Electromagnetic properties of the proton in a unified $\mathbf{N-\Delta}$ Model

K. Savvidy, Trento, July 2013, ECT

Based on arxiv:1305.3847 with Y. Zhang
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What is a proton?

Proton is (uud),
neutron is (ddd).
$\Delta$ is the same, but
why not a $N^{++}$
with (uuu)?
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Pauli exclusion!
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Pauli exclusion!

Since all the quarks are in s-wave, replace:

\[ \Psi_\alpha(x) \Psi_\beta(y) \Psi_\gamma(z) \rightarrow \Psi_{\alpha\beta\gamma}(x) \]
Spin 3/2 - textbook case

- $\Psi^\mu_a$ - has 16 components
- $1 \times 1/2 = 3/2 + 1/2$

- Spin 3/2: 4 components + 4 antiparticle
- Spin 1/2: $\chi^1 = \gamma_\mu \Psi^\mu_a$ 2 + 2
- Spin 1/2: $\chi^2 = p_\mu \Psi^\mu_a$ 2 + 2

\[ \underline{8 + 8} \]
The langrangian has free kinetic and mass terms:

\[ \mathcal{L} = -\bar{\psi}_\lambda [p_\mu \Gamma^{\mu\lambda\rho} - m \Theta^{\lambda\rho}] \psi^\rho, \]

\[ \Gamma^{\mu\lambda\rho} = \gamma^\mu \eta^{\lambda\rho} + \xi (\gamma^\lambda \eta^{\mu\rho} + \gamma_\rho \eta^{\lambda\mu}) + \zeta \gamma^\lambda \gamma^\mu \gamma^\rho, \]

\[ \Theta^{\lambda\rho} = \eta^{\lambda\rho} - z \gamma^\lambda \gamma^\rho, \]

\[ \xi = 2z - 1, \quad \zeta = 6z^2 - 4z + 1, \]

It follows from eom that

\[ p_\mu \Psi^\mu_a = 0 \quad \text{and if } z = \frac{1}{3} \text{ then } \gamma_\mu \Psi^\mu_a = 0 \]

else a physical spin 1/2 particle of mass \[ M = \frac{m}{6z - 2}. \]
Propagator

\[-iS(p) = \frac{(\phi + m) \Pi_3}{p^2 - m^2 + i\epsilon} - \frac{(\phi + M) \Pi_{11}}{p^2 - M^2 - i\epsilon} \frac{2M^2}{m^2} \]

\[+ \left[ \Pi_{22} - (\Pi_{21} + \Pi_{12}) / B + \Pi_{11} \frac{3}{B^2} \right] \frac{3}{2 (M + 2m)} \]

\[B = \frac{3m}{2M + m} \]
Vertex

~ Elementary, minimal interaction:

\[ \Gamma_{\mu \lambda \rho} = \gamma^\mu \eta^\lambda \rho + \xi (\gamma^\lambda \eta^\mu \rho + \gamma_\rho \eta^\lambda^\mu) + \zeta \gamma^\lambda \gamma^\mu \gamma_\rho \]

~ Ward identity:

\[ -i k_\mu \Gamma_{\mu \lambda \rho} = S^\lambda_\rho (p + k)^{-1} - S_\rho^\lambda (p)^{-1} \]

~ Need also non-minimal interactions!
Extended vertex:

We add as yet undetermined electromagnetic formfactors:

\[
\tilde{\Gamma}^{\mu\lambda}_{\rho} = \Gamma^{\mu\lambda}_{\rho} + \frac{i}{2M} \sum_{n} F_{n}(k^{2}) (\Gamma_{n})^{\mu\lambda}_{\rho},
\]

If amplitudes are to be gauge invariant, the Ward identity should still hold. For that, it is sufficient to have \( k_{\mu}(\Gamma_{n})^{\mu\lambda}_{\rho} = 0 \) and thus \( \Gamma_{n} \) should be of the form:

\[
(\Gamma_{n})^{\mu\lambda}_{\rho} = (\Sigma_{n})^{\mu\nu\lambda}_{\rho} k_{\nu}, \tag{1}
\]

where \( (\Sigma_{n})^{\mu\nu\lambda}_{\rho} \) is antisymmetric in \( \mu \) and \( \nu \).
Formfactors

\[(\Sigma_1)_{\mu\nu\lambda\rho} = -\frac{1}{2} \tau^{\mu\nu\lambda\rho},\]
\[(\Sigma_2)_{\mu\nu\lambda\rho} = \sigma^{\mu\nu} \eta^{\lambda\rho},\]
\[(\Sigma_3)_{\mu\nu\lambda\rho} = -\frac{1}{9} \gamma^{\lambda} \sigma^{\mu\nu} \gamma^{\rho},\]
\[(\Sigma_4)_{\mu\nu\lambda\rho} = \frac{1}{12} \left( \gamma^{\lambda} \gamma^{\mu} \eta^{\nu\rho} - \gamma^{\lambda} \gamma^{\nu} \eta^{\mu\rho} + \gamma^{\mu} \gamma^{\rho} \eta^{\nu\lambda} - \gamma^{\nu} \gamma^{\rho} \eta^{\mu\lambda} \right),\]
\[(\Sigma_5)_{\mu\nu\lambda\rho} = \frac{-i}{12} \left( \gamma^{\lambda} \gamma^{\mu} \eta^{\nu\rho} - \gamma^{\lambda} \gamma^{\nu} \eta^{\mu\rho} - \gamma^{\mu} \gamma^{\rho} \eta^{\nu\lambda} + \gamma^{\nu} \gamma^{\rho} \eta^{\mu\lambda} \right).\]
M1 transitions

\[ \gamma_{NN} \]

\[ \frac{\mu_p}{\mu_N} = 1 + \lambda_p = 1 + \frac{4M(m + M)}{3m^2} + \frac{2M^2}{3m^2}(F_1 + F_2 + F_3 + F_5) \]

\[ \gamma_{\Delta\Delta} \]

\[ \frac{\mu_{\Delta^+}}{\mu_N} = \frac{M}{m} + \left( -\frac{1}{2}F_1 + F_2 \right). \]

When all the form factors are set to zero, \( \mu_{\Delta^+} = \frac{e}{2m} \), so the g-factor of \( \Delta^+ \) is \( \frac{2}{3} \), which agrees with expectations for that of an elementary spin 3/2 particle Belinfante (1953)
For $\gamma N\Delta$ transition:

\[
\begin{align*}
 &\begin{array}{ccc}
 +3/2 & \leftrightarrow & +1/2 \\
 +1/2 & \leftrightarrow & -1/2 \\
 -1/2 & \leftrightarrow & -3/2 \\
 \Delta & \Delta & \Delta \\
 \end{array} \\
 &\begin{array}{ccc}
 \bar{\nu}_4(p_1, \sigma_1) & \tilde{\Gamma}^{\mu} & u_2(p_2, \sigma_2) \\
 & \left(\begin{array}{cc}
 \sqrt{3} & 0 \\
 0 & 1 \\
 0 & 0 \\
\end{array}\right)_{\sigma_1, \sigma_2} & A_\mu^L \\
 & + \mathcal{O}(q^2) \\
\end{array}
\end{align*}
\]

\[
G = \frac{1}{12\sqrt{2}} (8 + 2F_1 + 8F_2 + F_5 - iF_4)
\]
Loop Corrections

\[ m \rightarrow m - iW/2 \]
\[ \Gamma^{\mu\lambda}_{\rho} = \gamma^{\mu} \eta^{\lambda}_{\rho} + \xi (\gamma^{\lambda} \eta^{\mu}_{\rho} + \gamma^{\rho} \eta^{\lambda\mu}) + \zeta \gamma^{\lambda} \gamma^{\mu} \gamma^{\rho} \]
Similar to the effective Lagrangian proposed in \{0611327\},

\[
\mathcal{L}_{\text{pol}} = \frac{i\pi}{M} (\bar{\psi} \gamma^\mu \partial_\nu \psi - \partial_\nu \bar{\psi} \gamma^\mu \psi)(\alpha_B \ F_{\mu\rho} \ F^{\rho\nu} + \beta_B \ \tilde{F}_{\mu\rho} \ \tilde{F}^{\rho\nu})
\]

we include the following interaction Lagrangian to model "bare" polarizability:

\[
\mathcal{L}_{\text{pol}} = \frac{i\pi}{M} (\bar{\psi}_\lambda \ \Gamma^{\mu\lambda}_{\ \rho} \ \partial_\nu \ \psi^\rho - \partial_\nu \ \bar{\psi}_\lambda \ \Gamma^{\mu\lambda}_{\ \rho} \ \psi^\rho)(\alpha_B \ F_{\mu\rho} \ F^{\rho\nu} + \beta_B \ \tilde{F}_{\mu\rho} \ \tilde{F}^{\rho\nu})
\]
Total polarizability

\[ \bar{\beta} = \frac{4\alpha|G|^2}{(m_{\Delta} - M_p)M_p^2} + \beta_B \]
Fitting data

Figure y: Fixed cm\(\text{m}\) angle cross section and the data points where the parameters from the fitting to all the 7\(\nu\)y data points are used for the theoretical cross section curves. The x-axis is cm\(\text{m}\) frame scattering angle and y-axis is cm\(\text{m}\) frame differential cross section in units of nanobarns. Labels are the same as in Figs xs. The quoted photon energy of the data points included in this plot may differ from the nominal values by at most yMeVs.

The form factors and thus the parameters \(\mu\) and \(G_n\) are generally functions of \(k^2\) where \(k\) is photon momentums. For real Compton scattering, \(k^2\) is always positive, so the form factors should be constants in this paper. However, in the case of the bare polarizabilities, it is possible that they vary with energy and/or scattering angle \([x8]\). This would make fitting with constant bare polarizabilities unsuccessful.

The strategy we propose to deal with the possible variation of bare polarizabilities is as follows: First, we fit only the peak range data points and fix the form factors \(\mu\) and \(G_n\) from this fitting. Then we fit the low energy data points varying only the bare polarizabilities. For the peak range, we use only the MAMI\(\text{u}uvn\) experiment \([vu]\) which contains \(x6\) data points with photon

Yellow: MAMI, Red: Saskatoon, Grey: LEGS
Fitting data

\[ \chi^2/\text{dof} \sim 1.9 \text{ (for fit to MAMI data alone)} \]
Fitting data

$\chi^2/dof \sim 1.9$ (for fit to MAMI data alone)
Figure 5: Projections of the 6-dimensional 95th confidence region into planes spanned by several pairs of parameters. Plots in each column share the same x-axis parameter as indicated at the top of each column. Plots in each row share the same y-axis parameter as indicated at the left of each row. Range for $F_1$ is $-6\mu$ to $-6\mu$; $G$: $-79\mu$ to $-88\mu$; $\mu$: $-79\mu$ to $-88\mu$; $F_6$: $-87\mu$ to $-87\mu$. A good fit is achieved at $F_1 = -7\mu$. $\mu = vy$ with $\chi^2 \sim 8$. It is notable that $F_1$, $F_6$, $\mu$, and $G$ have not changed much from the complete fit of all data points, yet $\chi^2$ per datapoint is much smaller. This may be indicative of the fact that experimental data prior to this latest and more precise measurements may not be consistent with each other. In the past, one of the strategies for...
Fitting data

\[ \bar{\alpha} + \bar{\beta} = 11.3 \pm 0.9 \pm 2.3 (95\% C.L.) \]
\[ \bar{\alpha} - \bar{\beta} = 7.8 \pm 3.3 \pm 2.0 (95\% C.L.) \]
\[ \chi^2/\text{dof} \sim 2.8 \text{ (fit to data below 140MeV)} \]
Conclusion

- A study of properties of the $\Delta$, but instead got to know the Proton!
- Pure theory is good, but phenomenology is still rather crude.
- $\chi$PT calculations of pion loop effects can and should be done within this framework.

Thank you!