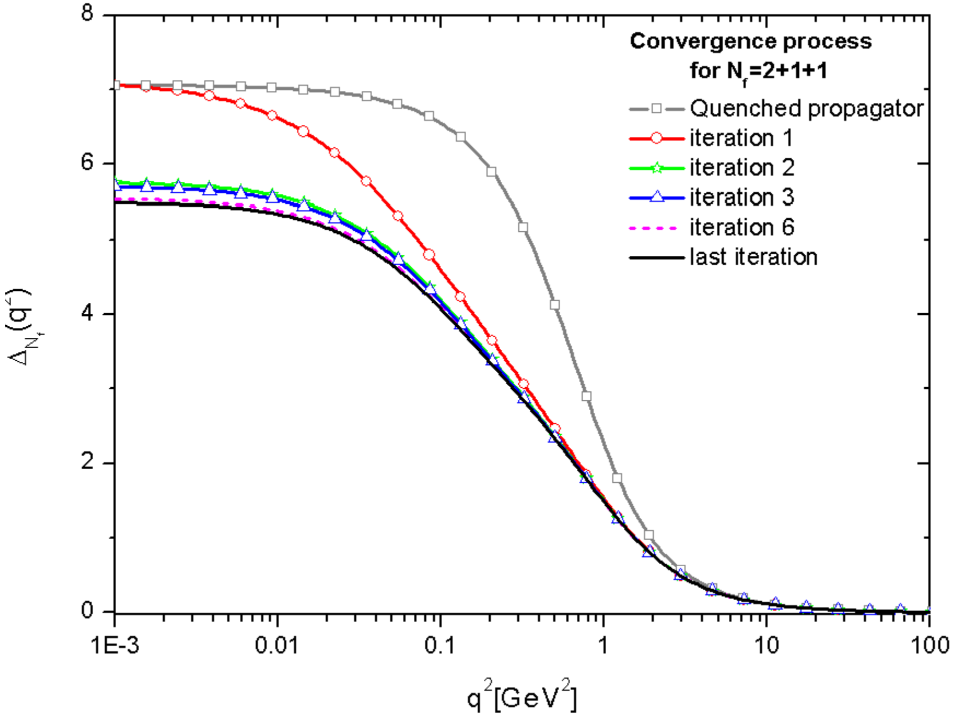
Supression in the intermediate region is clear



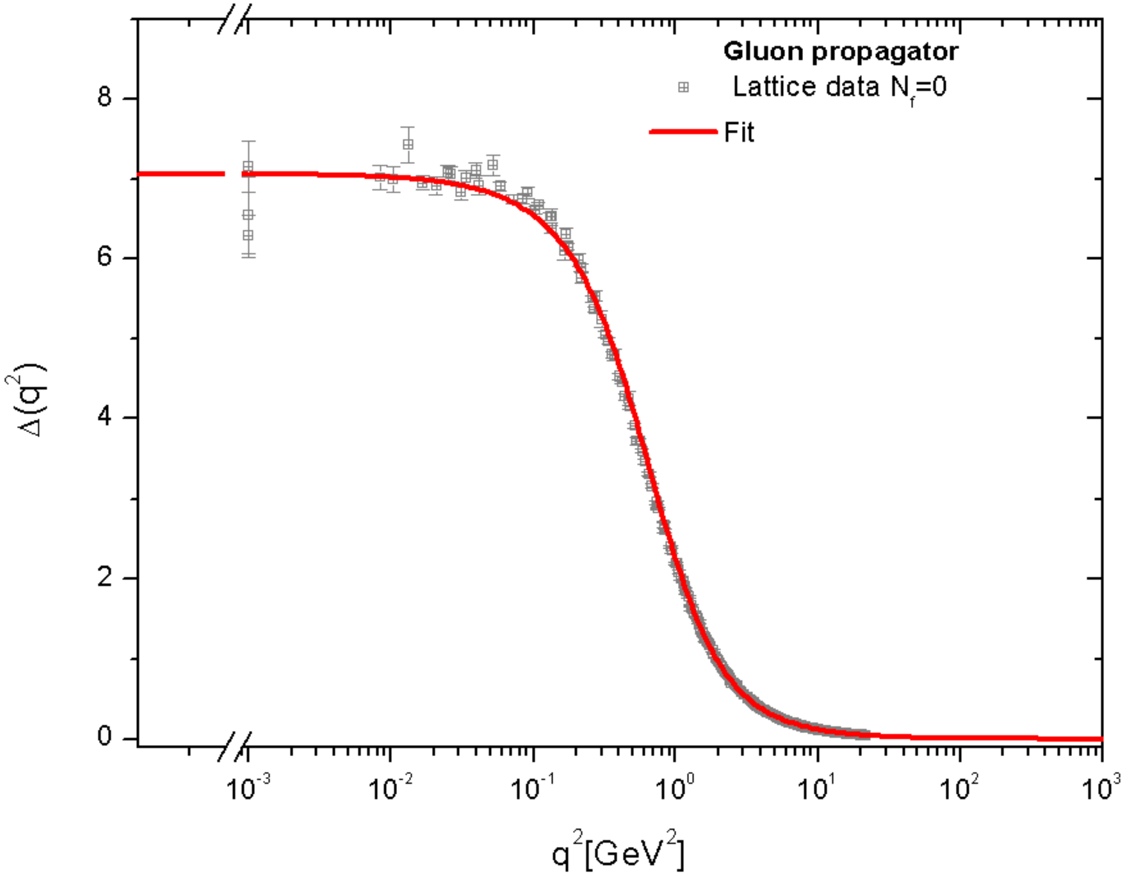
Full treatment: gluon mass equation turned on

- Convergence process for $\lambda(q^2) \neq 0$



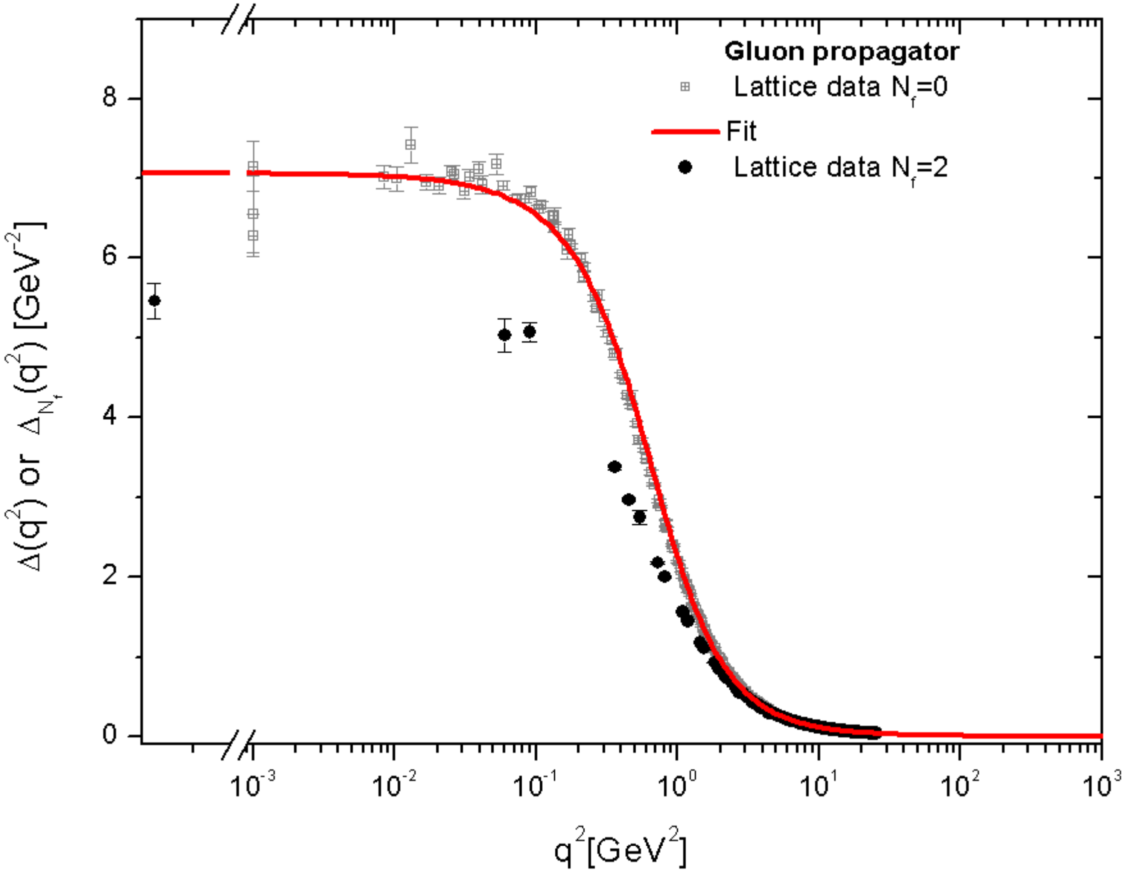


Comparison with the lattice



Quenched lattice propagator

The effect of two quarks

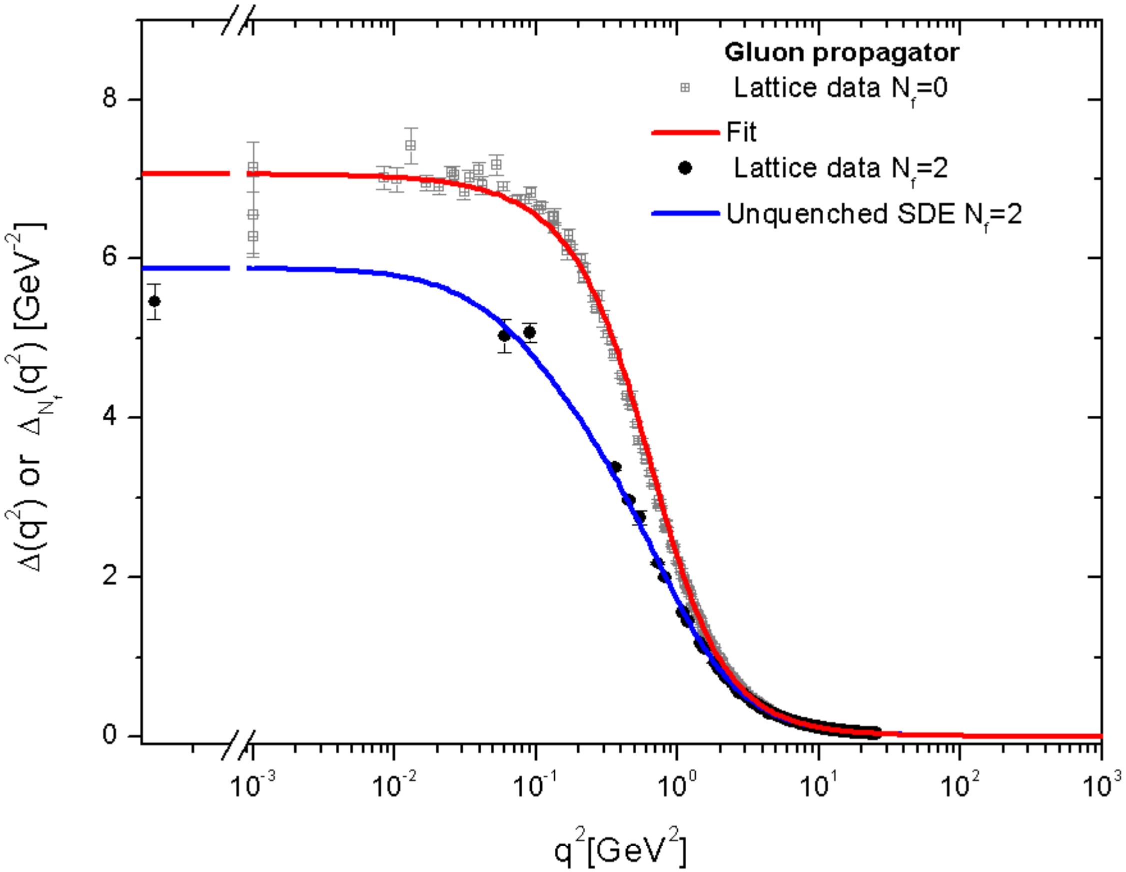


Unquenched lattice propagator ●

n_f = 2

Quark	Mass
up/down	41.2 MeV

The effect of two quarks

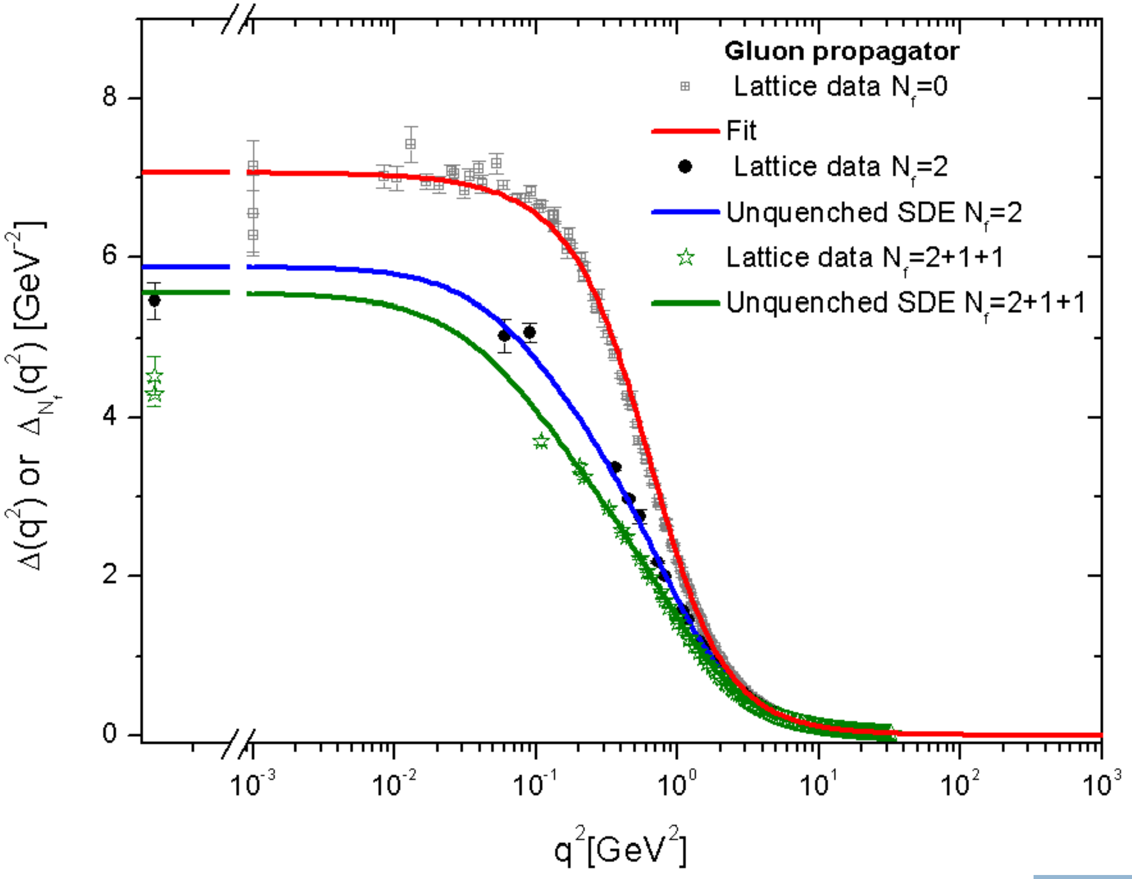


Unquenched SDE result

n_f = 2

Quark	Mass
up/down	41.2 MeV

The effect of adding heavier quarks

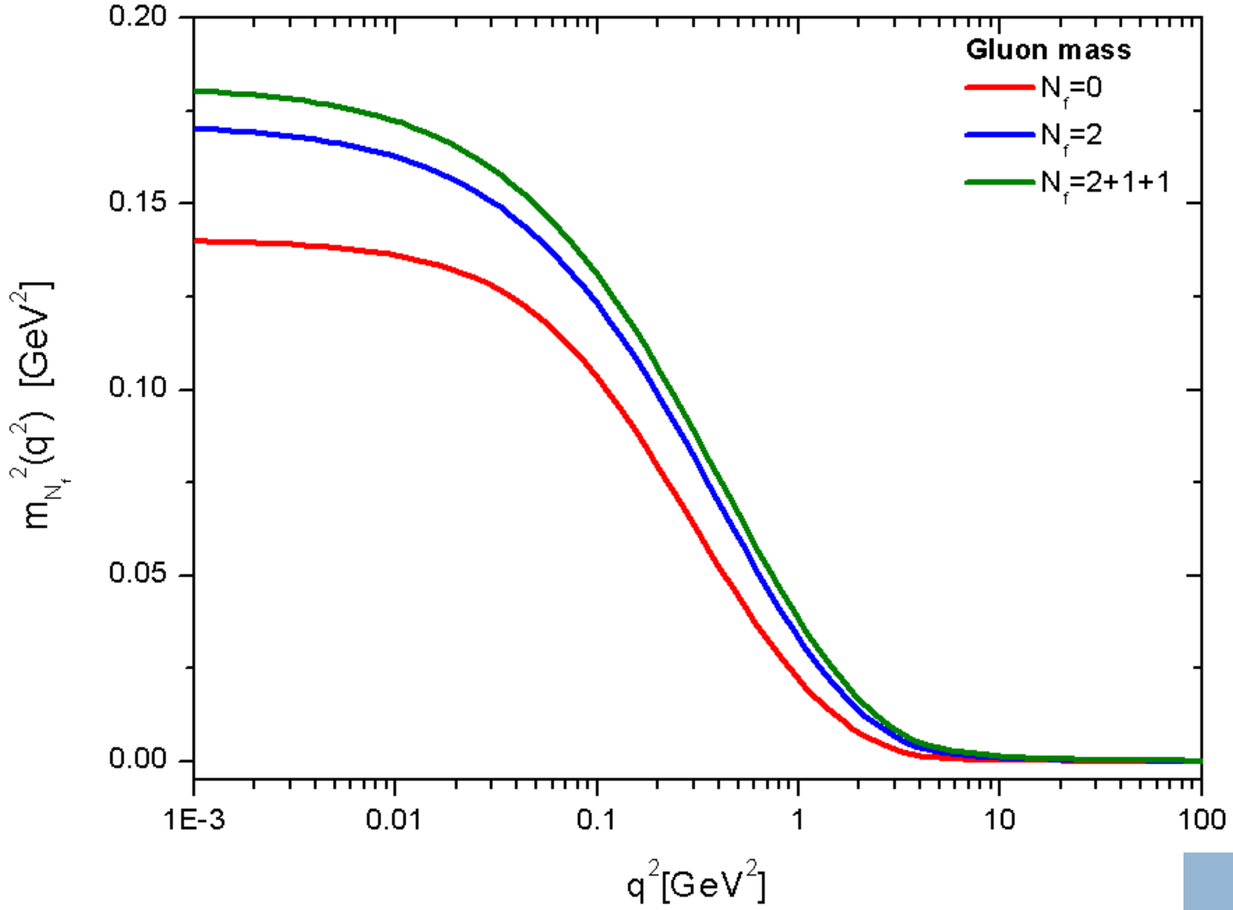


n_f = 2+1+1

Unquenched SDE result

Quark	Mass
up/down	41.2 MeV
strange	95 MeV
charm	1.51 GeV

The dynamical gluon mass



N_f	m(0)
0	375 MeV
2	413 MeV
2+1+1	425 MeV

A. C. Aguilar, D. Binosi and J. P. arXiv:1304.5936 [hep-ph].

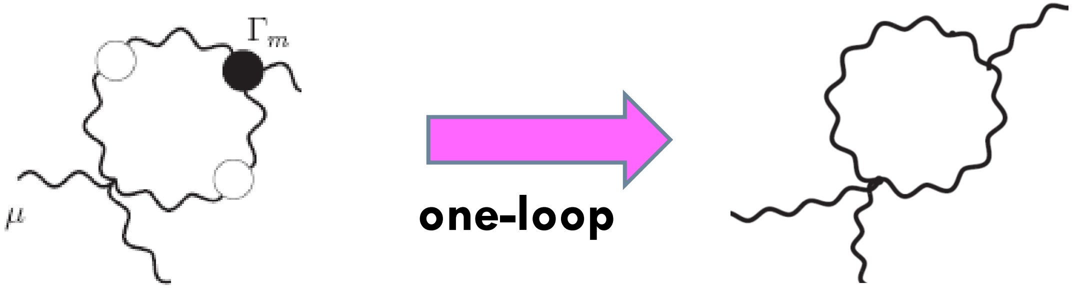
Conclusions

- **New SDE-based** method for **estimating quark effects**
- **Unquenched** gluon propagator computed as a **deviation from the quenched one.**
- **Quark loops suppress** intermediate **and infrared region.**
- **Gluon mass increases.**
- Good **agreement with lattice.**
- **Apply this method to TC models**, or QCD-like theories with fermions in higher representation.

Additional Slides

Simplifying the kernel

- **Perturbative one-loop expression for Y(k²)**



$$Y_{\mathrm{R}}(k^2) = -\frac{1}{(4\pi)^2}\frac{5}{4}\log\frac{k^2}{\bar{\mu}^2}.$$

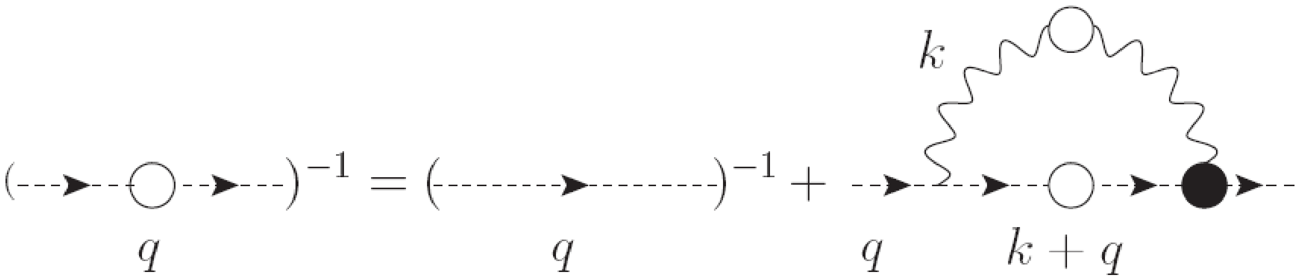
- **Rescaling**

$$Y_R(k^2) \to C\,Y_R(k^2)$$

- **C arbitrary parameter, models additional corrections.**

Unquenched effects on the ghost propagators

- The ghost SDE is given by

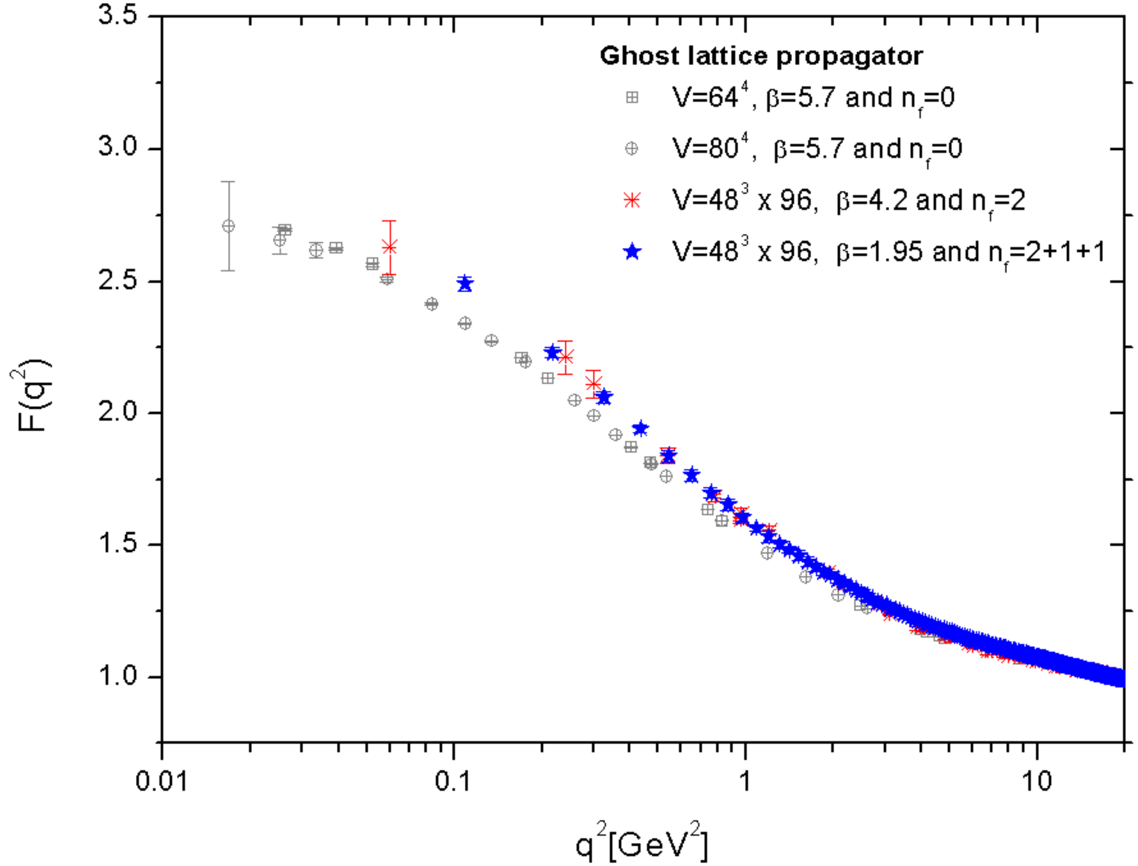


increases

$$iD^{-1}(q^2) = q^2 + ig^2 C_{\text{A}} \int_k \Gamma^{\mu} \Delta_{\mu\nu}(k) \mathbf{\Gamma}^{\nu}(k,q) D(q+k).$$

Suppresses

N_f	α_s (4.3 GeV)
0	0.222
2	0.285
2+1+1	0.331

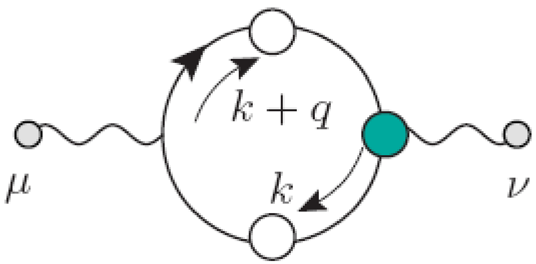

Ghost lattice propagator
V=64^4, β=5.7 and n_f=0
V=80^4, β=5.7 and n_f=0
V=48^3 x 96, β=4.2 and n_f=2
V=48^3 x 96, β=1.95 and n_f=2+1+1
F(q^2)
q^2[GeV^2]
3.5
3.0
2.5
2.0
1.5
1.0
0.01
0.1
1
10

Including the quarks

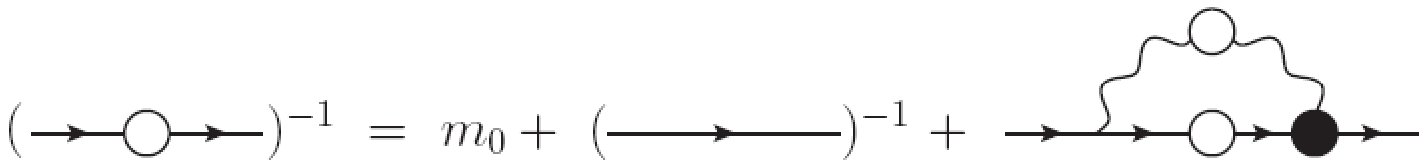
$$\Delta^{-1}_{N_f}(q^2) \;=\; [1+G(q^2)]^{-2}\widehat{X}(q^2) \;+\; \overbrace{m^2_{N_f}(q^2) - m^2(q^2)}^{\lambda(q^2)} \;+\; \underbrace{\Delta^{-1}(q^2)}_{\text{lattice}}$$

(a)

$\widehat{X}(q^2)P_{\mu\nu}(q) \;=$



(b)



(c)

$$1 + G(q^2) \approx \underbrace{F^{-1}(q^2)}_{\text{lattice}}$$

(d)

Contribution at zero momentum

- In the limit $q^2 \rightarrow 0$

$$\widehat{X}(0) = -\frac{2g^2}{d-1}\int_k \frac{1}{A^2(k^2-\mathcal{M}^2)^2}\left\{A\left[(2-d)k^2+d\mathcal{M}^2\right]+2A'k^2(k^2+\mathcal{M}^2)-4k^2B'\mathcal{M}\right\}.$$

by virtue of the identity

$$\int_k k^2 f'(k) + \frac{d}{2}\int_k f(k) = 0,$$

A. C. Aguilar and J.P., Phys. Rev. D 81, 034003 (2010).

setting

$$f(k) = [A(k)(k^2 - \mathcal{M}^2(k))]^{-1}$$

$$\widehat{X}(0) = 0$$

⇓

- No **direct** influence on the value of $\Delta(0)$;
- However modifies it **indirectly,** due to the **change** in **the overall shape of $\Delta(q^2)$** throughout the entire range of momenta.