Real time correlation function from analytic continuation

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In collaboration with N. Strodthoff, J. M. Pawlowski

Based on
Formalism
Pawlowski, Strodthoff
Phys.Rev. D92 (2015) no.9, 094009

Finite temperature
Pawlowski, Strodthoff, NW
arxiv:1711.07444
Why real time correlation functions?

Motivation

- Resonances
- Transport peak
- Decay thresholds

Spectral function

Frequency
Why real time correlation functions?

Spectral function

Light meson spectrum

Resonances + Thresholds

Motivation

Spectral function

R. Williams, C. S. Fischer, and W. Heupel
R. Williams, C. S. Fischer, and W. Heupel
Why real time correlation functions?

Spectral function

Transport peak (+ Resonances)

Transport coefficients

PRL, 115 (2015) no.11, 112002
Christiansen, Haas, Pawlowski, Strodthoff
**QCD from the FRG**

Flow equation for QCD

\[ \frac{d\Gamma_k}{dt} = \frac{1}{2} \]

- **Gluon**
- **Ghost**
- **Quark**
- **Mesons**

**Functional Renormalization Group**

\[ \Gamma_{k=\Lambda} = S \]

- Non-perturbative first principle method
- Access to physical mechanisms
- No sign problem
  - Chemical potential
  - Real time

Bound states efficiently taken into account via Dynamical Hadronization

Collaborative effort **fQCD collaboration:**

QCD from the FRG

Aiming at apparent convergence

Numerics heavy $\Rightarrow$ requirements

- Automatable workflow
- Suitable (Lorentz invariant) regulator required

Cyrol, Mitter, Pawlowski, Strodthoff, arxiv:1708.03482
**Motivation**

QCD from the FRG

Gluon dressing $p^2 G_{AA}(p)$

Quark dressings $1/Z(q)$ and $M(q)$

Cyrol, Mitter, Pawlowski, Strodthoff, arxiv:1708.03482
Towards real time QCD from first principles
Use analyticity constrains and KMS condition to obtain real time correlation functions from Matsubara formalism.
Replace sum by contour integral:

$$T \sum_n f(2\pi n T) = -\frac{1}{2} \oint_C dz \ f(z)[1 + 2n_B(iz)]$$

Illustrative example

Bosonic occupation number

$$\sum_T \frac{1}{(q_0 + p_0)^2 + (\epsilon_{q+p}^1)^2} \frac{1}{(q_0)^2 + (\epsilon_{q}^2)^2}$$

Continuation procedure

Nicolas Wink (ITP Heidelberg)  Phase diagram of strongly interacting matter (Trento 2017)
$\frac{1}{i} \sum_{\pm} \left( \text{Res}_1^{\pm} \cdot [1 + 2n_B(-ip_0 + \epsilon_{q+p}^1)] + \text{Res}_2^{\pm} \cdot [1 + 2n_B(\epsilon_q^2)] \right)$

$p_0 = 2\pi m T \quad m \in \mathbb{Z}$

Identify ambiguity of the analytic continuation

$n_B(ip_0) = 1$

Mathematically rigorous


Analyticity off the imaginary axis

Correct decay behaviour at infinity

Unique physical analytic continuation identified by setting $n_B(ip_0) = 1$ everywhere
Remarks

- Numerically accessible
- Corresponds to a contour deformation at vanishing temperature
- Considering poles is sufficient
- Branch cuts can be mapped to poles via spectral/integral representations

\[ G(p_0, \vec{p}) = \int_{\eta > 0} \frac{2\eta \rho(\eta, \vec{p})}{p_0^2 + \eta^2} \]

Strodthoff, PRD 95 (2017) no.7, 076002
Pawlowski, Strodthoff, NW, arxiv:1711.07444

Jung, Pawlowski, von Smekal, NW, work in progress
Generalisation to the FRG

Regulator poles

\[ R_k(q^2) \]

No new conceptual problems

Lorentz invariant

No changes


Additional poles

Foerchinger, JHEP 1205 (2012) 021

\[ \frac{1}{q^2 + m^2} \]
\[ \frac{1}{q^2 + m^2 + R_k(q^2)} \]
\[ \frac{1}{q^2 + m^2 + R_k(q^2 + m_r)} \]
Application to the O(N)-Model

Effective description of the lightest mesons

Calculate spectral functions of the O(N) model

\[ \rho(\omega, \vec{p}) = -2 \, \text{Im} \, G_R(\omega, \vec{p}) \]

Truncation:

\[ \Gamma_k = \sum_{T, q} \Delta \Gamma^{(2)}_{\sigma} + \Delta \Gamma^{(2)}_{\pi} + V(\sigma) \]

Vacuum: \[ \Delta \Gamma_{\pi}^{(2)} = \Gamma_{\pi}^{(2)}(q^2) - \Gamma_{\pi}^{(2)}(0) \]

Finite Temperature: \[ \Delta \Gamma_{\pi}^{(2)} = Z_{\pi} q^2 \]

Results O(N)-model
Application to the O(N)-Model

![Graph showing phase diagram of strongly interacting matter with different curves for Pion mass, Sigma mass, Order parameter, Curvature Mass, Pole Mass (Spectral function), and Pole Mass (Padé).](image)

Pawlowski, Strodthoff, NW, arxiv:1711.07444
Application to the O(N)-Model

Temperature evolution of the spectral function

Pawlowski, Strodthoff, NW, arxiv:1711.07444
Application to the O(N)-Model

Temperature evolution of the spectral function

![Graph showing temperature evolution of the spectral function](attachment:image.png)

Spectral Function [1/MeV²]

Temperature $T = 0$ MeV
- Pion
- Sigma

Frequency $\omega$ [MeV]

Pawlowski, Strodthoff, NW, arxiv:1711.07444
Application to the $O(N)$-Model

Imaginary part of the retarded two-point function

Pion

Results $O(N)$-model

Pawlowski, Strodthoff, NW, arxiv:1711.07444
Application to the O(N)-Model

Sigma meson

Imaginary part of the retarded two-point function

Pawlowski, Strodthoff, NW, arxiv:1711.07444
Application to the O(N)-Model

Finite temperature spectral function for various external momenta

Pion meson

Results O(N)-model

Pawlowski, Strodthoff, NW, arxiv:1711.07444
Application to the O(N)-Model

Finite temperature spectral function for various external momenta

Pion meson

\[ P(\omega, \vec{p}) \]

\[ \text{Pion} \quad T = 55 \text{ MeV} \quad \varepsilon = 0.55 \text{ MeV} \]

\[ |\vec{p}| = 46 \text{ MeV} \]

\[ |\vec{p}| = 414 \text{ MeV} \]

\[ |\vec{p}| = 644 \text{ MeV} \]

\[ |\vec{p}| = 276 \text{ MeV} \]

Pawlowski, Strodthoff, NW, arxiv:1711.07444
Application to the O(N)-Model

Finite temperature spectral function for various external momenta

![Graph](image)

Sigma meson

Parameters:
- $T = 193 \text{ MeV}$
- $\varepsilon = 0.74 \text{ MeV}$

Pawlowski, Strodthoff, NW, arxiv:1711.07444
Application to the O(N)-Model

Sigma meson

Finite temperature spectral function for various external momenta

Pawlowski, Strodthoff, NW, arxiv:1711.07444
Application to the O(N)-Model

In medium breaking of Lorentz invariance

$$\lim_{\vec{p} \to 0} \lim_{p_0 \to 0} \Gamma^{(2)}(p_0, \vec{p}) \neq \lim_{p_0 \to 0} \lim_{\vec{p} \to 0} \Gamma^{(2)}(p_0, \vec{p})$$

Sigma meson

$$T = 138 \text{ MeV}$$

Pawlowski, Strodthoff, NW, arxiv:1711.07444
Summary & Outlook

• Conceptually easy and numerically accessible algorithm

• Benchmark for finite temperature spectral function in the O(N)-model

• Bound states

• Transport coefficients

• Yang-Mills/QCD in the near future!
Application to the O(N)-Model

Vacuum spectral function

![Graph showing vacuum spectral function with different approximations: LPA, LPA', LPA' + Y, and Full. Peaks at ω = 150 MeV for π and ω = 250 MeV for σ.]
Retarded Greens function

\[
\lim_{\epsilon \to 0} G(-i(\omega + i\epsilon))
\]

Take limit analytically

Numerical extrapolation
Application to the O(N)-Model

Results O(N)-model

Pawlowski, Strodthoff, NW, arxiv:1711.07444

Nicolas Wink (ITP Heidelberg)
Phase diagram of strongly interacting matter (Trento 2017)
Application to the O(N)-Model

\[ G_{\text{Euclidean}}(t,0) \left[ \text{1/MeV}^2 \right] \]

- Full Range
- Pole & unitarity cut
- Landau cut

Fit parameters:
- \( m_{\text{pole}} = 152 \text{ MeV} \)
- \( m_{\text{cut}} = 92 \text{ MeV} \)
- \( B_{\text{cut}} = 137 \text{ MeV}^2 \)

Pawlowski, Strodthoff, NW, arxiv:1711.07444
Application to the O(N)-Model

![Graph showing spectral function vs frequency for different cases at T = 55 MeV and ε = 0.46 MeV. The graphs compare Pion and Sigma with 4D and 3D regulators.](attachment:image.png)
Application to the O(N)-Model

![Graph showing wave function renormalization vs. temperature.](image)

Pawlowski, Strodthoff, NW, arxiv:1711.07444