Linear density response theory of neutron matter from the QMC equation of state

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WALK ON THE NEUTRON-RICH SIDE

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- It is possible to use ab initio calculations for ground state properties, while for excited state we still need to use mean field approximation.
- This work follows a previous work focused on the isospin channel (asymmetry of nuclear matter).

Overview

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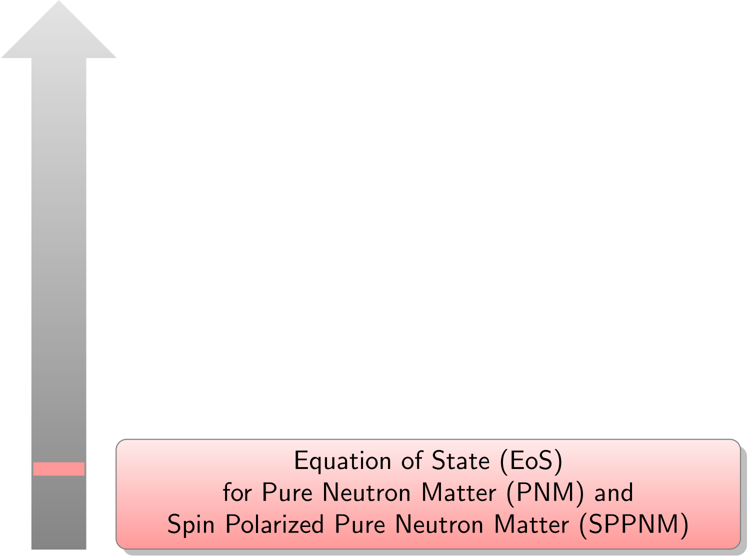

Equation of State (EoS)
for Pure Neutron Matter (PNM) and
Spin Polarized Pure Neutron Matter (SPPNM)

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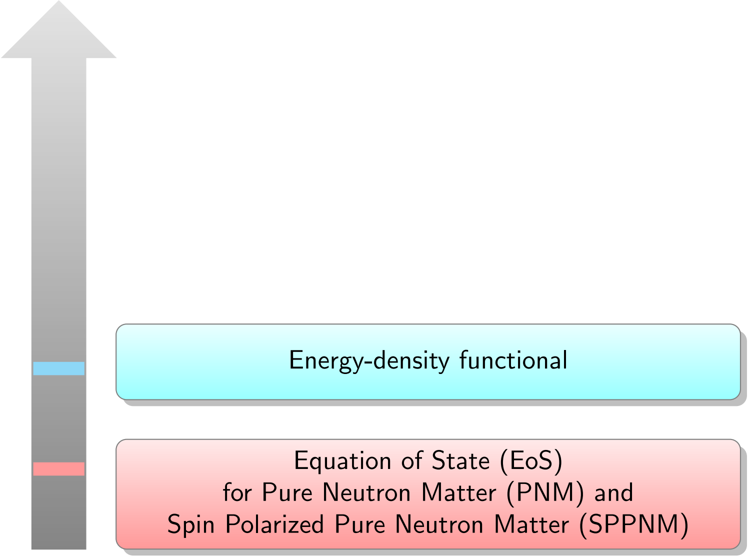

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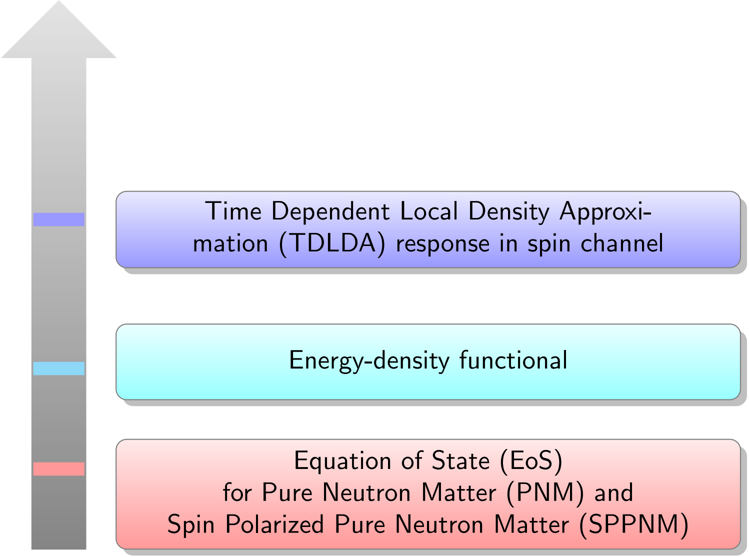

Time Dependent Local Density Approximation (TDLDA) response in spin channel

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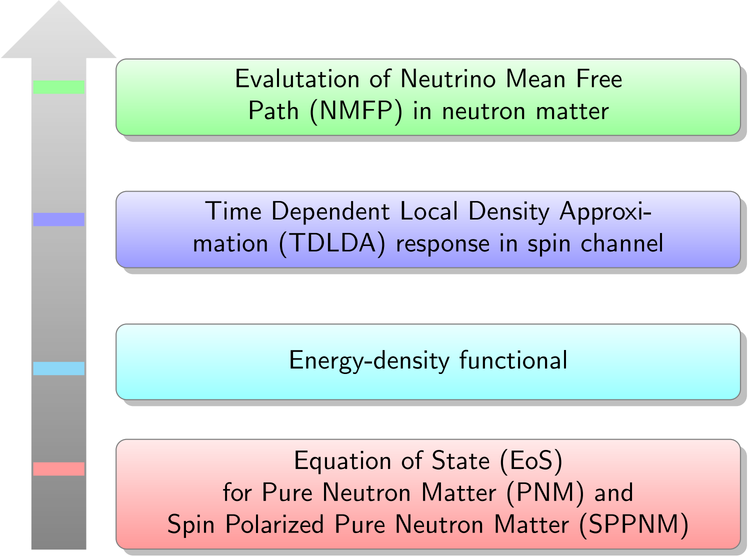

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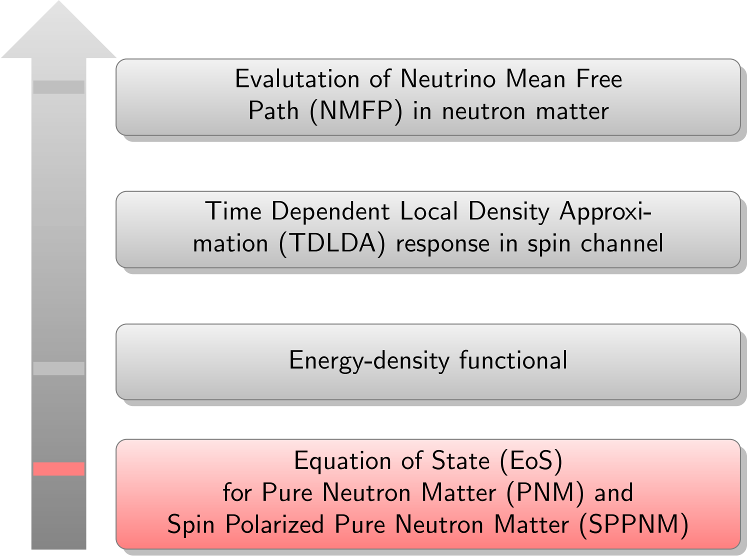

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Neutron matter

Neutron matter can be modeled as a periodic system of N neutrons interacting by an Hamiltonian of the form:

$$H = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \cdots \ .$$

In our calculations we used two kinds of potentials:

- phenomenological AV8'+UIX.
- chiral EFT N2LO local (D2,E1 and with R_0=R3N=1.0 fm) [Lynn et al. PRL 116, 062501 (2016)].

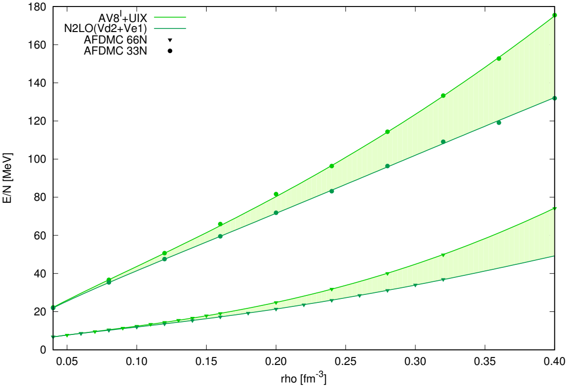
Procedure

The EoS has been computed by means Auxiliary Field Diffusion Monte Carlo (AFDMC) calculations. Trial wavefunction used in the Monte Carlo algorithm has the form:

$$\psi_T(\mathbf{R}, S) = \phi_{\mathcal{S}}(\mathbf{R})\phi_{\mathcal{A}}(\mathbf{R}, S) \ ,$$

where the first term is a Jastrow operatorial correlation function and the second term is a Slater determinant of plane waves. Computations have been carried out with 33 and 66 neutrons for PNM and SPPNM respectively.

Neutron matter with AV8'+ UIX and N2LO



Phenomenological PNM results are from Gandolfi et al. Eur. Phys. J. A, 50(2) (2014), while Chiral EFT PNM results from S. Gandolfi calculations.

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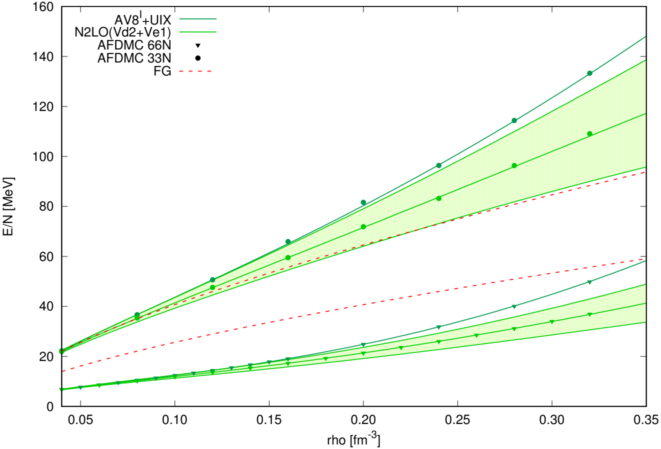

Error estimates of Chiral EFT have been computed acconding to Epelbaum et al. Eur. Phys. J. A, 51, 53 (2015).

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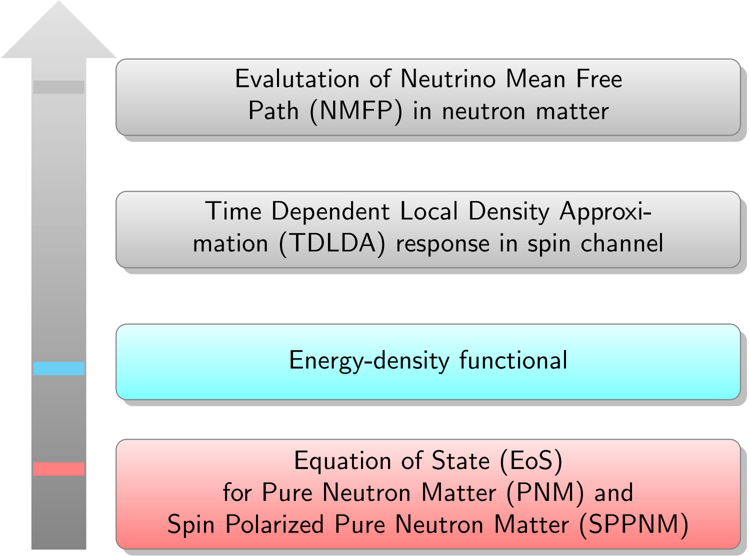

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Energy density functional

The interaction part of the EDF is assumed to be of the form:

$$\epsilon_V(\rho, m) = \epsilon_0(\rho) + m^2 \left[\epsilon_1(\rho) - \epsilon_0(\rho)\right] \ ,$$

where:

$$\epsilon_q(\rho) = \epsilon_q^0 + a_q(\rho - \rho_0) + b_q(\rho - \rho_0)^2 + c_q(\rho - \rho_0)^3$$

The saturation density is assumed to be $\rho_0 = 0.16 \text{ fm}^{-3}$.

This parametrization reproduces very well the AFDMC calculations in a wide range of density ρ (from $\rho_0/2$ to $3\rho_0$) and for both $m = 0, 1$.

General density excitations in nuclear matter

We are interested in studying the density response of the system. For nucleons the response can be splitted in different channels, described by the following operators:

$$O_F = \sum_i O_F(i) = \sum_i \tau^{\pm} e^{i\mathbf{q}\cdot\mathbf{r_i}} \quad \text{"Fermi"}$$

$$\mathbf{O}_{GT} = g_A \sum_i \mathbf{O}(i) = \sum_i \sigma_i \tau_i^{\pm} e^{i\mathbf{q}\cdot\mathbf{r_i}} \quad \text{"Gamow-Teller"}$$

$$\begin{aligned} O_{NV} &= \sum_i O_{NV}(i) \quad \text{"Neutral-vector"} \\ &= \sum_i \left[-\sin^2\theta_W + \tfrac{1}{2}(1 - 2\sin^2\theta_W)\tau_i^z \right] e^{i\mathbf{q}\cdot\mathbf{r_i}} \end{aligned}$$

$$\mathbf{O}_{NA} = g_A \sum_i \mathbf{O}_{NA}(i) = g_A \sum_i \tfrac{1}{2}\tau_i^z \sigma_{\mathbf{i}} e^{i\mathbf{q}\cdot\mathbf{r_i}} \quad \text{"Neutral-axial-vector"}$$

Weinberg-Salam model

The relation between weak scattering processes and nuclear density response descends from the *Weinberg-Salam Lagrangian* coupling a nucleon of mass m with neutrinos through weak currents.

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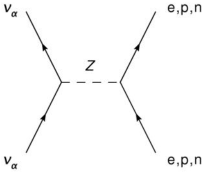
$$\mathcal{L}_W = \frac{G_W}{\sqrt{2}} \bar{\psi}_\nu(x)\gamma_\mu(1-\gamma_5)\psi_\nu(x)\frac{1}{2}\bar{\psi}_n(x)\gamma^\mu(1-C_A\gamma_5)\psi_n$$



ν scattering rate

The WS Lagrangian couples neutrinos to *density* and *spin density* fluctuations of neutrons.

In the **non-relativistic limit** the baryonic current can be approximated by:

$$\bar{\psi}_n(x)\gamma^{\mu}(1 - C_A\gamma_5)\psi_n \sim \psi_n^{\dagger}(x)\psi_n(x)\delta_0^{\mu} - C_A\psi_n^{\dagger}(x)\sigma_i\psi_n(x)\delta_i^{\mu}.$$

We have two contribution a density fluctation and a spin-density fluctuation operators.
The scattering rate from a system of neutrons of a neutrino with 4-momentum $q^{\mu} \equiv (q^0, \vec{q})$ can be computed from the Fermi golden rule, averaging on the initial (neutron and/or proton) states and summing over all the final states.

ν scattering rate

The result gives the *neutrino scattering rate*. For a neutrino of incident energy E, the contribution to the scattering rate σ in a given channel can be writtten as:

$$\sigma = \frac{G^2}{2}\frac{1}{E}\int dq \int d\omega (E-\omega) q \left(1 + \frac{E^2 + (E-\omega)^2 - q^2}{2E(E-\omega)}\right) S(q,\omega),$$

where $S(q,\omega)$ is the *dynamical structure factor (DSF)* for the excitation operators describing the process. These in turn can be written as a combination of the DSF relative to a density, and spin-density excitations.

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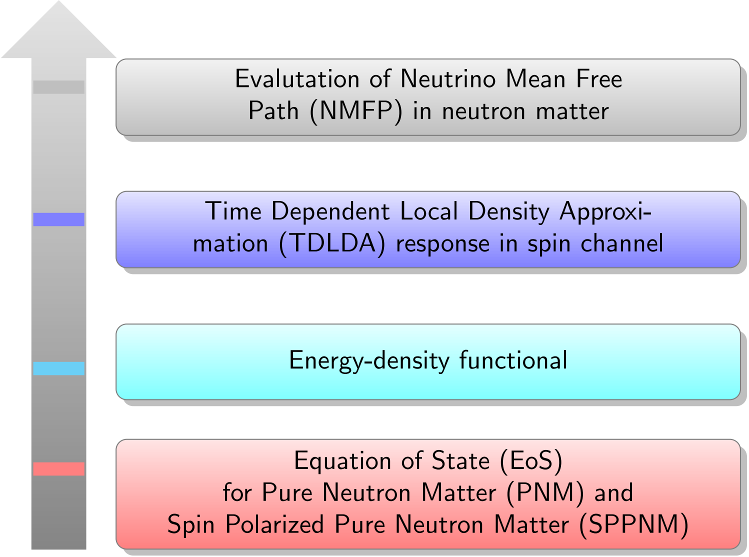

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Time Dependent Local Density Approximation

We use here the Time Dependent Local Spin Density Approximation (TDLSDA) approach to compute the response function and the DSF.

We have worked out the response function in the transverse and longitudinal spin channels.

TDLSDA

Following the *Kohn-Sham* method, we introduce a Local Spin Density Approximation (LSDA) for the homogeneous neutron matter defining the energy functional as:

$$E(\rho, m) = T_0(\rho, m) + \int \epsilon_V(\rho, m)\ \rho\ d\mathbf{r},$$

where $T_0(\rho, m)$ is the kinetic energy of the *non interacting* system with density $\rho = \rho_{n_\uparrow} + \rho_{n_\downarrow}$, and spin polarization $m = \rho_1/\rho$ with $\rho_1 = \rho_{n_\uparrow} - \rho_{n_\downarrow}$.

The Hohenberg-Kohn theorem provides a *variational principle on the energy-density functional*.

Longitudinal channel

Since are interested in homogeneous systems, the solutions of the KS-equations have the form (for both spin up and spin down):

$$\rho_n(\mathbf{r},t) = \rho_n + \delta\rho_n(\mathbf{r},t)\,,$$

where:

$$\delta\rho_n(\mathbf{r},t) = \delta\rho_n\left[e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)} + e^{-i(\mathbf{q}\cdot\mathbf{r}-\omega t)}\right].$$

The quantities $\delta\rho_n$ have to be determined from the KS equations. In order to determine $\delta\rho_{n_\uparrow}$ and $\delta\rho_{n_\downarrow}$, we insert $\rho_{n_\uparrow}(\mathbf{r},t)$,$\rho_{n_\downarrow}(\mathbf{r},t)$ in the KS equations, and linearize. After this procedure one obtains:

$$\lambda\chi^{n\uparrow}(q,\omega) = \lambda'_{n_\uparrow}\,\chi_0^{n_\uparrow}(q,\omega)$$
$$\lambda\chi^{n\downarrow}(q,\omega) = \lambda'_{n_\downarrow}\,\chi_0^{n_\downarrow}(q,\omega)$$

Longitudinal channel

The previous two equations, together with the definition of the linear response function, allow for solving for $\chi^{n_\uparrow}$ and $\chi^{n_\downarrow}$, given χ_0 (longitudinal response function of the free Fermi gas). Summing and subtracting $\chi^{n_\uparrow}$ and $\chi^{n_\downarrow}$ we obtain the density-density and vector-density/vector-density response functions for arbitrary spin polarization.

TDLSDA longitudinal response functions (scalar and vector)

$$\frac{\chi^s(q,\omega)}{V} = \frac{\frac{\chi_0^{n_\uparrow}}{V}[1-(V_{n_\downarrow n_\downarrow}-V_{n_\uparrow n_\downarrow})\frac{\chi_0^{n_\downarrow}}{V}] + \frac{\chi_0^{n_\downarrow}}{V}[1-(V_{n_\uparrow n_\uparrow}-V_{n_\downarrow n_\uparrow})\frac{\chi_0^{n_\uparrow}}{V}]}{(1-V_{n_\downarrow n_\downarrow}\frac{\chi_0^{n_\downarrow}}{V})(1-V_{n_\uparrow n_\uparrow}\frac{\chi_0^{n_\uparrow}}{V}) - V_{n_\uparrow n_\downarrow}\frac{\chi_0^{n_\uparrow}}{V}V_{n_\downarrow n_\uparrow}\frac{\chi_0^{n_\downarrow}}{V}} ,$$

$$\frac{\chi^v(q,\omega)}{V} = \frac{\frac{\chi_0^{n_\uparrow}}{V}[1-(V_{n_\downarrow n_\downarrow}+V_{n_\uparrow n_\downarrow})\frac{\chi_0^{n_\downarrow}}{V}] + \frac{\chi_0^{n_\downarrow}}{V}[1-(V_{n_\uparrow n_\uparrow}+V_{n_\downarrow n_\uparrow})\frac{\chi_0^{n_\uparrow}}{V}]}{(1-V_{n_\downarrow n_\downarrow}\frac{\chi_0^{n_\downarrow}}{V})(1-V_{n_\uparrow n_\uparrow}\frac{\chi_0^{n_\uparrow}}{V}) - V_{n_\uparrow n_\downarrow}\frac{\chi_0^{n_\uparrow}}{V}V_{n_\downarrow n_\uparrow}\frac{\chi_0^{n_\downarrow}}{V}} .$$

Transverse channel

A similar derivation can be done for the transverse channel.
In this case the LSDA-KS equations are:

$$\left[-\frac{1}{2}\nabla^2_{\mathbf{r}} + \frac{1}{2}\omega_L\sigma_z + v(\mathbf{r}) + w(\mathbf{r})\sigma_z \right] \varphi_i^\sigma(\mathbf{r}) = \varepsilon_{i,\tau}\,\varphi_i^\sigma(\mathbf{r})$$

The second term in the l.h.s is an effective vector potential accounting for the equilibrium spin polarization (due to the presence of strong magnetic fields). The parameter ω_L can be related to spin imbalance by imposing that the variation of the LSDA energy with respect to m be zero.

Transverse channel

The resulting interaction (single particle) Hamiltonian, when a density fluctuation is considered, becomes:

$$H_{\mathrm{int}} \sim \sigma^- e^{i\mathbf{q}\cdot\mathbf{r}-\imath\omega t} + \sigma^+ e^{-i\mathbf{q}\cdot\mathbf{r}+\imath\omega t}.$$

The response function is defined as the difference of the respose relative σ^- and σ^+ and, eventually the final result for the response function is:

$$\frac{\chi_t(q,\omega)}{V} = \frac{\chi_t^0(q,\omega)}{1-\frac{2}{V}\mathcal{W}(\rho,m)\chi_t^0(q,\omega)},$$

where the $\chi_t^0(q,\omega)$ is the transverse response of the free Fermi gas.

Excitation strengths and sum rules

- From the response function it is possible to determine the dynamic structure factor via the relation:

$$S^{s,v}(q,\omega) = -\frac{1}{\pi}\Im m[\chi^{s,v}]$$

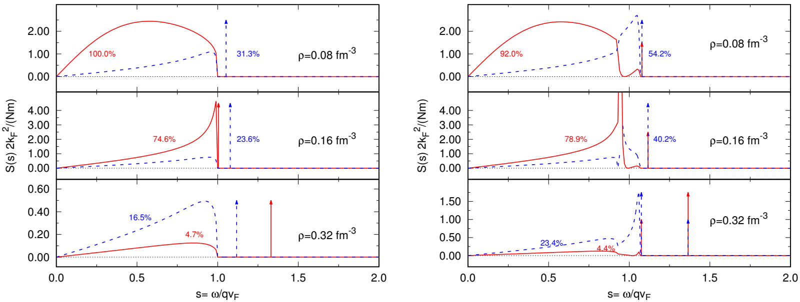
- From the DSF it is also possible to compute the energy weighted sum rules:

$$m_k^{s,v} = \int_0^\infty d\omega\, \omega^k S^{s,v}(q,\omega) = \sum_n \omega_{no}^k |\langle 0|F^{s,v}|n\rangle|^2$$

  In particular the ratio m_{-1}/m_0 gives the *compressibility* of the system.

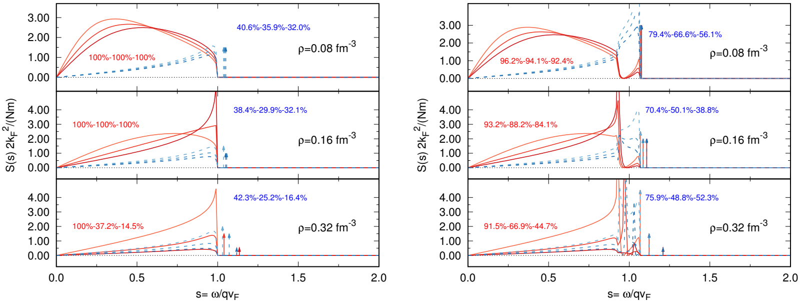
- The poles of $\chi(q,\omega)$ give the spectrum and the dispersion $\omega(q)$ of the collective excitations, for which we can also evaluate the strength.

Longitudinal response (AV8'+UIX)



TDLSDA Dynamical structure factor for **density** (solid lines) and **spin density** (dashed lines) in the longitudinal channel. Left panel: results for PNM. Right panel: results for m=0.2. Arrows indicate the location of the collective excitations. The percentages represent the fraction of the total strength pertinent to the particle-hole excitations.

Longitudinal response (Chiral EFT)



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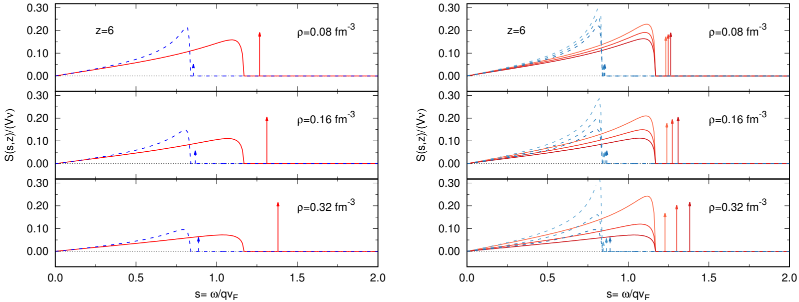
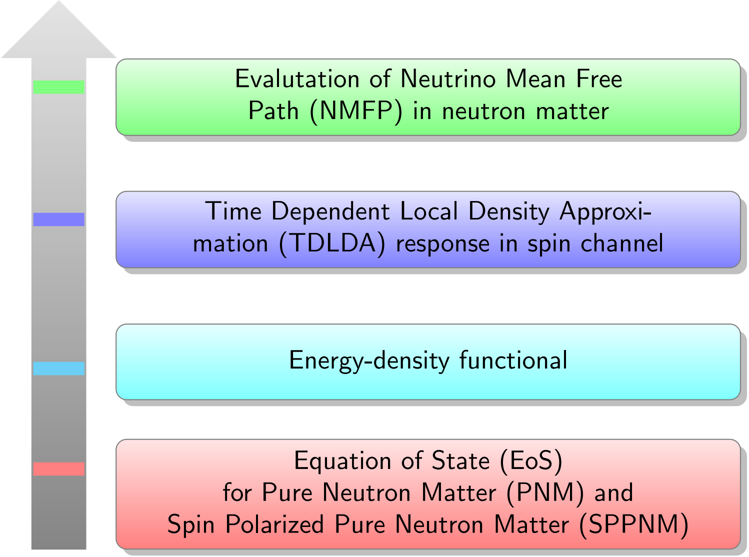
Transverse response

$$\nu = mk_F/(2\pi^2)$$



Excitation strengths for $z = 3q/(2k_F m) = 6$. The full and dashed lines indicate the particle/hole and collective strengths in the $\Delta S_z = -1$ ($s > 0$ - red) and $\Delta S_z = +1$ ($s < 0$ - blue) channels.

Overview



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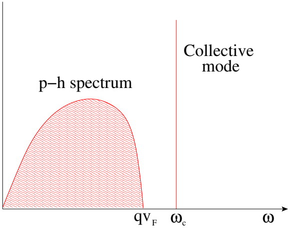
Neutrino mean free path

As previously discussed, the scattering rate of neutrinos can be obtained by computing the integral:

$$\sigma = \frac{G^2}{2}\frac{1}{E}\int dq \int d\omega (E-\omega) q \left(1+\frac{E^2+(E-\omega)^2-q^2}{2E(E-\omega)}\right) S(q,\omega)$$

The neutrino mean free path λ is related to σ by the following relation:

$$\lambda = \frac{1}{\sigma\rho}$$



The scattering rate is made up of two contributions:

- Contribution from the particle-hole excitations
- Contribution from the collective mode

Neutrino mean free path

The integration must be performed on the values of momentum kinematically accessible to neutrinos.

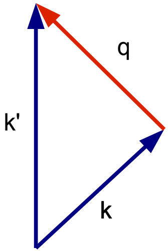
Notation:

$$k^{\mu} = (k^0, \vec{k}) \quad k'^{\mu} = (k'^0, \vec{k}')$$

are the incoming and outgoing 4-momenta of the neutrino.

$$q^{\mu} = (\omega, \vec{q})$$

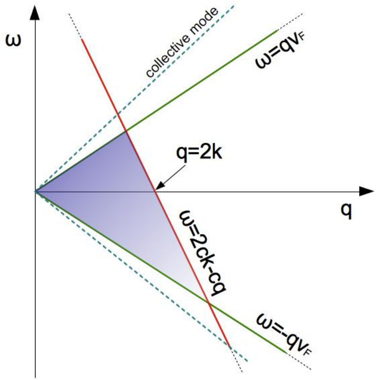
is the transferred 4-momentum.



Neutrinos are assumed to be *ultra-relativistic*.

Neutrino mean free path

Kinematic limits



The transferred momentum must satisfy the following inequality:

$$|\omega| < cq < |\omega - 2ck|$$

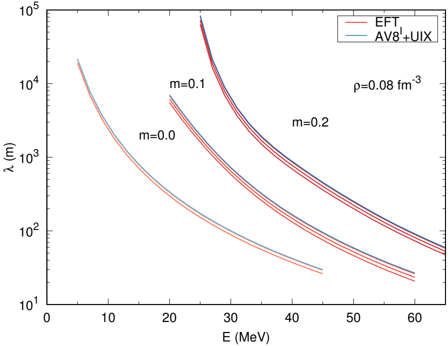
This implies that:

$$\omega < c(2k - q)$$

This represents the integration bound, that has to be intersected with the limits coming from $S(q,\omega)$.

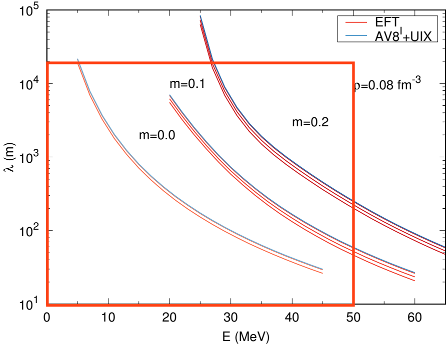
This holds for non-degenerate neutrinos.

Neutrino mean free path (transverse)



Neutrino mean free paths at different spin polarizations.

Neutrino mean free path (transverse)



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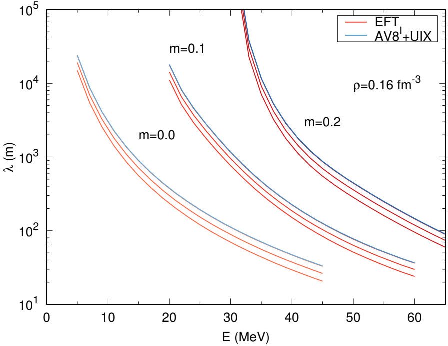

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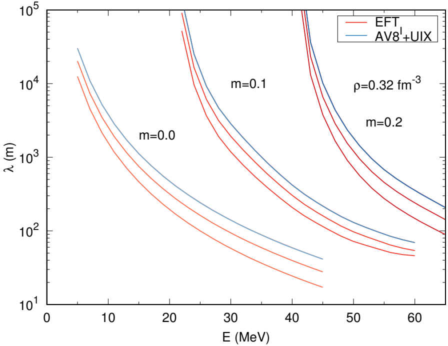

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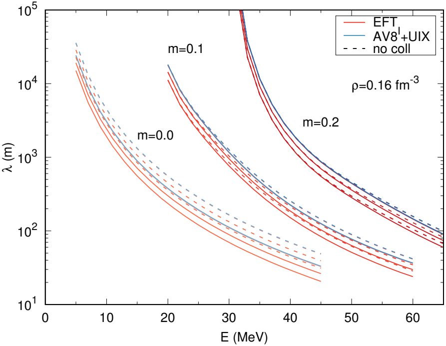
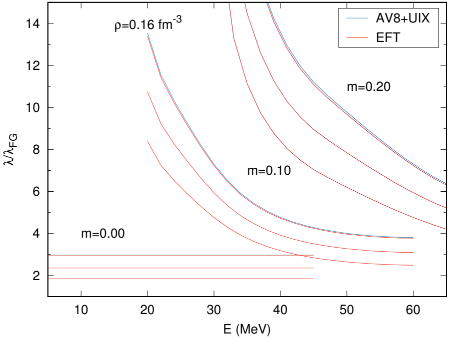


It is interesting to look at the contribution of the collective modes.

Neutrino mean free path (transverse)



Ratio of the NMFP in an interacting neutron matter and in a free Fermi gas at density $\rho/\rho_0 = 1$.

Conclusions

- We computed the response function in the longitudinal and transverse channel in pure neutron matter, starting from accurate QMC calculations of (spin polarized) neutron matter.
- The time dependent local density approximation was successfully applied to estimate the response function of arbitrary spin polarized neutron matter.
- We computed the contribution of transverse channel to the suppression of the neutrino mean free path in neutron matter (longitudinal channels is in progress). At the NS core conditions matter is essentially transparent, while relevant effects could be seen in the NS crust.