Recent Developments in (d,p) Reaction Theory (at Surrey)

N.K. Timofeyuk

in collaboration with
R.C. Johnson, G.W. Bailey and J.A. Tostevin (University of Surrey)
S.J. Waldecker (University of Tennessee, Chattanooga)
Exact amplitude of $A(d,p)B$ reaction

$$T_{(d,p)} = \left\langle \chi_{pB}^{(-)} \psi_p \psi_B \right| \left( V_{np} + \sum_{i \in B} V_{ip} - U_{pB} \right) \right| \Psi^{(+)} \right\rangle$$

Where $\Psi^{(+)}$ is exact many-body wave function

$\psi_B$ is internal wave function of $B$ (exact solution of $H_B \psi_B = E_B \psi_B$)

$\psi_p$ is the proton spin function

$\chi_{pB}$ is the distorted wave in the $p - B$ channel obtained from a two-body model with **ARBITRARY** potential $U_{pB}$
Including deuteron breakup: \[ \Psi^{(+)} \approx \psi^{(+)}_{\text{Anp}}(\mathbf{R}, r_{np}) \psi_A \]

\[ \psi^{(+)}_{\text{Anp}}(\mathbf{R}, r_{np}) \] is obtained from three-body Schrödinger equation:

\[ \left( T_R + T_{np} + V_{np} + U_{nA} + U_{pA} - E \right) \psi^{(+)}_{\text{Anp}}(\mathbf{R}, r_{np}) = 0 \]

\[ U_{nA} \text{ and } U_{pA} \text{ are } nA \text{ and } pA \text{ optical potentials} \]

\[ U_{nA} \text{ optical potentials taken in adiabatic model (until 2013) at half the deuteron incident energy } E_d / 2 \]

\[ T_{(d,p)} = \left\langle \chi_p^{(-)} \psi_p \psi_B \right| V_{np}(r_{np}) \left| \psi^{(+)}_{\text{Anp}}(\mathbf{R}, r_{np}) \right| \psi_A \right\rangle \]
Solving the 3-body Schrödinger equation using Weinberg state expansion. Johnson-Tandy model.  

\[ \psi_{Anp}^{(+)}(R, r_{np}) = \sum_{i=1}^{\infty} \varphi_i(r_{np}) \chi_i(R) \]

Weinberg basis:  
\[ (T_{np} + \alpha_i V_{np} + \varepsilon_d) \varphi_i(r_{np}) = 0, \quad \alpha_1 = 1, \varphi_1 = \varphi_d, \quad \langle \varphi_i | V_{np} | \varphi_j \rangle = -\delta_{ij} \]

Assumption: only first term of the expansion is important for (d,p) reactions. It can be found from one equation if all the coupling to the other channels are neglected.

\[ (T_R + V(R) - E_d) \chi_1^{(+)}(R) = 0 \]

First Weinberg component  
\[ \chi_1 = -\langle \varphi_1 | V_{np} | \psi_{Anp}^{(+)} \rangle \]

adiabatic potential  
\[ V(R) = \int dr_{np} \frac{\varphi_1(r_{np})^2 V_{np}(r_{np})}{\langle \varphi_1 | V_{np} | \varphi_1 \rangle} \left( V_{nA}(R + \frac{r_{np}}{2}) + V_{pA}(R - \frac{r_{np}}{2}) \right) \]

n-A and p-A optical potentials are needed for deuteron channel.
Rapid convergence of the Weinberg expansion of the deuteron stripping amplitude


Weinberg components are constructed from CDCC solutions. Application to $^{132}\text{Sn}(d,p)^{133}\text{Sn}$ reactions.
Nonlocal two-body model

\[(T - E) \Psi_N(r) = -\int dr' V(r, r') \Psi_N(r')\]

Perey-Buck potential;

\[V(r, r') = U_N \left( \frac{|r + r'|}{2} \right) \exp \left( -\frac{(r - r')^2}{\beta^2} \right) = H(s) U_N \left( \frac{|r + r'|}{2} \right) \]

\[s = r - r'\]

\[(T - E) \Psi_N(r) = -\int ds H(s) U_N(r + \frac{s}{2}) \Psi_N(r + s)\]

\[U_N(r + \frac{s}{2}) \approx U_N(r)\] gives lowest order potential

\[U_N(r + \frac{s}{2}) \approx U_N(r) + s \cdot \nabla U_N(r)\] includes first order corrections
Using energy-independent nonlocal potentials in ADWA

Deuteron wave function is found from equation:

\[
(T_R + U_C(R) - E_d)\chi(R) = -\int ds \, ds' H(s) \left[ \phi_1(x + \alpha_1 s) U_{nA} \left( \frac{x}{2} - R \right) + \phi_1(x - \alpha_1 s) U_{pA} \left( -\frac{x}{2} - R \right) \right] \phi_0(x) \chi \left( \frac{\alpha_2 s}{2} + R \right)
\]

\[
\phi_1(r) = \frac{V_{np}(r) \phi_0(r)}{\langle \phi_0 | V_{np} | \phi_0 \rangle}
\]

a) We can solve it exactly
b) we can solve it in lowest order approximation  \[U_{NA} \left( \frac{x}{2} - R \right) \approx U_{NA}(R)\]
The difference between $U_{JS}$ and $U_{loc}^0$ depends on $s$-wave Hulthen $s+d$ AV18

<table>
<thead>
<tr>
<th>$s$-wave Hulthen</th>
<th>$s+d$ AV18</th>
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<tbody>
<tr>
<td>106 MeV</td>
<td>218 MeV</td>
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Motivation – d-state within a Nonlocal Adiabatic model

Nonlocal potentials can be sensitive to high momentum components in the deuteron wavefunction

If nonlocal Adiabatic potentials are sensitive to these components the d-state effect may be significant
\[(T_R^{(L')} + U_c(R) - E_d) \chi_{L'L}(R) + R \sum_{L''} \int_0^\infty dR'R' \left[ V_L^{J}(R, R') \delta_{L'L''} + V_L^{J}(R, R') \right] \chi_{L''L}(R') = 0\]

Trivially-equivalent local potential

\[\chi^{J}_{L'L}(R)\]
Results - differential cross sections (S = 1)

18F(d,p)19F at 14.9 MeV
E_x = 6.080 MeV, l = 2

14C(d,p)15C at 17.06 MeV
GS, l = 0

26Al(d,p)27Al at 12 MeV
E_x = 3.004 MeV, l = 0

40Ca(d,p)41Ca at 11.8 MeV
GS, l = 3

Pure s-state Hulthen
AV18 s+d
A.S.Adekola et.al (2011)

A.M.Mukhamedzhanov et.al (2011)

V.Margerin et.al (2015)

U.Schmidt-Rohr et.al (1964)
How does contribution from d-state depend on:
- orbital momentum of transferred neutron
- separation energy of transferred neutron?

\[
\frac{\sigma(\text{only } s - \text{wave Hulthen})}{\sigma(s + d - \text{waves AV18})}
\]

![Graph showing the percentage difference over neutron separation energy](image-url)
Short-range physics in low-energy \((d,p)\) reactions

\(^{26}\text{Al}(d,p)^{27}\text{Al}, \ E_d = 12 \text{ MeV}\)

(destruction of \(^{26}\text{Al}\) in Wolf-Rayet stars and AGB stars)

\((d,p)\) cross sections are determined by vertex function

\[ V_{np}\varphi_d \]

**Used NN models:**
Hulthen, AV18, Reid, CD-Bonn, chiral EFT (N4LO)

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Optical potential in $A + n + p$ three-body model

$$\Psi = \phi_{g.s.}\chi_0(r, R) + \sum_{i \neq 0} \phi_i \chi_i(r, R)$$

Ground-state channel function can be found from three-body model

$$(T_3 + V_{np} + \langle \phi_{g.s.} \mid V_{opt} \mid \phi_{g.s.} \rangle - E_3) \chi_0 = 0$$

$$V_{opt} = U_{nA} + U_{pA} + U_{nA} \frac{Q}{e} U_{pA} + U_{pA} \frac{Q}{e} U_{nA} + ...$$

$$U_{NA} = v_{NA} + v_{NA} \frac{Q}{e} U_{NA}$$

Optical potential for 3-body system has two-body and three-body terms

R.C. Johnson and N.K. Timofeyuk, PRC 89, 024605 (2014)
Two-body force in a three-body system

\[ (T_3 + V_{np} + \langle \varphi_{g.s.} | U_{nA} + U_{pA} | \varphi_{g.s.} \rangle - E_3) \chi_0 = 0 \]

\[ U_{nA} = v_{nA} + v_{nA} \frac{Q}{E_3 + i0 - T_3 - V_{np} - (H_A - E_A)} U_{nA} \]

Comparison to N-A optical potential:

\[ U_{nA} = v_{nA} + v_{nA} \frac{Q}{E_N + i0 - T_{nA} - (H_A - E_A)} U_{nA} \]

Two-body force in three-body system differs from two-body optical potential!
N-A two-body force in A(d,p)B reactions

\[ T_{(d,p)} = \left\langle \chi_{pB}^{-} \psi_p \psi_B \right| V_{np} (r_{np}) \right| \psi_{npA}^{(+)} (R, r_{np}) \psi_A \right\rangle \]

- The most important contribution to the (d,p) reaction comes from small n-p separations.

- Adiabatic approximation can be made by retaining only first Weinberg component

  \[ \psi_{npA}^{(+)} (R, r_{np}) \approx \psi_{dA}^{(+)} (R) \phi_d (r_{np}) \]

- In the adiabatic approximation the distorted wave function satisfies the equation:

\[
\left( T_R + \left\langle \phi_d \phi_{g.s.}^A \right| V_{np} (U_{nA} + U_{pA}) \right| \phi_d \phi_{g.s.}^A \right\rangle - E_d \right) \chi_{dA}^{(+)} (R) = 0
\]

What is needed for transfer reactions in the N-A force averaged over \( V_{np} \)
Averaging procedure gives

\[
\left\langle \varphi_d \varphi^A_{g.s.} | V_{np} U_{NA} | \varphi_d \varphi^A_{g.s.} \right\rangle \approx \left\langle \varphi^A_{g.s.} \left| v_{NA} + v_{NA} \frac{Q}{E_{eff} + i0 - T_N - (H_A - E_A)} U_{NA} \right| \varphi^A_{g.s.} \right\rangle
\]

Two-body N-A potential for (d,p) reaction is equal to the non-local energy-dependent complex optical N-A potential taken at the fixed effective energy

\[
E_{eff} = \frac{1}{2} E_d + \frac{1}{2} \frac{\left\langle \varphi_d | V_{np} T_{np} | \varphi_d \right\rangle}{\left\langle \varphi_d | V_{np} | \varphi_d \right\rangle}
\]

n-p kinetic energy in deuteron equal approximately 57 MeV

Three-body problem for (d,p) reactions should be solved with energy-independent nonlocal nucleon potentials taken at effective energy equal to half the deuteron energy plus a shift.
Non-local energy-dependent dispersive optical potential for $N^{+40}$Ca

\[
\Sigma(\mathbf{r}, \mathbf{r}'; E) = U_{HF}^{vol1}(\tilde{\mathbf{r}}) H(x; \beta_{vol1}) + U_{HF}^{vol2}(\tilde{\mathbf{r}}) H(x; \beta_{vol2}) + U_{wb}^{\rho}(\tilde{\mathbf{r}}) H(x; \beta_{wb}) + U_{dy}^{sur+}(\tilde{\mathbf{r}}; E) H(x; \beta_{sur+}) + U_{dy}^{sur-}(\tilde{\mathbf{r}}; E) H(x; \beta_{sur-}) + U_{dy}^{vol+}(\tilde{\mathbf{r}}; E) H(x; \beta_{vol+}) + U_{dy}^{vol-}(\tilde{\mathbf{r}}; E) H(x; \beta_{vol-}) + U_{so}(r; E)
\]

\[
\tilde{\mathbf{r}} = |\mathbf{r} + \mathbf{r}'|/2 \text{ and } \mathbf{x} = \mathbf{r}' - \mathbf{r}
\]

\[
H(x; \beta) = \exp\left(-x^2/\beta^2\right)/(\pi^{3/2}/\beta^3)
\]

This potential simultaneously reproduces the data above and below the Fermi energy
Local-equivalent potential in the lowest order

\[ U_{\text{loc}}(r) = \sum_i U_{NA,i}(r) \exp \left[-\frac{\mu_N \beta_i^2}{2\hbar^2}(E - U_{\text{loc}}(r)) \right] \]

First-order corrections:

\[ \Delta U_N = \frac{\hbar^2}{\mu_N} \left[ \left(\frac{\nabla f}{f}\right)^2 - \frac{1}{2} \frac{\nabla^2 f}{f} \right] f(r) = \exp \left[\frac{\mu_N \beta_{\text{eff}}(r)}{4\hbar^2} U_{\text{loc}}\right] \]

Seth Waldecker
Deuteron channel wave function in adiabatic model with NLDOM:

\[
(T_R + U_C(R) - E_d)\chi(R) = -\sum_{N=n,p} \sum_i \int ds\,dx\, \phi_1(x + \alpha_1 s) U_{NA,i}(\frac{x}{2} - R) H_i(s) \phi_0(x) \chi(\frac{\alpha_2 s}{2} + R)
\]

\[
\phi_1(r) = \frac{V_{np}(r) \phi_0(r)}{\langle \phi_0 | V_{np} | \phi_0 \rangle}
\]

- lowest order potential obtained from:
  \[
  U_{NA,i}(\frac{x}{2} - R) \approx U_{NA,i}(R)
  \]

- first order corrections derived from:
  \[
  U_{NA,i}(r - \frac{x}{2}) \approx U_{NA,i}(R) - \frac{x}{2} \cdot \nabla U_{NA,i}(R)
  \]

when local-energy approximation is used:

\[
T(r) = E - U_{loc}(r)
\]
\[(T_R + U_C - E_d)\varphi = -(U_{loc}^0 + \Delta U)\varphi\]

\[\Delta U = \Delta U_0 + \Delta U_1\]

**\(\Delta U_0\)**
Correction to local-energy approximation in the lowest order

**\(\Delta U_1\)**
First order corrections
Overlap integral $\langle ^{40}\text{Ca}|^{41}\text{Ca} \rangle$

Spectroscopic factor:

NLDOM 0.73
LDOM 0.72

NLDOM overlap is very well approximated with WS potentials with $r_0=1.252$ fm and $a=0.718$ fm without any Perey factor.

The WS shown are standard obtained with $r_0=1.25$ fm and $a=0.65$ fm.
Lowest order + corrections for proton channel + corrections for deuteron channel + spin-orbit potential

Spectroscopic factor used:
NLDOM 0.73

\(^{40}\text{Ca}(d,p)^{41}\text{Ca}\)

\(E_d = 11.8\ \text{MeV}\)

Spectroscopic factor used:
NLDOM 0.73
Results with different optical potentials.

Spectroscopic factor used: NLDOM 0.73

$^{40}\text{Ca}(d,p)^{41}\text{Ca}$

$E = 11.8 \text{ MeV}$

The same optical potentials are used both in the entrance and exit channels.
Cross sections are too large $\Rightarrow$ something is missing.

What is missing?

Main assumptions of the (d,p) theory we used:

- The (d,p) amplitude contains projection to A+n+p channel. Projections onto all excited states are neglected.
- Only p-A and n-A potentials were used to calculate d+A potential. Multiple scattering effects (A+n+p force) are neglected.
- (d,p) amplitude contains $V_{np}$ only.
- Adiabatic approximation is used.
- Deuteron wave function contains only s-wave state.
- Hulthen wave deuteron function was used to generate d+A potential
Summary

• We can now calculate (d,p) cross section in the adiabatic model (which is a cheap way to include deuteron breakup) with non-local energy-independent and energy-dependent potentials.

• Cross sections in nonlocal model are determined by the n-p kinetic energy within the short range of their interaction where contribution from deuteron d-state is important. This gives rise to sensitivity of (d,p) cross sections to high n-p momenta

• Using NLDOM potential we can fix contribution from single-particle physics. But the NLDOM cross sections are very large, which indicates that some absorption (presumably from multiple scattering effect) is missing.