In-Medium Vector-Meson Spectral Functions
with the Functional Renormalization Group

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Based on:
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Outline

I) Introduction and Motivation

II) Theoretical Framework

▶ Functional Renormalization Group
▶ QCD effective model including vector mesons
▶ Flow equations for 2-point functions
▶ Analytic continuation

III) Results

▶ Phase diagram
▶ Vacuum vector-meson spectral functions
▶ In-medium vector-meson spectral functions

IV) Summary and Outlook
I) Introduction and Motivation

[motivation]

[courtesy L. Holicki]
Introduction: Exploring the Phase Structure of QCD

Exploration of phase structure of QCD-matter through heavy ion collisions

Useful probes: photons and dileptons, negligible interaction with hadronic medium
Dileptons and Vector Meson Spectral Functions

- Vector mesons can decay directly into lepton pairs
- Interesting: light vector mesons $\rho$, $\omega$ and $\phi$
- Lifetime $\tau_\rho \approx 1.3$ fm/c, smaller than lifetime of fireball ($\approx 10$ fm/c)

**Dilepton rate:**

$$ \frac{dN_{ll}}{d^4x d^4q} \sim \text{Im} \Pi_{\text{em}}^{\mu\nu}(M, q; \mu, T) $$

For low energy regime $M \leq 1$ GeV (VMD):

$$ \text{Im} \Pi_{\text{em}}^{\mu\nu} \sim \text{Im} D_\rho^{\mu\nu} + \frac{1}{9} \text{Im} D_\omega^{\mu\nu} + \frac{2}{9} \text{Im} D_\phi^{\mu\nu} $$

$\Rightarrow$ Aim: In-medium spectral functions of vector mesons and order parameter for chiral phase transition within the same framework
II) Theoretical Framework

[courtesy L. Holicki]
Flow equation for the effective average action $\Gamma_k$:

$$\partial_k \Gamma_k = \frac{1}{2} \text{STr} \left( \partial_k R_k \left[ \Gamma_k^{(2)} + R_k \right]^{-1} \right)$$


- $\Gamma_k$ interpolates between bare action $S$ at $k = \Lambda$ and effective action $\Gamma$ at $k = 0$
- Regulator $R_k$ acts as a mass term and suppresses fluctuations with momenta smaller than $k$
- The use of 3D regulators allows for a simple analytic continuation procedure
Modelling Vector Mesons

Sakurai (1960): vector mesons as gauge bosons of local gauge symmetry $SU(2)$

- Electromagnetic-hadronic interaction via exchange of vector mesons
- Current field identity (CFI):

$$j^\mu_{em} = \frac{m^2_\rho}{g_\rho} \rho^\mu + \frac{m^2_\omega}{g_\omega} \omega^\mu + \frac{m^2_\phi}{g_\phi} \phi^\mu$$

⇒ Vector Meson Dominance (VMD)

Lee and Nieh (1960s): Gauged linear sigma model, local gauge symmetry $SU(2)_L \times SU(2)_R$

⇒ $\rho$ meson and chiral partner $a_1$ meson as gauge bosons
Gauged Linear Sigma Model with Quarks

- Local gauge symmetry $SU(2)_L \times SU(2)_R$: Low-energy model of two-flavor QCD

- Ansatz for the effective average action $\Gamma_k \equiv \Gamma_k[\sigma, \pi, \rho, a_1, \psi, \bar{\psi}]$ (LPA):

$$\Gamma_k = \int d^4 x \left\{ \bar{\psi} \left( \partial - \mu \gamma_0 + h_S (\sigma + i \vec{\tau} \vec{\pi} \gamma_5) + i h_V (\gamma_\mu \vec{\tau} \vec{\rho}^\mu + \gamma_\mu \gamma_5 \vec{\tau} \vec{a}_1^\mu) \right) \psi + U_k (\phi^2) - c \sigma \\
+ \frac{1}{2} (\partial_\mu \Phi)^2 + \frac{1}{8} \text{Tr} (\partial_\mu V_\nu - \partial_\nu V_\mu)^2 + \frac{1}{4} m_{V,k}^2 \text{Tr} V_\mu V_\mu - ig V_\mu \Phi \partial_\mu \Phi \\
- \frac{1}{2} g^2 (V_\mu \Phi)^2 - \frac{i}{2} g \text{Tr} \partial_\mu V_\mu [V_\mu, V_\nu] - \frac{1}{4} g^2 \text{Tr} V_\mu V_\nu [V_\mu, V_\nu] \right\}$$

- Mesonic fields: $\phi \equiv (\vec{\pi}, \sigma)$ and $V_\mu \equiv \vec{\rho}_\mu \vec{T} + \vec{a}_{1,\mu} \vec{T}^5$

- Scale dependent quantities: $U_k, m_{k,V}^2$
Flow Equations for 2-Point Functions

\[ \partial_k \Gamma^{(2)}_{\rho,k} = -\rho \rho \pi \pi - \frac{1}{2} \rho \pi \pi - \frac{1}{2} \rho \psi \psi \]

\[ \partial_k \Gamma^{(2)}_{a_1,k} = a_1 \sigma \sigma a_1 + a_1 \pi \pi a_1 - \frac{1}{2} a_1 \pi \pi - \frac{1}{2} a_1 \sigma \sigma - 2 a_1 \psi \psi \]

- Neglect vector mesons inside the loops
- Vertices extracted from ansatz of the effective average action \( \Gamma_k \)
- Tadpole diagrams give \( \omega \)-independent contributions
Two-step Analytic Continuation Procedure

1) Use periodicity in external imaginary energy
   \[ ip_0 = i2n\pi T: \]
   \[ n_{B,F}(E + ip_0) \rightarrow n_{B,F}(E) \]

2) Substitute \( p_0 \) by continuous real frequency \( \omega \):
   \[ \Gamma^{(2),R}(\omega) = -\lim_{\epsilon \rightarrow 0} \Gamma^{(2),E}(ip_0 \rightarrow -\omega - i\epsilon) \]

Spectral function is then given by

\[ \rho(\omega) = -\text{Im}(1/\Gamma^{(2),R}(\omega))/\pi \]

III) Results

[courtesy L. Holicki]
Phase Diagram of the Model

- Chiral order parameter $\sigma_0$ decreases towards higher $T$ and $\mu$

- A crossover is observed at $T \approx 175$ MeV and $\mu = 0$

- Critical endpoint (CEP) at $\mu \approx 292$ MeV and $T \approx 10$ MeV

- We will study spectral functions along $\mu = 0$ and $T \approx 10$ MeV

Phase Diagram of the Model

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Flow of Euclidean Masses

- Starting at cutoff scale
  \( \Lambda = 1500 \text{ MeV} \) in chirally symmetric phase

- Chiral symmetry breaking sets in at \( k_{\chi_{SB}} \approx 700 \text{ MeV} \)

- Euclidean masses in the vacuum:

  \[
  m_\sigma = 557 \text{ MeV} \\
  m_\pi = 140 \text{ MeV} \\
  m_\rho = 1298 \text{ MeV} \\
  m_{a_1} = 1676 \text{ MeV} \\
  m_\psi = 300 \text{ MeV}
  \]

Euclidean (curvature) masses determine thresholds for the decay processes.

Euclidean masses in the vacuum:

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- \( m_\pi = 140\) MeV
- \( m_\rho = 1298\) MeV
- \( m_{a_1} = 1676\) MeV
- \( m_\psi = 300\) MeV

Mass degeneration of chiral partners for high \( T \) and \( \mu \)

Decay Channels of the $\rho$ Meson

$\rho^* \rightarrow \pi + \pi$

$\omega \geq \sqrt{(2m_\pi)^2 + \vec{p}^2}$

$\rho^* \rightarrow \psi + \bar{\psi}$

$\omega \geq \sqrt{(2m_\psi)^2 + \vec{p}^2}$

Decay channels of the $a_1$ Meson

\[ a_1^* \to \sigma + \pi \]
\[ a_1^* + \pi \to \sigma \]
\[ a_1^* + \sigma \to \pi \]
\[ a_1^* \to \psi + \bar{\psi} \]

\[ \omega \geq \sqrt{(m_\sigma + m_\pi)^2 + \vec{p}^2} \]
\[ \omega \leq (m_\sigma - m_\pi) \sqrt{1 + \frac{\vec{p}^2}{\Delta m^2}} \]
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Pole mass $\sim$ zero-crossing of the real part:

$$m_\rho^p = 789 \text{ MeV}, \quad m_{a_1}^p = 1275 \text{ MeV}$$

Decay channels accessible when imaginary part is non-zero

$\rho$ and $a_1$ meson unstable due to mesonic and quark-antiquark decay channels

Real and Imaginary Part of $\Gamma^{(2), R}$

Pole mass $\sim$ zero-crossing of the real part:

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Decay channels accessible when imaginary part is non-zero

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Vacuum Spectral Functions

\[ T = 10 \text{ MeV}, \mu = 0 \text{ MeV} \]

1: \( \rho^* \rightarrow \pi + \pi \)
2: \( \rho^* \rightarrow \bar{\psi} + \psi \)
3: \( a_1^* + \pi \rightarrow \sigma \)
4: \( a_1^* \rightarrow \pi + \sigma \)
5: \( a_1^* \rightarrow \bar{\psi} + \psi \)

$T$-dependence of Spectral Functions in Linear Scale

$T$-dependence of the Spectral Functions
Discussion and $T$-dependent Pole Masses

- Degeneration of $\rho$ and $a_1$ spectral functions in chirally symmetric phase
- Broadening of spectral functions with increasing $T$
- Pole masses do not vary much, no dropping $\rho$ mass

$\Rightarrow$ Consistent with broadening/melting-$\rho$-scenario

μ-dependence of the Spectral Functions
Space-like Processes of the $\rho$ Meson

$\rho^* + \pi \to \pi$

$0 \leq \omega \leq |\vec{p}|$

$\rho^* + \psi \to \psi$

$\rho^* + \bar{\psi} \to \bar{\psi}$

$0 \leq \omega \leq |\vec{p}|$

Space-like Processes of the $a_1$ meson

\[ a_1^* + \sigma \rightarrow \pi \]

\[ a_1^* + \pi \rightarrow \sigma \]

\[ a_1^* + \psi \rightarrow \psi \]

\[ a_1^* + \overline{\psi} \rightarrow \overline{\psi} \]

\[ 0 \leq \omega \leq |\vec{p}| \]

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Momentum-dependence of $\rho$ Spectral Function

- shown for $\mu = 0$ and $T = 100$ MeV

- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies

- space-like region ($\omega < \vec{p}$) is non-zero at finite $T$ due to space-like processes

Temperature-dependence of $\rho$ Spectral Function
Momentum-dependence of $a_1$ Spectral Function

- shown for $\mu = 0$ and $T = 100$ MeV

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Improved Truncation: LPA’ (Preliminary)

- Introduce wave function renormalization to include dressing of momentum dependence of meson propagators:

\[
\Gamma_{k}^{(\text{meson,kin})} = \int d^{4}x \left\{ \frac{1}{2} Z_{S,k} (\partial_{\mu} \Phi)^{2} + \frac{1}{8} Z_{V,k} \text{Tr} (\partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu})^{2} \right\}
\]

- Renormalize physical quantities, e.g. curvature masses:

\[
m_{\alpha}^{2} \quad \mapsto \quad \frac{m_{\alpha}^{2}}{Z_{\alpha,k}} \equiv M_{\alpha}^{2}
\]

- Curvature masses vs. pole masses ($\rho$ meson):

<table>
<thead>
<tr>
<th>Truncation</th>
<th>$m_{\rho}^{\text{curv}}$</th>
<th>$m_{\rho}^{\text{pole}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{S,k} = 1, Z_{V,k} = 1$</td>
<td>1298 MeV</td>
<td>789 MeV</td>
</tr>
<tr>
<td>$Z_{S,k} = 1, Z_{V,k}$</td>
<td>914 MeV</td>
<td>786 MeV</td>
</tr>
</tbody>
</table>
Summary and Outlook

- Analytic continued flow equations for vector meson two-point functions based on the gauged linear sigma model within the FRG

- Chiral order parameter and in-medium spectral functions obtained by the same theoretical framework

- Degeneration of $\rho$ and $a_1$ spectral functions indicating restoration of chiral symmetry, consistent with broadening/melting-$\rho$-scenario

Work in progress:

- Improve truncation (e.g. include wave function renormalization for all mesons)

- Improve phenomenology (e.g. include vector mesons inside the loops)
Outlook: Vector Meson Decay Channels

\[ \partial_k \Gamma^{(2)}_{\rho,k} = -\frac{1}{2} \rho \rho \pi \pi - \frac{1}{2} \rho \rho \rho \rho - \frac{1}{2} \rho \rho a_1 a_1 + \rho \pi \pi \rho - 2 \rho \psi \psi \rho \]

\[ \partial_k \Gamma^{(2)}_{a_1,k} = -\frac{1}{2} a_1 a_1 \pi \pi - \frac{1}{2} a_1 a_1 \sigma \sigma - \frac{1}{2} a_1 a_1 \rho \rho - \frac{1}{2} a_1 a_1 a_1 a_1 - 2 a_1 \psi \psi a_1 \]
Summary and Outlook

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Aim: Provide spectral functions for transport simulations to compute dilepton spectra!