# NONABELIAN GAUGE FIELDS BEYOND PERTURBATION THEORY.

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Progress in physics was usually related to the introduction of new symmetries.

Recent examples are given by gauge theories. QED may be formulated in the Coulomb gauge, however much more transparent formulation is presented by the quantization in a manifestly covariant gauge. Yang-Mills theory became really popular only after its formulation in the Lorentz covariant terms and explicit proof of its renormalizability. The gauge invariance of the Higgs model allows to give a manifestly renormalizable theory describing a massive gauge theory.

Transform the fields

$$\varphi = \frac{\partial^n \varphi'}{\partial t^n} + f(\frac{\partial^{n-1} \varphi'}{\partial t^{n-1}}, \dots \frac{\partial \varphi'}{\partial t}) = \tilde{f}(\varphi') \tag{1}$$

The spectrum is changed. What about the unitarity?

The new Lagrangian is invariant with respect to the supertransformations

$$\delta\varphi'_a = c_a\varepsilon$$
$$\delta c_a = 0; \quad \delta\bar{c}_a = \frac{\delta L}{\delta\varphi_a}(\varphi')\varepsilon \tag{2}$$

On mass shell these transformations are nilpotent and generate a conserved charge $Q$. In this case there exists an invariant subspace of states annihilated by $Q$, which has a semidefinite norm. (A.A.S.,1991). For asymptotic space this condition reduces to

$$Q_0|\phi>_{as} = 0 \tag{3}$$

The scattering matrix is unitary in the subspace which contains only excitations of the original theory. However the theories described by the $L$ and the $\tilde{L}$ are different, and only expectation values of the gauge invariant operators coincide.

Using this method one can construct a renormalizable formulation of nonabelian gauge theories free of the Gribov ambiguity.

In fact it is not necessary to introduce higher derivatives. Necessary ingredients are new ghost excitations, and new symmetry of the Lagrangian.

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To deal with gauge theory one has to impose the gauge condition,selecting a unique representative in a gauge equialent class. Differential gauge conditions: $L(A_\mu, \varphi) = 0 \rightarrow$ Gribov ambiguity.

Algebraic gauge conditions: $\tilde{L}(A_\mu, \varphi) = 0 \rightarrow$ absence of the manifest Lorentz invariance and other problems.

Coulomb gauge

$$
\begin{aligned}
\partial_i A_i &= 0 \\
A_i' &= (A^{\Omega})_i \\
\triangle \alpha^a + ig\varepsilon^{abc}\partial_i(A_i^b \alpha^c) &= 0 \qquad (4)
\end{aligned}
$$

This equation has nontrivial solutions rapidly decreasing at spatial infinity$\rightarrow$Gribov ambiguity.

In perturbation theory the only solution is $\alpha = 0$.

A remedy: 1. Use of this phenomenon to try to explain confinement e.t.c. (Series of works by D.Zwanziger and others.)

2.To avoid the Gribov problem by using new (equivalent) formulation of the Yang-Mills theory using more ghost fields.

Let us consider the classical ($SU(2)$)Lagrangian

$$L = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - m^{-2} (D^2 \tilde{\phi})^* (D^2 \tilde{\phi}) + (D_\mu e)^* (D_\mu b) + (D_\mu b)^* (D_\mu e)$$
$$+ \alpha^2 (D_\mu \tilde{\phi})^* (D_\mu \tilde{\phi}) - \alpha^2 m^2 (b^* e + e^* b) \quad (5)$$

where $\phi$ is a two component complex doublet, and

$$\tilde{\phi} = \phi - \widehat{\mu}; \quad \widehat{\mu} = (0, \mu\sqrt{2} g^{-1}) \quad (6)$$

$\mu$ is an arbitrary constant. $D_\mu$ denotes the usual covariant derivative. To save the place we consider here the group $SU(2)$.

In the proper parametrization the Lagrangian (5) is invariant with respect to "shifted"gauge transformations

$$
\begin{aligned}
A^a_\mu &\to A^a_\mu + \partial_\mu \eta^a - g\epsilon^{abc} A^b_\mu \eta^c \\
\phi^a &\to \phi^a + \frac{g^2}{4\mu} \phi^a \phi^b \eta^b + \mu \eta^a
\end{aligned}
\tag{7}
$$

The field $\phi^a$ is shifted by an arbitrary function, therefore one can put $\phi^a = 0$. This gauge is algebraic, but Lorentz invariant. It may be used beyond perturbation theory as well.

This Lagrangian is also invariant with respect to the supersymmetry transformations

$$\begin{aligned} \phi &\rightarrow \phi - b\epsilon \\ e &\rightarrow e - \frac{D^2(\phi - \widehat{\mu})}{m^2}\epsilon \\ b &\rightarrow b \end{aligned} \tag{8}$$

where $\epsilon$ is a constant Hermitean anticommuting parameter. This symmetry plays a crucial role in the proof of decoupling of unphysical excitations. It holds for any $\alpha$, but for $\alpha = 0$ these transformations are also nilpotent.

In the case under consideration the nilpotency of the asymptotic charge requires $\alpha = 0$, and the massive theory with $\alpha \neq 0$ is gauge invariant but not unitary. It may seem strange as usually the gauge invariance is a sufficient condition of unitarity, because one can pass freely from a renormalizable gauge to the unitary one, where the spectrum includes only physical excitations. In the present case there is no "unitary"gauge. Even in the gauge $\phi^a = 0$, there are unphysical excitations.

The shift of the variables $\phi$ produces the term

$$\alpha^2(D_\mu\widehat{\mu})^*(D_\mu\widehat{\mu}) = \frac{\alpha^2\mu^2}{2}A_\mu^2 \qquad (9)$$

which gives a mass to the vector field. The term

$$m^{-2}(D^2\widehat{\mu})^*(D^2\widehat{\mu}) = \frac{\mu^2}{2m^2}[(\partial_\mu A_\mu)^2 + \frac{g^2}{2}(A^2)^2] \qquad (10)$$

makes the theory renormalizable for any $\alpha$. To avoid complications due to the presence of the Yang-Mills dipole ghosts at $\alpha = 0$ we put $\mu^2 = m^2$.

Invariance of the Lagrangian (9) with respect to the gauge transformation (7) and the supersymmetry transformations (8) makes the effective Lagrangian invariant with respect to the simultaneous BRST transformations corresponding to (7) and the supersymmetry transformations (8). The effective Lagrangian may be written in the form

$$L_{ef} = L + s_1 \overline{c}^a \phi^a = L(x) + \lambda^a \phi^a - \overline{c}^a (\mu c^a - b^a) \tag{11}$$

One can integrate over $\bar{c}, c$ in the path integral determining expectation value of any operator corresponding to observable. It leads to the change $c^a = b^a\mu^{-1}$. After such integration the effective Lagrangian becomes invariant with respect to the transformations which are the sum of the BRST transformations and the supersymmetry transformations (8) with $c^a = b^a\mu^{-1}$

According to the Neuther theorem the invariance with respect to the supertransformations mentioned above generates a conserved charge $Q$, and the physical asymptotic states may be chosen to satisfy the equation

$$\hat{Q}_0|\psi>_{as}= 0 \tag{12}$$

Due to the conservation of the Neuther charge this condition is invariant with respect to dynamics.

This symmetry guarantees the decoupling of all unphysical excitations at $\alpha = 0$ and the transitions between the states, annihilated by the charge $Q$ include only three dimensionally transversal components of the Yang-Mills field. Therefore we succeeded to formulate the Yang-Mills theory in such a way that in a topologically trivial sector Gribov ambiguity is absent and the infrared regularization valid beyond perturbation theory is easily constructed. This approach opens also interesting possibilities to consider topologically nontrivial sectors and study the confinement problem.

# Conclusion.

A renormalizable manifestly Lorentz invariant formulation of the non-Abelian gauge theories which allows a canonical quantization without Gribov ambiguity (including Higgs model is possible.)

In perturbation theory the scattering matrix and the gauge invariant correlators coincide with the standard ones.

On the basis of this approach infrared regularization of Yang-Mills theory beyond perturbation theory is constructed (Slavnov, 2013).

In this approach soliton exitations in Yang-Mills theory seem to be possible. The problem of color confinement by means of topologically nontrivial solitons is under consideration.