Two-photon exchange in elastic electron-nucleon scattering

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However, this "electron microscopy" interpretation is not correct for relativistic quark constituents and due to quark and nucleon recoil.

When the target is static ($m_{\text{constituent}}, m_{\text{target}} \gg Q$), the 3-dim Fourier transform of the form factors gives the spatial distribution of electric charge and magnetization.

$\gamma$

$\gamma$

quarks
size and shape of non-relativistic many-body systems

Sizes of nuclei as revealed through elastic electron scattering

Shapes of deformed nuclei as revealed through inelastic electron scattering
**spin-1/2 electromagnetic form factors**

- Elastic e p -> e p scattering is our electron microscope to investigate nucleon structure.

- In 1-photon exchange approximation: nucleon structure parameterized by 2 form factors.

\[ A_{\lambda \lambda'}^\mu = \langle p + \frac{1}{2} q, \lambda' | J^\mu(0) | p - \frac{1}{2} q, \lambda \rangle = \bar{u}(p + \frac{1}{2} q, \lambda') \left[ F_1(Q^2) \gamma^\mu + F_2(Q^2) i \frac{\sigma^{\mu \nu} q_\nu}{2m} \right] u(p - \frac{1}{2} q, \lambda) \]

  **Dirac**  
  **Pauli**  
  \( F_1 \) helicity conserving, \( F_2 \) helicity flip FFs

- Alternatively, the Sachs form factors

\[ G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \quad \text{with } \tau = \frac{Q^2}{4 m^2} \]
measurement of nucleon Form Factors: Rosenbluth separation method

One-photon exchange elastic electron-nucleon cross section

\[ \sigma_R = G_M^2 + \frac{\varepsilon}{\tau} G_E^2 \]

SLAC:
Andivahis et al. (1994)

\[ \tau \equiv \frac{Q^2}{4M^2} \]
\[ \frac{1}{\varepsilon} \equiv 1 + 2(1 + \tau) \tan^2 \frac{\theta}{2} \]

\[ G_E^2 / \tau \]
polarization transfer method

\[ \vec{e} + p \rightarrow e + \vec{p} \]

Akhiezer, Rekalo (1974)

\[ d\sigma_{pol} = d\sigma_{unpol}(1 + h S_x P_t + h S_z P_l) \]

in one-photon exchange approximation:

\[ P_t = -\sqrt{\frac{2\varepsilon(1 - \varepsilon)}{\tau} \frac{G_E G_M}{\tau \sigma_R}} \]

\[ P_l = \sqrt{1 - \varepsilon^2} \frac{G_M^2}{\tau \sigma_R} \]

\[ \frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1 + \varepsilon)} \frac{G_E}{G_M}} \]
Rosenbluth vs polarization transfer measurements of $G_E/G_M$ of proton

Two methods, two different results!

Rosenbluth data
SLAC, Jlab (Hall A, C)

Polarization data
Jlab (Hall A, C)

GEpI Jones et al. (2000), Punjabi et al. (2005)
GEpII Gayou et al. (2002)
GEpIII Puckett et al. (2010)
Speculation: there are radiative corrections to Rosenbluth experiments that are important and are not included. Missing correction: few %, linear in $\varepsilon$, not strongly $Q^2$ dependent.

$G_E$ term is proportionally smaller at large $Q^2$. Effect more visible at large $Q^2$. 

$Q^2 = 4 \text{ GeV}^2$

$\left( \frac{G_M}{\mu_p G_D} \right)_{\text{exp}} = 1.055$

$\left( \frac{G_M}{\mu_p G_D} \right)_R = 1.03$

$\left( \frac{\mu_p G_E}{G_M} \right)_{\text{exp}} = 0.48$

$\left( \frac{\mu_p G_E}{G_M} \right)_R = 0.89$

Diff 2.4%

Diff 46%
comments on radiative corrections

\[ d\sigma = d\sigma_0 (1 + \delta) \]

\[ E_e = 4 \text{ GeV}, \quad Q^2 = 4 \text{ GeV}^2 \quad (\varepsilon = 0.60) \]

- **Electron and photon corrections**: known and well understood
  \[ \delta (e \text{ vertex+brems}) = -0.199 \]
  \[ \delta (e \text{ vac pol}) = 0.023 \]
  \[ \delta (\mu \text{ vac pol}) = 0.007 \]

- **Soft bremsstrahlung**: compositeness of nucleon only enters through on-shell FFs

- **Nucleon vertex and box diagrams**: involve hard and soft photons
  \[ \delta (Z) = -0.027 \]
  \[ \delta (Z^2) = -0.015 \]

- **Soft photons**: are included in radiative correction

- **IR div cancelled by electron-proton bremsstrahlung interference**: \[ \delta (\text{TOT}) = -0.211 \]
description of radiative tail: state-of-art

Bernauer et al. (2013)
status of 2γ-exchange corrections

- Tsai (1961), Mo & Tsai (1968)
  
  box diagram calculated using only nucleon intermediate state and using $q_1 \approx 0$ or $q_2 \approx 0$ in both numerator and denominator (calculate 3-point function)
  
  -> gives correct IR divergent terms

- Maximon & Tjon (2000)
  
  same as above, but make the above approximation only in numerator (calculate 4-point function) + use on-shell nucleon form factors in loop integral

- Blunden, Melnitchouk, Tjon (2003), Kondratyuk, Blunden (2007)
  
  further improvement by keeping the full numerator, insert higher resonances

  
  similar as previous from dispersive approach

- Bystritsky, Kuraev, Tomasi-Gustafsson (2006)
  
  assumption that dominant region comes from $q_1 \approx q_2 \approx q/2$ (obtained TPE is very small)
extracted $2\gamma$-correction on cross section

- soft photons in box: Maximon-Tjon
- Coulomb distortions: Feshbach correction:
  \[ \delta_F = Z \alpha \pi \sin(\theta/2) / (1 + \sin(\theta/2)) \]
- remainder TPE correction: simple parameterization
  \[ \delta_{\text{TPE}} = - (1 - \varepsilon) a \ln(b Q^2 + 1) \]

Graphs showing different corrections:
- solid: Bernauer fit to data
- dashed: Borisyuk, Kobushkin (2006)
- dotted: Arrington, Blunden, Melnitchouk (2011)
general formalism
2-photon exchange
**elastic eN scattering beyond 1γ-exchange**


Kinematical invariants:

\[ T^{\text{non-flip}}_{h', \lambda'_N, h \lambda_N} = \frac{e^2}{Q^2} \bar{u}(k', h') \gamma_\mu u(k, h) \]
\[ \times \bar{u}(p', \lambda'_N) \left( \tilde{G}_M \gamma_\mu - \tilde{F}_2 \frac{P^\mu}{M} + \tilde{F}_3 \gamma \cdot (K P^\mu) \right) u(p, \lambda_N) \]

\[ \tilde{G}_M(\nu, Q^2) = G_M(Q^2) + \delta \tilde{G}_M \]
\[ \tilde{F}_2(\nu, Q^2) = F_2(Q^2) + \delta \tilde{F}_2 \]
\[ \tilde{F}_3(\nu, Q^2) = 0 + \delta \tilde{F}_3 \]

Equivalently

\[ \tilde{G}_E = \tilde{G}_M - (1 + \tau) \tilde{F}_2 \]
\[ \tilde{G}_E(\nu, Q^2) = G_E(Q^2) + \delta \tilde{G}_E \]

\[ P \equiv \frac{p + p'}{2}, \quad K \equiv \frac{k + k'}{2} \]
\[ Q^2 = -(p - p')^2 \]
\[ \nu = K \cdot P = (s - u)/4 \]

For real part

\[ Y^M_{2\gamma}(\nu, Q^2) \equiv \Re \left( \frac{\delta \tilde{G}_M}{G_M} \right) \]
\[ Y^{E}(\nu, Q^2) \equiv \Re \left( \frac{\delta \tilde{G}_E}{G_M} \right) \]
\[ Y^{\gamma}(\nu, Q^2) \equiv \frac{\nu}{M^2} \Re \left( \frac{\tilde{F}_3}{G_M} \right) \]

3 independent observables
observables including $2\gamma$-exchange

\[
\frac{\sigma_R}{G_M^2} = 1 + \frac{\epsilon}{\tau} \frac{G_E^2}{G_M^2} \\
+ 2 Y_{2\gamma}^M + 2\epsilon \frac{G_E}{\tau G_M} Y_{2\gamma}^E + 2\epsilon \left(1 + \frac{G_E}{\tau G_M}\right) Y_{2\gamma}^3 \\
+ \mathcal{O}(\epsilon^4)
\]

\[
-\sqrt{\frac{\tau(1+\epsilon)}{2\epsilon}} \frac{P_t}{P_l} = \frac{G_E}{G_M} \\
+ Y_{2\gamma}^E - \frac{G_E}{G_M} Y_{2\gamma}^M + \left(1 - \frac{2\epsilon}{1+\epsilon G_M}\right) Y_{2\gamma}^3 \\
+ \mathcal{O}(\epsilon^4)
\]

\[
\frac{P_l}{P_l^{\text{Born}}} = 1 \\
-2\epsilon \left(1 + \frac{\epsilon}{\tau} \frac{G_E^2}{G_M^2}\right)^{-1} \left\{ \frac{\epsilon}{1+\epsilon} \left(1 - \frac{G_E^2}{\tau G_M^2}\right) + \frac{G_E}{\tau G_M} \left[Y_{2\gamma}^E - \frac{G_E}{G_M} Y_{2\gamma}^M\right] \right\} \\
+ \mathcal{O}(\epsilon^4)
\]
extraction of 2γ-amplitudes: data

Rosenbluth data: Jlab (Hall A)

Q^2=2.64 GeV^2

Polarization data: Jlab (Hall C)

Q^2=2.5 GeV^2

Qattan et al. (2005)

Meziane et al. (2011)
extraction of $2\gamma$-amplitudes: fit

$Q^2 = 2.64 \text{ GeV}^2$

Extracted $2\gamma$ amplitudes are in the (expected) 2-3% range

Guttmann, Kivel, Meziane, Vdh (2011)
e+/e- ratio: test of real part of 2γ-amplitude

**SLAC data points**

Prediction from phenomenological extraction

\[ R = \frac{\sigma_{e^+p}}{\sigma_{e^-p}} \]

Arrington (2004)

Guttmann, Kivel, Meziane, Vdh (2011)

New experiments underway/planned:
VEPP-3, CLAS, Olympus@DESY, MUSE
recent results for e+/e- ratio

black data points:
$Q^2 = 0.21 \text{ GeV}^2$

Jlab/CLAS

Moteabbed et al. (2013)

black data points:
$\varepsilon = 0.95$, $Q^2 = 0.23 \text{ GeV}^2$

$\varepsilon = 0.50$, $Q^2 = 1.43 \text{ GeV}^2$

VEPP-3

Gramolin et al. (2011)
Projected data for the Olympus@DESY experiment
normal spin asymmetries in elastic eN scattering

- directly proportional to the imaginary part of 2-photon amplitudes
- spin of beam OR target NORMAL to scattering plane
- order of magnitude estimates:
  - target: \( A_n \sim \alpha_{em} \sim 10^{-2} \)
  - beam: \( B_n \sim \alpha_{em} \cdot m_e \sim 10^{-6} - 10^{-5} \)
to $O(\alpha_{em})$

\[
A = \frac{\text{Im} \left( T_{f,i}^{*1\gamma} \text{Abs} T_{f,i}^{2\gamma} \right)}{|T_{f,i}^{1\gamma}|^2}
\]

De Rujula et al. (1971)

1\text{γ exchange}

\[
s = (k + p)^2
\]

function of elastic nucleon form factors

2\text{γ exchange}

absorptive part of double virtual Compton scattering

\[
\text{Abs} T^{2\gamma} = e^4 \int \frac{d^3k_1}{(2\pi)^3 2E_{k_1}} \bar{u}(k', h') \gamma_\mu (\gamma \cdot k_1 + m_e) \gamma_\nu u(k, h) \\
\times \frac{1}{Q_1^2 Q_2^2} W^{\mu\nu}(p', \lambda'; p, \lambda_N)
\]
# Beam Normal Spin Asymmetry: Experiments

<table>
<thead>
<tr>
<th>Expt.</th>
<th>$E$(GeV)</th>
<th>$Q^2$ GeV$^2$</th>
<th>$B_n$(ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAMPLE</td>
<td>0.192</td>
<td>0.10</td>
<td>-16.4±5.9</td>
</tr>
<tr>
<td>A4</td>
<td>0.570</td>
<td>0.11</td>
<td>-8.59±0.89</td>
</tr>
<tr>
<td>A4</td>
<td>0.855</td>
<td>0.23</td>
<td>-8.52±2.31</td>
</tr>
<tr>
<td>HAPPEX</td>
<td>3.0</td>
<td>0.11</td>
<td>-6.7 ± 1.5</td>
</tr>
<tr>
<td>G0</td>
<td>3.0</td>
<td>0.15</td>
<td>-4.06 ± 1.62</td>
</tr>
<tr>
<td>G0</td>
<td>3.0</td>
<td>0.25</td>
<td>-4.82 ± 2.85</td>
</tr>
<tr>
<td>E-158(ep)</td>
<td>46.0</td>
<td>0.06</td>
<td>-3.5 -&gt; -2.5</td>
</tr>
</tbody>
</table>
beam normal spin asymmetry: theory

\[ E_e = 0.300 \text{ GeV} \]
\[ \Theta_e = 145 \text{ deg} \]

\[ E_e = 0.570 \text{ GeV} \]
\[ \Theta_e = 35 \text{ deg} \]

\[ E_e = 0.855 \text{ GeV} \]
\[ \Theta_e = 35 \text{ deg} \]

data: MAMI A4

target normal spin asymmetry

\[ A_n = \sqrt{\frac{2 \varepsilon (1 + \varepsilon)}{\tau}} \left( G_M^2 + \frac{\varepsilon}{\tau} G_E^2 \right)^{-1} \times \left\{ -G_M \operatorname{Im} \left( \delta \tilde{G}_E + \frac{\nu}{M^2} \tilde{F}_3 \right) + G_E \operatorname{Im} \left( \delta \tilde{G}_M + \left( \frac{2\varepsilon}{1 + \varepsilon} \right) \frac{\nu}{M^2} \tilde{F}_3 \right) \right\} \]

neutron

Neutron SSA [%]

Q^2 [GeV^2]

preliminary

JLAB Hall A data

Averett et al. (2013)
model calculations of $2\gamma$-exchange corrections
hadronic calculation of $2\gamma$-contribution

Blunden, Tjon, Melnitchouk (2003, 2005)
partonic calculation of $2\gamma$-contribution: handbag model

\[ H_{h,\lambda} = \frac{(e e_q)^2}{Q^2} \bar{u}(k', h) \gamma_\mu u(k, h) \cdot \bar{u}(p'_q, \lambda) \left( \tilde{f}_1 \gamma^\mu + \tilde{f}_3 \gamma \cdot K P_q^\mu \right) u(p_q, \lambda) \]

calculation for $e_\mu \rightarrow e_\mu$ was verified explicitly \textit{van Nieuwenhuizen (1971)}

to reproduce the IR divergent contribution at nucleon correctly, i.e. to satisfy the Low Energy Theorem \rightarrow need diagrams with two active quarks
2γ-calculation in **handbag model**

\[
\begin{align*}
\delta \tilde{G}_M^{\text{hard}} &= C \\
\delta \tilde{G}_E^{\text{hard}} &= -\left(\frac{1 + \varepsilon}{2 \varepsilon}\right) (A - C) + \sqrt{\frac{1 + \varepsilon}{2 \varepsilon}} \cdot B \\
\tilde{F}_3 &= \frac{M^2}{\nu} \left(\frac{1 + \varepsilon}{2 \varepsilon}\right) (A - C)
\end{align*}
\]

**GPD integrals**

\[
\begin{align*}
A &\equiv \int_{-1}^{1} dx \frac{1}{x} \left[ (\hat{s} - \hat{u}) \tilde{f}_1^{\text{hard}} - \hat{s} \hat{u} \tilde{f}_3 \right] \sum_q e_q^2 \left( H^q + E^q \right) \\
B &\equiv \int_{-1}^{1} dx \frac{1}{x} \left[ (\hat{s} - \hat{u}) \tilde{f}_1^{\text{hard}} - \hat{s} \hat{u} \tilde{f}_3 \right] \sum_q e_q^2 \left( H^q - \tau E^q \right) \\
C &\equiv \int_{-1}^{1} dx \frac{1}{x} \tilde{f}_1^{\text{hard}} \text{sgn}(x) \sum_q e_q^2 \tilde{H}^q
\end{align*}
\]

“magnetic” GPD  
“electric” GPD  
“axial” GPD
2γ-calculation in handbag model

2γ-exchange at large $Q^2$: QCD factorization approach

Kivel, Vdh (2009)
Borisyuk, Kobushkin (2009)

dominant region: both photons are highly virtual

$$q_1^2 \sim q_2^2 \sim q^2 = (p' - p)^2 \equiv -Q^2$$

$$\varphi_N(x_i) \ast T_H(x, x', \varepsilon, Q^2) \ast \varphi_N(x_i')$$

helicity conserving amplitudes

1-γ exchange

$$G_M \sim \alpha^2_S/Q^4$$

2-γ exchange (2 amplitudes)

$$\delta\tilde{G}_M \sim \tau\tilde{F}_3 \sim \alpha_{em}\alpha_S/Q^4$$

- all integrals are IR-finite with asymptotic DA
proton distribution amplitude

\[ 4 \left\langle 0 \left| \varepsilon^{ijk} u^i_\alpha (a_1 \lambda n) u^j_\beta (a_2 \lambda n) d^k_\sigma (a_3 \lambda n) \right| p \right\rangle \]

\[ = V \ p^+ \left[ \left( \frac{1}{2} \bar{n} \cdot \gamma \right) C \right]_{\alpha \beta} [\gamma_5 N^+]_\sigma \]

\[ + A \ p^+ \left[ \left( \frac{1}{2} \bar{n} \cdot \gamma \right) \gamma_5 C \right]_{\alpha \beta} [N^+]_\sigma \]

\[ + T \ p^+ \left[ \frac{1}{2} i \sigma_{\perp \bar{n}} C \right]_{\alpha \beta} [\gamma \gamma_5 N^+]_\sigma \]

Braun et al. (2000)

lattice nucleon DA \( \mu^2 = 4 \text{ GeV}^2 \)

at leading twist:

\[ V(x_i) \approx 120 x_1 x_2 x_3 f_N \left[ 1 + r_+(1 - 3x_3) \right] \]

\[ A(x_i) \approx 120 x_1 x_2 x_3 f_N \ r_-(x_2 - x_1) \]

\[ T(x_i) \approx 120 x_1 x_2 x_3 f_N \left[ 1 + \frac{1}{2} (r_- - r_+) (1 - 3x_3) \right] \]

Chernyak, Ogloblin, Zhitnitsky (1988)

Braun, Lenz, Wittmann (2006)

Lattice QCDSF (2008)
hard spectator scattering: results

Rosenbluth plots

correction on $P_t / P_l$ small!

Data set SLAC NE11, 1994

$\sigma_R / (\mu_p G_D)^2$

$G_D = 1 / (1 + Q^2 / 0.71)^2$

blue: 1-photon results

Kivel, Vdh (2009)
2γ-exchange at large $Q^2$: soft spectator scattering (SCET)

Handbag model: one active quark
relatively successful at intermediate/large $Q^2$

However:
- not a systematic approach as QCD factorization
- Separation soft/hard contributions cannot be done systematically
- theoretical problems with GPD formulation at large $-t$

Description of soft spectator scattering involves 3 different scales associated with the virtualities of the scattering particles:

- **Hard**: $q_h^2 \sim Q^2$ (hard subprocess)
- **Hard-collinear**: $q_{hc}^2 \sim Q\Lambda$ (hard-collinear subprocess)
- **Soft**: $q_s^2 \sim \Lambda^2$ (soft nonperturbative content)

$Q^2 = 9 - 25\text{GeV}^2$
$\Lambda \approx 0.3\text{GeV}$
$Q\Lambda \approx 0.9 - 1.5\text{GeV}^2$
$q_s^2 \gg Q\Lambda \approx m_N^2$
**2γ-exchange: soft spectator approach**

Kivel, Vdh (2013)

- **soft spectator contribution**
- **hard spectator contribution**

**QCD fact**

**WACS:** \[
\frac{d\sigma}{dt} \approx \frac{d\sigma^{KN}}{dt} |F_1|^2
\]

- Data: JLab (2005)
- \(\mathcal{F}_1\)
- \(\frac{d\sigma}{dt}\)
- \(\frac{d\sigma^{KN}}{dt}\)
- \(|F_1|^2\)

![Graphs and Plots]

- Data: JLab (2005)
- Data: JLab (2011)
- \(P_t/P_t^B\)
- \(\mathcal{E}\)}
$2\gamma$-exchange at zero/low $Q^2$
**size of proton:**

**electric charge radius**

**Corrections to Lamb shift:**

300 $\mu$eV below expectation

**Proton structure corrections:**

$\Delta E = -35.1$ $\mu$eV

Pachucki (1999)

$\Delta E = -34.5$ $\mu$eV

Martynenko (2006)

$\Delta E = (-33 \pm 2)$ $\mu$eV


**$\mu$H data:**

R$_E$ = 0.8409 ± 0.0004 fm

Pohl et al. (2010)

Antognini et al. (2013)

7$\sigma$ difference

**ep-data:**

R$_E$ = 0.8772 ± 0.0046 fm

Bernauer et al. (2010)

Zhan et al. (2011)
Proton e.m. form factors

**MAMI cross section data in range** \( Q^2 = 0.003 - 1 \text{ GeV}^2 \)

Bernauer et al. (2010, 2013)

\[ R_E = 0.879 \pm 0.005_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.002_{\text{model}} \pm 0.004_{\text{group}} \text{ fm} \]

**\( R_E \):** good agreement between Bernauer et al. and Zhan et al.

\[ R_M = 0.777 \pm 0.013_{\text{stat}} \pm 0.009_{\text{syst}} \pm 0.005_{\text{model}} \pm 0.002_{\text{group}} \text{ fm} \]

**\( R_M \):** disagreement between both!

MAMI \( G_M \) data ~ 1.5 % larger than global fit of Zhan et al. \( R_M = 0.867 \pm 0.020 \text{ fm} \)

**BUT:** Bernauer et al. central value is dependent on applied TPE corrections
\[ R_M = 0.803 \text{ fm} \text{ (using Borisyuk, Kobushkin TPE corrections)} \]
\[ R_M = 0.799 \text{ fm} \text{ (using Arrington et al. TPE corrections)} \]

**ONGOING:** TPE re-evaluation in dispersive approach at low \( Q^2 \)

Tomalak, Vdh
electron scattering has reached level of precision where effects of two-photon exchange (beyond the soft limit) are clearly visible

- cross section (Rosenbluth separation) vs Pt/Pt
- epsilon dependence of Pt
- beam and target normal spin asymmetries
- $e^+ / e^-$ cross section ratios

challenge for theory: as both soft and hard scales are involved

progress in theory: hadronic, handbag, hard and soft rescattering (SCET)...

crucial input for 2γ hadronic corrections to Lamb shift, HFS,...
also for improving precision on radius extractions from present and upcoming electron scattering experiments

New experiments being analyzed: CLAS@JLAB, Olympus@DESY,...
or on horizon: $R_E$ using ISR (MAMI), $R_E$ in Hall B (Jlab), $\mu p$ MUSE (PSI),...