Dipole response of medium-mass nuclei in the Self-Consistent Green’s Function approach (and effective charges)

Prospects on the microscopic description of odd mass nuclei and other multi-quasiparticle excitations with beyond-mean-field and related methods

ECT* (25-29 September 2017)

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(University of Surrey)
Outline

• Extension of the Algebraic Diagrammatic Construction (ADC) method with three-nucleon interactions

• Dipole Response Function and Polarisability in Oxygen and Calcium isotopes

• Effective charges in Oxygen and Nickel isotopes from realistic nuclear interactions
• Extension of the Algebraic Diagrammatic Construction (ADC) method with three-nucleon interactions

• Dipole Response Function and Polarisability in Oxygen and Calcium isotopes

• Effective charges in Oxygen and Nickel isotopes from realistic nuclear interactions
Motivations: Role of 3N forces in nuclear phenomena
Impact of 3N forces in No-Core Shell Model calculations

Excitation spectra of $^{10}$B in NCSM


- Chiral 2N force alone predicts incorrect spin of $^{10}$B ground-state
- Crucial effect of 3NF in reproducing correct order of the states
  (Ground state and two lowest $1^+$ states)
Impact of 3N forces in SCGF calculations

**Binding energies of Nitrogen and Fluorine isotopes**


![Graph showing binding energies of nitrogen and fluorine isotopes](image)

Energy per nucleon in symmetric nuclear matter


![Graph showing energy per nucleon](image)

Microscopic mechanism behind neutron drip lines explained by repulsive effect of 3N forces

- Increased spin-orbit splitting between $d_{3/2}^+$ and $d_{5/2}^+$ quasiparticles orbitals
- 3N force reproduce saturation
Method: Self-consistent Green’s function formalism
Green’s functions for nuclear physics


Microscopic nuclear Hamiltonian

\[ \hat{H} = \sum_{\alpha} \varepsilon^0_\alpha a^\dagger_\alpha a_\alpha - \sum_{\alpha\beta} U_{\alpha\beta} a^\dagger_\alpha a_\beta + \frac{1}{4} \sum_{\alpha\gamma\beta\delta} V_{\alpha\gamma,\beta\delta} a^\dagger_\alpha a^\dagger_\gamma a_\delta a_\beta + \frac{1}{36} \sum_{\alpha\gamma\epsilon\beta\delta\eta} W_{\alpha\gamma\epsilon,\beta\delta\eta} a^\dagger_\alpha a^\dagger_\gamma a^\dagger_\epsilon a_\eta a_\delta a_\beta \]

Green’s function (Lehmann representation)

\[ g_{\alpha\beta}(\omega) = \sum_n \frac{\langle \Psi^+_0 | a_\alpha | \Psi^{A+1}_n \rangle \langle \Psi^{A+1}_n | a_\beta^\dagger | \Psi^+_0 \rangle}{\omega - \varepsilon^+_n + i\eta} + \sum_k \frac{\langle \Psi^+_0 | a_\beta^\dagger | \Psi^{A-1}_k \rangle \langle \Psi^{A-1}_k | a_\alpha | \Psi^+_0 \rangle}{\omega - \varepsilon^-_k - i\eta} \]

Dyson equation

\[ G_{\alpha\beta}(\omega) = G^{(0)}_{\alpha\beta}(\omega) + \sum_{\gamma\delta} G^{(0)}_{\alpha\gamma}(\omega) \Sigma^{*}_{\gamma\delta}(\omega) G_{\delta\beta}(\omega) \]
Green’s functions for nuclear physics


Microscopic nuclear Hamiltonian

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\]

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g_{\alpha\beta}(\omega) = \sum_n \frac{\langle \Psi^A_0 | a_\alpha | \Psi^{A+1}_n \rangle \langle \Psi^{A+1}_n | a_\beta^+ | \Psi^A_0 \rangle}{\omega - \varepsilon^+_n + i\eta} + \sum_k \frac{\langle \Psi^A_0 | a_\beta^+ | \Psi^{A-1}_k \rangle \langle \Psi^{A-1}_k | a_\alpha | \Psi^A_0 \rangle}{\omega - \varepsilon^-_k - i\eta}
\]

Dyson equation

\[
G_{\alpha\beta}(\omega) = G_{\alpha\beta}^{(0)}(\omega) + \sum_{\gamma\delta} G_{\alpha\gamma}^{(0)}(\omega) \Sigma_{\gamma\delta}^*(\omega) G_{\delta\beta}(\omega)
\]

Gorkov formalism: C. Barbieri, T. Duguet, V. Somà
Green’s functions for nuclear physics

“What a nucleon does in the nucleus”
(i.e. in a strongly interacting many-fermions systems)?

Spectroscopic information ($^{56}$Ni)

Binding energies and driplines (O)

- Ground state properties
- Spectroscopic informations
- One- (two-, ...) body operators matrix elements
Green’s functions for nuclear physics

“What a nucleon does in the nucleus” (i.e. in a strongly interacting many-fermions systems)?

Spectroscopic information ($^{56}$Ni)

Bubble (?) nucleus ($^{34}$Si)

Correct reduction of splitting $E_{1/2} - E_{3/2}$ from $^{37}$S to $^{35}$Si

Such a sudden reduction of 50% is unique

Any correlation with the bubble?!


T. Duguet at al, PRC 95, 034319 (2017)
Green’s functions for nuclear physics

Microscopic nuclear Hamiltonian

\[
\hat{H} = \sum_{\alpha} \varepsilon_\alpha^0 a_\alpha^\dagger a_\alpha - \sum_{\alpha \beta} U_{\alpha \beta} a_\alpha^\dagger a_\beta + \frac{1}{4} \sum_{\alpha \gamma \beta \delta} V_{\alpha \gamma, \beta \delta} a_\alpha^\dagger a_\gamma^\dagger a_\delta a_\beta + \frac{1}{36} \sum_{\alpha \gamma \epsilon, \beta \delta \eta} W_{\alpha \gamma \epsilon, \beta \delta \eta} a_\alpha^\dagger a_\gamma^\dagger a_\epsilon a_\delta a_\beta
\]

Green’s function (Lehmann representation)

\[
g_{\alpha \beta}(\omega) = \sum_n \left\langle \Psi_0^A | a_\alpha | \Psi_n^{A+1} \right\rangle \left\langle \Psi_n^{A+1} | a_\beta^\dagger | \Psi_0^A \right\rangle \frac{1}{\omega - \varepsilon_n^+ + i\eta} + \sum_k \left\langle \Psi_0^A | a_\beta^\dagger | \Psi_k^{A-1} \right\rangle \left\langle \Psi_k^{A-1} | a_\alpha | \Psi_0^A \right\rangle \frac{1}{\omega - \varepsilon_k^- - i\eta}
\]

Dyson equation

\[
G_{\alpha \beta}(\omega) = G_{\alpha \beta}^{(0)}(\omega) + \sum_{\gamma \delta} G_{\alpha \gamma}^{(0)}(\omega) \Sigma_{\gamma \delta}^*(\omega) G_{\delta \beta}(\omega)
\]

Self-energy: effective potential affecting the s.p. propagation in the nuclear medium
Inclusion of 3N forces via effective interactions

With 3N forces, # of self-energy diagrams is too cumbersome…

**BASIC IDEA:**
Effective interaction concept generalises the HF approximation of the two-body forces to the N-body forces and with respect to the correlated propagator.

Example of two-body effective interaction

\[
\tilde{V} = \frac{1}{4} \sum_{\alpha \gamma, \beta \delta} \sum_{\epsilon \eta} W_{\alpha \gamma \epsilon, \beta \delta \eta} G_{\eta \epsilon}(t-t^+) a_{\alpha}^\dagger a_{\gamma}^\dagger a_{\delta} a_{\beta} 
\]

Density matrices defined wrt correlated wave functions

\[ \rho_{\eta \epsilon} \]
Inclusion of 3N forces via effective interactions

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Example of diagram with effective interaction:

Four interaction-reducible diagrams with 2N and 3N Fermions (solid lines)

Interactions (dashed lines)
Inclusion of 3N forces via effective interactions

With 3N forces, # of self-energy diagrams is too cumbersome…

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Example of diagram with effective interaction

Four interaction-**reducible** diagrams with 2N and 3N
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Example of diagram with effective interaction
1p Interaction-irreducible second-order self-energy diagram
Interaction-irreducible Self-Energy with 3N forces


\[ G_{\alpha\beta}(\omega) = G_{\alpha\beta}^{(0)}(\omega) + \sum_{\gamma\delta} G_{\alpha\gamma}^{(0)}(\omega) \Sigma^*_{\gamma\delta}(\omega) G_{\delta\beta}(\omega) \]

Second-order diagrams with 3N forces

Third-order diagrams with 3N forces
Interaction-irreducible Self-Energy with 3N forces


\[ G_{\alpha\beta}(\omega) = G^{(0)}_{\alpha\beta}(\omega) + \sum_{\gamma\delta} G^{(0)}_{\alpha\gamma}(\omega) \Sigma_{\gamma\delta}^*(\omega) G_{\delta\beta}(\omega) \]

Second-order diagrams with 3N forces

Diagrams with effective 2N forces


Diagram with irreducible 3N forces

(F.R., C. Barbieri, Proceeding of NTSE (2016))

Third-order diagrams with 3N forces
Algebraic Diagrammatic Construction method at order 3

J. Schirmer and collaborators:

Self-energy expansion is treated NON-perturbatively:
Entire classes of self-energy diagrams (ladder and ring) are summed at infinite order by means of a geometric series
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$\mathbf{M}$: matrices coupling the single-particle propagator to more complex intermediate configurations
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$\mathbf{C}$: interaction matrix linked only to internal fermion lines

Dyson-ADC(n)
$\mathbf{M}$: matrices coupling the single-particle propagator to more complex intermediate configurations.

$\mathbf{C}$: interaction matrix linked only to internal fermion lines.

Propagator (intermediate state configurations)

The set of ladder diagrams is a geometric series:

\[
\mathbf{M}^\dagger \frac{1}{\hbar \omega - E_{2p1h}} \mathbf{M} + \mathbf{M}^\dagger \frac{1}{\hbar \omega - E_{2p1h}} \mathbf{C} \frac{1}{\hbar \omega - E_{2p1h}} \mathbf{M} + \mathbf{M}^\dagger \frac{1}{\hbar \omega - E_{2p1h}} \mathbf{C} \frac{1}{\hbar \omega - E_{2p1h}} \mathbf{C} \frac{1}{\hbar \omega - E_{2p1h}} \mathbf{M} + \ldots
\]

Sum

\[
\mathbf{M}^\dagger \frac{1}{\hbar \omega - E_{2p1h} - \mathbf{C}} \mathbf{M}
\]
How does ADC(n) work practically (I)

General form of the irreducible self-energy

\[ \Sigma_{\alpha\beta}(\omega) = \mathcal{M}^\dagger \frac{1}{\hbar \omega - E_{ph} - C} \mathcal{M} \]

\( \mathcal{M} \) in powers of interactions

First order in the interaction

\[ \mathcal{M} = \mathcal{M}^{(I)} + \mathcal{M}^{(II)} + \mathcal{M}^{(III)} + \ldots \]
How does ADC(n) work practically (I)

General form of the irreducible self-energy

\[ \Sigma_{\alpha\beta}(\omega) = \mathcal{M}^\dagger \frac{1}{\hbar \omega - E_{ph} - C} \mathcal{M} \]

First order in the interaction

\[ \varepsilon_{2p1h}, \varepsilon_{3p2h}, \ldots \]

Formal expansion of \( \mathcal{M} \) in powers of interactions

\[ \mathcal{M} = \mathcal{M}^{(I)} + \mathcal{M}^{(II)} + \mathcal{M}^{(III)} + \ldots \]

Explicit expressions for \( \mathcal{M} \) and \( C \) are found by comparing with derived expressions of self-energy Goldstone diagrams up to the same order.

\[ \mathcal{M}^\dagger \frac{1}{\hbar \omega - E_{ph} - C} \mathcal{M} = \mathcal{M}^{(I)} \frac{1}{\hbar \omega - E_{ph}} \mathcal{M}^{(I)} \]

\[ + \mathcal{M}^{(II)} \frac{1}{\hbar \omega - E_{ph}} \mathcal{M}^{(I)} + \mathcal{M}^{(I)} \frac{1}{\hbar \omega - E_{ph}} \mathcal{M}^{(II)} + \mathcal{M}^{(I)} \frac{1}{\hbar \omega - E_{ph}} C \frac{1}{\hbar \omega - E_{ph}} \mathcal{M}^{(I)} \]

+ fourth order + \ldots
How does ADC(n) work practically (I)

General form of the irreducible self-energy

\[ \Sigma_{\alpha\beta}(\omega) = M^\dagger \frac{1}{\hbar \omega - E_{ph} - C} M \]

First order in the interaction

\[ \epsilon_{2p1h}, \epsilon_{3p2h}, \ldots \]

Formal expansion of \( M \) in powers of interactions

\[ M = M^{(I)} + M^{(II)} + M^{(III)} + \ldots \]

Explicit expressions for \( M \) and \( C \) are found by comparing with derived expressions of self-energy Goldstone diagrams up to the same order.

\[
M^\dagger \frac{1}{\hbar \omega - E_{ph} - C} M = M^{(I)}^\dagger \frac{1}{\hbar \omega - E_{ph}} M^{(I)} \\
+ M^{(II)}^\dagger \frac{1}{\hbar \omega - E_{ph}} M^{(I)} + M^{(I)}^\dagger \frac{1}{\hbar \omega - E_{ph} C} \frac{1}{\hbar \omega - E_{ph}} M^{(I)} + \text{fourth order} + \ldots
\]
Features of Self-Energy in ADC(n)

\[ \Sigma_{\alpha\beta}(\omega) \]

- Compatible with the Lehmann representation
- Principle of Causality
- Hermitian
- Non perturbative resummation

Dyson equation is solved as eigenvalue problem
poles and residues of the propagator are found as eigenvalues and eigenvectors of the Self-Energy Hermitian matrix

Complete set of ADC(3) working equations can be found in:
(F.R., C. Barbieri, Proceeding of NTSE (2016))

Work in progress:
Implementation in BcDor Code
• Extension of the Algebraic Diagrammatic Construction (ADC) method with three-nucleon interactions

• Dipole Response Function and Polarisability in Oxygen and Calcium isotopes

• Effective charges in Oxygen and Nickel isotopes from realistic nuclear interactions
Electric Dipole Polarizability $\alpha_D$

In general:

$\alpha_D \propto E1$ electromagnetic response

(quality of the nuclear wave function correlations)

Recent studies:

- Reinhard et al, PRC 81 051303(R) 2010
- Piekarewicz et al, PRC 85 041302(R) 2012

$\alpha_D$ as input quantity for constraining the isovector part of the nuclear interaction

Theory input for determining the Radius of Neutron stars
Electromagnetic response in SCGF

**PHOTOABSORPTION CROSS SECTION**

\[ \sigma_\gamma(E) = 4\pi^2 \alpha E R(E) \]

**ELECTRIC DIPOLE POLARIZABILITY**

\[ \alpha_D = 2\alpha \int dE \frac{R(E)}{E} \]

Response \( R(E) \) depends on excited states of the nuclear system, when “probed” with dipole operator \( \hat{D} \)

\[ R(E) = \sum_\nu |< \psi_\nu^A | \hat{D} | \psi_0^A >|^2 \delta_{E_\nu, E} \]
Electromagnetic response in SCGF

\[ \sigma_\gamma(E) = 4\pi^2 \alpha E R(E) \]

PHOTOABSORPTION CROSS SECTION

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Nuclear structure correlations:
- \( g^{II} \) RPA level (first order)
- \( g^I \) “dressed” ADC(3)

S.p. matrix element of the dipole one-body operator
Results: cross section and dipole polarisability

$^{16}\text{O} \quad ^{22}\text{O} \quad ^{40}\text{Ca} \quad ^{48}\text{Ca}$
Results for Oxygen isotopes

- GDR position of $^{16}$O reproduced
- Hint of a soft dipole mode on the neutron-rich isotope

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>SCGF</th>
<th>CC/LIT</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O</td>
<td>0.50</td>
<td>0.57(1)</td>
<td>0.585(9)</td>
</tr>
<tr>
<td>$^{22}$O</td>
<td>0.72</td>
<td>0.86(4)</td>
<td>0.43(4)</td>
</tr>
</tbody>
</table>
Results for Calcium isotopes

σ from RPA response (discretized spectrum) vs σ from photoabsorption and Coulomb excitation

40Ca

\[ N_{\text{max}} = 13, \bar{\hbar}\omega = 20 \text{ MeV} \]

\[ \alpha_D = 1.785356 \text{ fm}^3 \]

48Ca

\[ N_{\text{max}} = 13, \bar{\hbar}\omega = 20 \text{ MeV} \]

\[ \alpha_D = 2.079218 \text{ fm}^3 \]

- GDR positions reproduced
- Total sum rule reproduced but poor strength distribution (Lack of correlations)

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>SCGF</th>
<th>CC/LIT</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>40Ca</td>
<td>1.79</td>
<td>1.47 (1.87)_{\text{thresh}}</td>
<td>1.87(3)</td>
</tr>
<tr>
<td>48Ca</td>
<td>2.08</td>
<td>2.45</td>
<td>2.07(22)</td>
</tr>
</tbody>
</table>
Comparison with CC-LIT
(Couple Cluster- Lorentz Integral Transform method)

In collaboration with M. Miorelli and S. Bacca (TRIUMF, University of Mainz)

- **CC-Singles-Doubles** (analogous to 2nd RPA)
- **LIT** reduces a continuum state problem to a bound-state-like problem

Different treatment of the correlations:

**SCGF**
Reference state correlated
RPA (first-order two-body correlator)

**CC-SD-LIT**
HF Reference state
Singles-Doubles
Role of the correlations included in the reference state

NNLOsat, $\hbar \omega = 20$ MeV, $N_{max} = 13$

SCGF SumR = 14.98 MeV fm$^3$ (HF)
SCGF SumR = 14.04 MeV fm$^3$ (DysADC3)
• Extension of the Algebraic Diagrammatic Construction (ADC) method with three-nucleon interactions

• Dipole Response Function and Polarisability in Oxygen and Calcium isotopes

• Effective charges in Oxygen and Nickel isotopes from realistic nuclear interactions
Purpose: motivations for calculating effective charges from realistic potentials
Open questions

• Universal validity of “standard values” of the effective charges
• Effective charges values for nuclei towards neutron drip line
• Orbital dependence of effective charges
• Physical content of the effective charges in term of modern realistic interactions and correlations described in a many-body approach
Methods: Particle-Vibration coupling in the Self-consistent Green function formalism
Theoretical effective charges 
(as opposed to the ones extracted from experiment)

Our purpose is to calculate effective charges without resorting to any measurement of electromagnetic observables.

Basic idea: calculate the core-polarization effect felt by the single-particle orbital of interest because of the energy-dependent effective potential, calculated at ADC(3) level.

Effective charge as the ratio between the transition strengths (with and without the core-polarization) of a given multipole field:

\[
\frac{\langle \tilde{\alpha} | \hat{\phi}(\lambda_{\mu\lambda}) | \tilde{\beta} \rangle}{\langle \alpha | \hat{\phi}(\lambda_{\mu\lambda}) | \beta \rangle} = 1 + \frac{\tilde{\sum}_{\alpha\beta}(\lambda_{\mu})}{\langle \alpha | \hat{\phi}(\lambda_{\mu\lambda}) | \beta \rangle}
\]

\[|\tilde{\alpha}\rangle \equiv \text{s.p. state with correlations induced by the nuclear interaction and electromagnetic operator}\]
Results: Theoretical effective charges of Oxygen and Nickel isotopes for E2 operator
Features of the calculation

- Medium-mass isotopes:
  - Oxygen isotopes in \( sd \) and \( psd \) valence space: \( ^{14}\text{O}, ^{16}\text{O}, ^{22}\text{O} \) and \( ^{24}\text{O} \)
  - Nickel isotopes in \( 0f1p0g_{9/2} \): \( ^{48}\text{Ni}, ^{56}\text{Ni}, ^{68}\text{Ni} \) and \( ^{78}\text{Ni} \)

- NN and 3N nuclear interaction \( \text{NNLO}_{\text{sat}} \) (Phys. Rev. C 91, 051301(R))

- Electric quadrupole operator \( \mathbf{E}^{2} \)
  \[ \hat{\phi}^{(2\mu)} = \sum_{i} r_{i}^{2} Y_{2\mu}(\hat{r}_{i}) \]

- Dyson equation solved with self-energy truncated at \( \text{ADC}(3) \) level:

- Nuclear many-body wave function expanded in HO wave functions with \( N_{\text{max}}=13 \) and \( \hbar\Omega=20 \text{ MeV} \)
Results for Oxygen isotopes

<table>
<thead>
<tr>
<th>νs½ νd½</th>
<th>νs½ νd½</th>
<th>νp½ νp½</th>
<th>νd½ νd½</th>
<th>νd½ νd½</th>
<th>14O</th>
<th>16O</th>
<th>22O</th>
<th>24O</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.27</td>
<td>0.19</td>
<td>0.12</td>
<td>0.12</td>
<td>0.65</td>
<td>0.3 - 0.5 (0.65)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.41</td>
<td>0.30</td>
<td>0.12</td>
<td>0.12</td>
<td>0.53</td>
<td>0.2 - 0.4 (0.53)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.41</td>
<td>0.49</td>
<td>(0.04 ± 0.01)</td>
<td>0.19</td>
<td>0.36</td>
<td>0.1 - 0.4 (0.31)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.48</td>
<td>0.49</td>
<td>0.33</td>
<td>0.33</td>
<td>0.31</td>
<td>0.1 - 0.3 (0.27)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.27</td>
<td>0.49</td>
<td>(0.37 ± 0.14)</td>
<td>0.36</td>
<td>0.36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.46</td>
<td>0.49</td>
<td>0.36</td>
<td>0.36</td>
<td>0.33</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>0.44</td>
<td>0.49</td>
<td>0.33</td>
<td>0.33</td>
<td>0.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Neutron-rich nuclei have weaker core polarisation (quench of neutron effective charge)
- Significant isotopic dependence especially for neutrons (compared with Bohr-Mottelson Eq. 6-386b with Sagawa parametrisation of PRC 70, 054316, 200)

\[
e^{\text{eff}}_{\pi} = e + a \frac{Z}{A} + b \frac{N-Z}{A} - \left( c + d \frac{Z}{A} \frac{N-Z}{A} \right)
\]

\[
e^{\text{eff}}_{\nu} = a \frac{Z}{A} + b \frac{N-Z}{A} + \left( c + d \frac{Z}{A} \frac{N-Z}{A} \right)
\]

- Single-particle state dependence also significant (yet to be studied and understood…)

Standard values of experimental effective charges in psd nuclei are e_p=1.3 and e_n=0.5
Results for Oxygen isotopes

- Neutron-rich nuclei have weaker core polarisation (quench of neutron effective charge)
- Significant isotopic dependence especially for neutrons (compared with Bohr-Mottelson Eq. 6-386b with Sagawa parametrisation of PRC 70, 054316, 200

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e^{\text{eff}}_{\pi} = e + a \frac{Z}{A} + b \frac{N-Z}{A} - \left( c + d \frac{Z}{A} \frac{N-Z}{A} \right)
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e^{\text{eff}}_{\nu} = a \frac{Z}{A} + b \frac{N-Z}{A} + \left( c + d \frac{Z}{A} \frac{N-Z}{A} \right)
\]

- Single-particle state dependence also significant (yet to be studied and understood…)
# Results for Nickel isotopes

<table>
<thead>
<tr>
<th></th>
<th>$^{48}$Ni</th>
<th>$^{56}$Ni</th>
<th>$^{68}$Ni</th>
<th>$^{78}$Ni</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu f_{5/2}$</td>
<td>0.58</td>
<td>0.51</td>
<td>0.40</td>
<td>0.44</td>
</tr>
<tr>
<td>$\nu f_{7/2}$</td>
<td>0.80</td>
<td>0.55</td>
<td>0.57</td>
<td>0.39</td>
</tr>
<tr>
<td>$\nu f_{9/2}$</td>
<td>0.51</td>
<td>0.43</td>
<td>0.33</td>
<td>0.39</td>
</tr>
</tbody>
</table>
| $\nu f_{5/2} \ 
u p_{1/2}$ | 0.52 | 0.45 | 0.34 | 0.39 |
| $\nu f_{7/2} \ 
u p_{3/2}$ | 0.54 | 0.44 | 0.34 | na |
| $\nu f_{9/2} \ 
u p_{5/2}$ | 0.64 | 0.48 | 0.43 | 0.28 |
| $\nu p_{1/2} \ 
u p_{3/2}$ | 0.47 | 0.38 | 0.29 | 0.29 |
| $\nu p_{3/2}$   | 0.45      | 0.37      | 0.27      | 0.34      |
| $\nu g_{9/2}$   | 0.52      | 0.43      | 0.34      | 0.34      |
| $\pi f_{5/2}$   | 1.14      | 1.14      | 1.07      | 1.13      |
| $\pi f_{7/2}$   | 1.14      | 1.17      | 1.09      | 1.08      |
| $\pi f_{9/2} \ 
u p_{1/2}$ | 1.07 | 1.07 | 1.09 | 1.14 |
| $\pi f_{7/2} \ 
u p_{3/2}$ | 1.09 | 1.08 | 1.04 | 1.18 |
| $\pi f_{9/2}$   | 1.16      | 1.12      | 1.07      | 1.11      |
| $\pi p_{1/2} \ 
u p_{3/2}$ | 1.19 | 1.19 | 1.17 | 1.20 |
| $\pi p_{3/2}$   | 1.12      | 1.10      | 1.07      | 1.11      |
| $\pi g_{9/2}$   | 1.11      | 1.08      | 1.06      | 1.11      |

*Note: The starred values indicate the preferred transitions for shell model calculations.*
Results for Nickel isotopes

<table>
<thead>
<tr>
<th></th>
<th>${}^{48}\text{Ni}$</th>
<th>${}^{56}\text{Ni}$</th>
<th>${}^{68}\text{Ni}$</th>
<th>${}^{78}\text{Ni}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu f_{\frac{3}{2}}$</td>
<td>0.58</td>
<td>0.51</td>
<td>0.40</td>
<td>0.44</td>
</tr>
<tr>
<td>$\nu f_{\frac{5}{2}}$</td>
<td>0.80</td>
<td>0.55</td>
<td>0.57</td>
<td>0.39</td>
</tr>
<tr>
<td>$\nu f_{\frac{3}{2}}$</td>
<td>0.51</td>
<td>0.43</td>
<td>0.33</td>
<td>0.39</td>
</tr>
<tr>
<td>$\nu f_{\frac{5}{2}}$</td>
<td>0.52</td>
<td>0.45</td>
<td>0.34</td>
<td>0.45</td>
</tr>
<tr>
<td>$\nu f_{\frac{3}{2}}$</td>
<td>0.54</td>
<td>0.44</td>
<td>0.34</td>
<td>0.39</td>
</tr>
<tr>
<td>$\nu f_{\frac{5}{2}}$</td>
<td>0.64</td>
<td>0.48</td>
<td>0.43</td>
<td>na</td>
</tr>
<tr>
<td>$\nu f_{\frac{3}{2}}$</td>
<td>0.47</td>
<td>0.38</td>
<td>0.29</td>
<td>0.28</td>
</tr>
<tr>
<td>$\nu f_{\frac{5}{2}}$</td>
<td>0.45</td>
<td>0.37</td>
<td>0.27</td>
<td>0.29</td>
</tr>
<tr>
<td>$\nu f_{\frac{3}{2}}$</td>
<td>0.52</td>
<td>0.43</td>
<td>0.34</td>
<td>na</td>
</tr>
<tr>
<td>$\nu g_{\frac{3}{2}}$</td>
<td>0.64</td>
<td>0.48</td>
<td>0.38</td>
<td>0.28</td>
</tr>
<tr>
<td>$\nu g_{\frac{1}{2}}$</td>
<td>0.47</td>
<td>0.38</td>
<td>0.27</td>
<td>0.29</td>
</tr>
</tbody>
</table>

- Recent consensus on smaller values (~ 1.1 - 1.3) of proton effective charges
- Isotopic dependence with quenching of neutron effective charges for neutron-rich nuclei
- No consensus on magnitude of value of neutron effective charges (results consistent with microscopic PVC model with Skyrme interaction in PRC 80, 014316)
Conclusions and Perspectives

- ADC(n) as a non-perturbative method for many-body physics
- Set of effective charges for Oxygen and Nickel isotopes calculated from realistic potential (ready to be used as input in Shell Model calculations)
- Expected isospin-dependence of neutron effective charges is found
- Dipole response and polarisability calculated from first principles
- Continuum to be included, but dipole polarisability seems quite insensitive to it
- Correlations: comparison with CC-LIT and extension of ADC to polarization propagator